

The disadvantage of using a marker is the No. of channels to be multiplexed is reduced by one, $(N-1)$ channel are multiplexed.

Exll 24 Voice signal are sampled uniformly and time multiplexed. The sampling operation uses samples with width 1 uses.

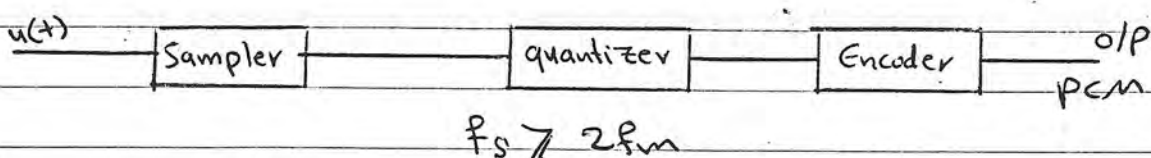
The multiplexing operation provides synch. by adding extra pulse duration of 1 μ sec. If $f_s = 8 \text{ KHz}$, calculate spacing between successive pulses?

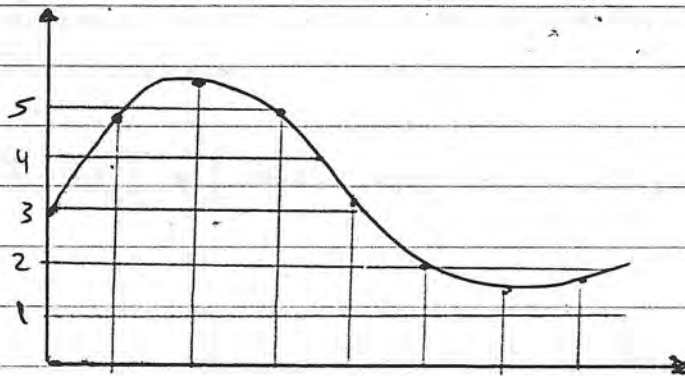
Solⁿ $T_s = \frac{1}{f_s} = \frac{1}{8000} = 125 \mu\text{sec}$

The pulse are separated by $\frac{125}{25} = 5 \mu\text{sec}$

Pulse Code Modulation (PCM):

Represent analog data to digital signal (two quantity - , +)





- any Sample which falls between two levels, it will be presented by the amplitude of Quantized level. This is some kind of approximation (close to the two values).
- If the sample is quantized to level (5), the encoder will represent 5 in binary. The encoder converts the quantized input to (n) bits.
- The quantization process results in a kind of error called "quantization error".

Transmission Bandwidth in PCM:-

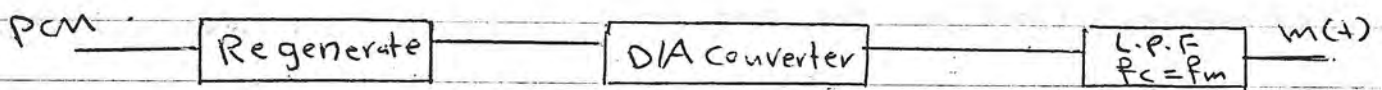
let the quantizer use (n) number of binary digits to represent each level. The No. of levels that can be represented by "n" digits will be $q = 2^n$

q :- Total number of digital level quantizer.
 n :- no. of bits.

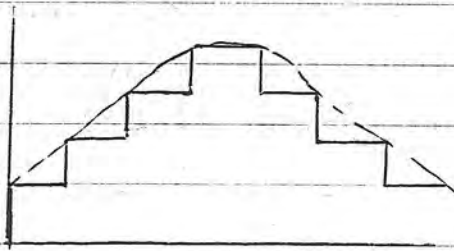
$$\begin{aligned} \text{No. of bit per Second} &= (\text{No. of bit per sample}) * (\text{No. of Sample/sec}) \\ &= n * f_s \end{aligned}$$

$$\therefore \text{Signaling rate in PCM (r)} = n f_s$$

$$B_T \geq \frac{1}{2} r$$



« PCM receiver »



« Reconstructed wave »

It is impossible to reconstruct the exact signal $m(t)$, because of the "quantization error"

This can be reduced by increasing the quantization levels bits per sample, but this will increase the signaling rate as well as B_T . The Noise introduced by the quantization should be in a tolerable limit.

