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University of Karbala, College of Education for Pure science, Department of Mathematics, Fourth Year, Dynamical Systems 14/11/2018 (part1) morning study.

Q1) Define each of the following

a) Orbit of x_0 under f

b) Super attracting fixed point of a function f

c) Non-hyperbolic fixed point

d) Eventually fixed point

e) Cobweb diagram

Q2) Find the fixed points (if exists), and find $f^{\circ 2}$ for the function $f(x) = |x-3|$, and sketch the graph of $f^{\circ 2}$.

Q3) Find the fixed points (if exists) and all 2-cycle (Period-2) points for the function $f(x) = -x$, and sketch the graph of $f^{\circ 2}$.

Q4) Determine whether the logistic family has an attracting fixed point for $\mu \in (0\,,3)$. Explain your answer.

Good luck

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14/11/2018 First Exam ① Dynamical Systems Fourth Y.

Q) Definitions

Orbit of x_0 $O(x_0) = \{x_0, f(x_0), f^{o2}(x_0), \dots, f^{on}(x_0), \dots\}$

Super attracting fixed point $f'(p) = 0$ (where $f(p) = p$)

Non-hyperbolic fixed point $|f'(p)| = 1$

Eventually fixed point Not fixed point but $\exists$ a fixed Point in it's orbit.

Cobweb diagram is a diagram that start with x_0 then move vertically to intersect the graph of f and then move horizontally to intersect the line $y = x$. and so on.

Q) Find the fixed pts, and f^{o2} for $f(x) = |x-3|$

Sol.

$$f(x) = |x-3| = \begin{cases} x-3 & ; x \geq 3 \\ 3-x & ; x < 3 \end{cases}$$

$x - 3 = x$ $\cancel{x}$. $3 - x = x \Rightarrow x = \frac{3}{2}$ fixed point

(2 m)

$$f^{o2}(x) = f(f(x)) = \begin{cases} f(x-3) & ; x \geq 3 \\ f(3-x) & ; x < 3 \end{cases}$$

$$= \begin{cases} x-3-3 & \text{if } x \geq 3 \text{ and } x-3 \geq 3 \\ 3-x+3 & \text{if } x \geq 3 \text{ and } x-3 < 3 \\ 3-x-3 & \text{if } x < 3 \text{ and } 3-x \geq 3 \\ 3-3+x & \text{if } x < 3 \text{ and } 3-x < 3 \end{cases}$$

$$\therefore f^{o2}(x) = \begin{cases} x-6 & \text{if } x \geq 6 \\ 6-x & \text{if } 3 \leq x < 6 \\ x & \text{if } 0 < x < 3 \\ -x & \text{if } x \leq 0 \end{cases}$$ (3m)

Q) Find the fixed point of $f(x) = -x$ and all 2-cycle (period-2) points of f.

Sol. $f(x) = x \Rightarrow -x = x \Rightarrow x = 0$ the fixed pt.

Now for any $x \neq 0$ we have that (2 m)

$O(x) = \{x, -x\}$ and $f^{o2}(x) = x$ (3m)

so $\mathbb{R} - \{0\}$ is the set of all 2-cycle points.

Q) Determine whether the logistic map (family) has an attracting fixed pt. for $\mu \in (0,3)$. Explain.

Sol. $L_\mu(x) = \mu x(1-x)$, $L'_\mu(x) = \mu - 2\mu x$

for fixed points we have that

$L_\mu(x) = x \Rightarrow x = \mu x(1-x) \Rightarrow \cdots \Rightarrow x = 0$ or $x = 1 - \frac{1}{\mu}$

for attracting f.p. $|L'_\mu(x)| < 1$ (2m)

$L'_\mu(0) = \mu \Rightarrow x = 0$ attracting for $0 < \mu < 1$

and $L'_\mu(1 - \frac{1}{\mu}) = \mu - 2\mu(1 - \frac{1}{\mu}) = \mu - 2\mu + 2$
$= 2 - \mu$

So $x = 1 - \frac{1}{\mu}$ is attracting if $|2 - \mu| < 1$

that is $-1 < 2 - \mu < 1 \Rightarrow 1 < \mu < 3$. (3m)