Quantization 30

When quantizing a signal $x(t)$, a new signal is created x_q . If a signal is limited to a higher and lower value V_H, V_L respectively, then we divide this range into (M) equal intervals each of size " S " called step size

$$S = \frac{V_H - V_L}{M}$$

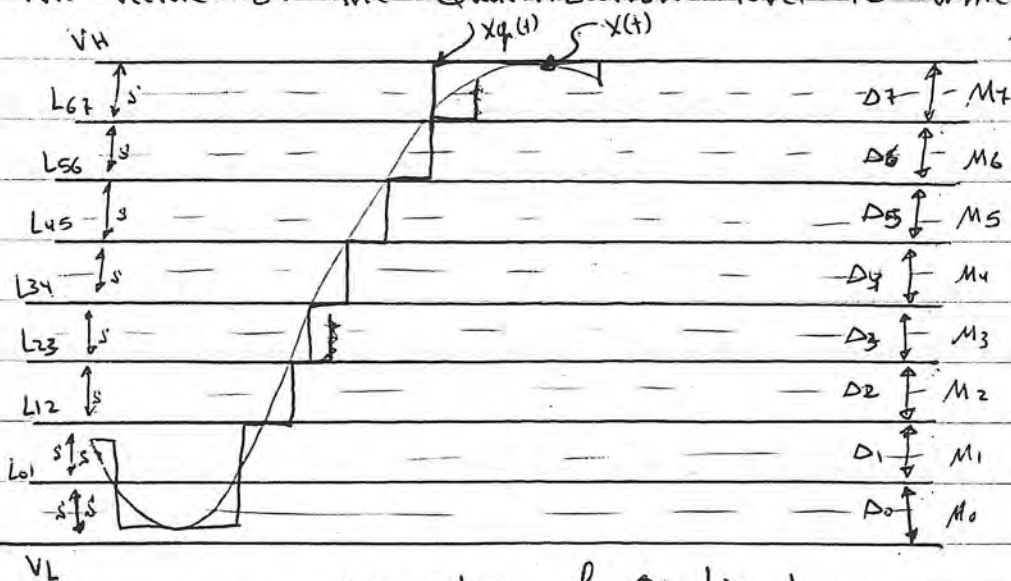
where M = is the No. of intervals



Taking $M=8$, There will 8 quantization levels, the quantized signal is as following:

When $x(t)$ is in the range Δ_0 , the signal $x_q(t)$ maintains constant level M_0 and soon

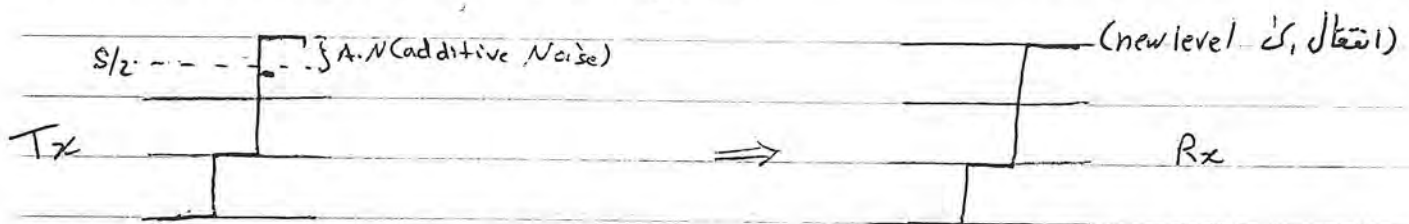
but, the transition in $x_q(t)$ from $x_q = m_0$ to $x_q = m_1$ is made abruptly, $x(t)$ passes the transition L_0 , which is midway between M_0 and m_1 . This means that "at every instant of time, $x_q(t)$ has the value of the Quantization level to which $x(t)$ is closest".



operation of Quantization

The Quantization error $[x(t) - x_q(t)]$ must have a magnitude less than or equal to $S/2$.

As long as the noise has an instantaneous amplitude less than $S/2$, the noise will not appear at the output. The figure below shows the error which results in the signal.



There are two kinds of noise, Additive noise and Quantization noise. The additive noise is decreased by putting repeaters (Quantization + amplifier) in decreased spacing. This will decrease the noise power and decrease the probability of error.

Increasing the step size will also decrease the error, but the difference $x - x_q$ and results in increasing "Quantization noise".

② Quantization noise?

Because of Quantization, inherent noise are introduced in the signal

$$\sum_{\text{error}} = x_q(nT_s) - x(nT_s)$$

Let an input $x(nT_s)$ amplitude is in the range $-x_{\max}$ to $+x_{\max}$. The total input samples are mapped into "q" levels.

$$\text{Total amplitude} = x_{\max} - (-x_{\max})$$

$$= 2x_{\max}$$

if the amplitude is divided into "q" level of Quantization, then the step "S"

$$S = \frac{x_{max} - (-x_{max})}{q}$$

$$= \frac{2x_{max}}{q}$$

if the signal is normalized to minimum and Maximum equal value S

$$x_{max} = 1$$

$$-x_{max} = -1$$

$$S = \frac{2}{q} \text{ for normalized signals}$$

For uniform Quantizer the maximum error is:-

$$\sum_{max} \leq \left| \frac{S}{2} \right|$$



Noise power:

$$\text{Noise power} = \frac{V_{noise}^2}{R}$$

where V_{noise}^2 is the mean square value of noise voltage.

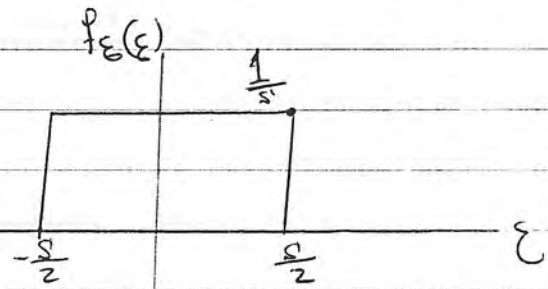
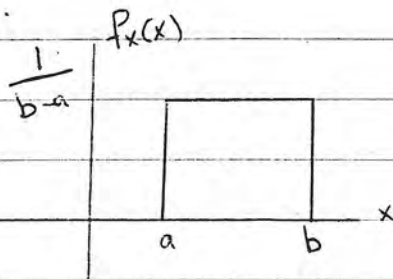
Since noise is defined by random Variable " ξ " and PDF $f_{\xi}(\xi)$, its mean square value is:-

$$\text{mean square value} = E\{\xi^2\} = \overline{\xi^2}$$

The mean square value of random variable "x" is given as:-

$$\overline{x^2} = E[x^2] = \int_{-\infty}^{\infty} x^2 \underbrace{f_x(x)}_{\text{PDF}} dx$$

POF



* So the above equation becomes

$$E[\epsilon^2] = \int_{-\frac{S}{2}}^{\frac{S}{2}} \epsilon^2 \cdot \frac{1}{S} \cdot d\epsilon$$

$$\text{Quantization noise power} = \frac{S^2}{12} \text{ for } R=1$$

Maximum signal to Quantization noise for linear Quantization:-

$$\frac{S'}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

$$= \frac{\text{Normalized signal power}}{S^2/12}$$

The Number of bit "n" and Quantization levels 'q' are related:

$$q = 2^n$$

$$S = \frac{2 \max}{2^n} \quad \text{Sub in } S/N$$

$$= \frac{\text{Normalized signal power}}{\left(\frac{2x_{\max}}{2^n}\right)^2 / 12}$$

$$S/N = \frac{3\bar{p}^2}{x_{\max}^2} 2^{2n}$$

if we assume the input $x(t)$ is Normalized
 $x_{\max} = 1$

$$S/N = 3 * 2^{2n} \bar{p}^2$$

if the destination signal "p" is Normalized
 $p \leq 1$

$$S/N \leq 3 * 2^{2n}$$

$$S/N \text{ db} = 4.8 + 6n$$

Expressing SN in decibels

$$\frac{S}{N} \text{ dB} = 10 \log [3 + 2^{2n}]$$

Ex11 A television signal with bandwidth of 4.2 MHz is transmitted using binary PCM, if the No. of Quantization levels is 512 calculated:

- ① The length of o/p code word
- ② BT
- ③ bitrate (signaling rate)
- ④ o/p signal to quantization noise ratio.

$$\text{Sol}^{\circ} \quad W = 4.2 \text{ MHz}$$

$$q = 512$$

$$① \quad q_n = 2^n \Rightarrow 512 = 2^n$$

$$\log 512 = \log 2^n$$

$$\log 512 = n \log 2$$

$$n = \frac{\log 512}{\log 2} = 9 \text{ bit (length of code word)}$$

$$② \quad BT \geq nW$$

$$BT \geq 9 \times 4.2 \times 10^6 \text{ Hz}$$

$$BT \geq 37.8 \text{ MHz}$$

$$\text{bandwidth} = f_{\max}$$

$$= \text{frequency ds1}$$

$$③ \quad r = n f_s$$

$$f_s \geq 2 f_m$$

$$= 2 n W$$

$$= 2 \times 9 \times 4.2 \times 10^6$$

$$= 75.6 \times 10^6 \text{ bit/sec}$$

$$④ \quad S/N \text{ dB} \leq [4.8 + 6n]$$

$$S/N \leq [4.8 + 6 \times 9]$$

$$S/N \leq 58.8 \text{ dB}$$

Ex|| The bandwidth of a signal input to the PCM is restricted to 4 KHz. The input variation is between $-3.8V$ to $+3.8V$ the average power of input is 3mW. A SN is required to be 20dB. Assuming uniform Quantization and binary output.

- ① No. of bits per Sample
- ② output of 30 such pcm coders are time Multiplexed. what is the minimum required transmission bandwidth for the Multiplexed signal?