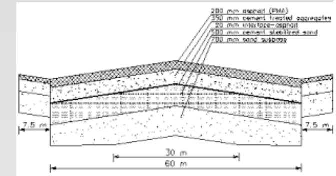
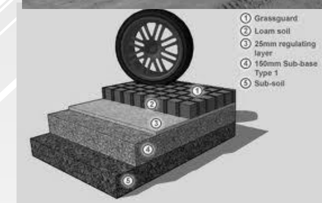
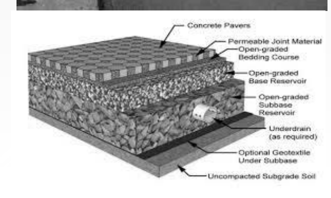
LECTURE 9:
Granular Materials










9.1 Introduction

Important parameters of unbound granular materials for the design of flexible pavements are the stiffness and strength characteristics. As has been shown in the lecture notes on soils and base course materials, these characteristics are strongly influenced by the stress conditions to which the material is subjected. Other important factors are the degree of compaction, the moisture content as well as characteristics of the material itself like gradation etc.
In this chapter some information will be given on how these characteristics can be estimated. The equations presented are developed for sands and base course materials made of mixtures of crushed concrete and crushed masonry.

9.2 Estimation of the resilient characteristics of sands and unbound base materials

The dependency of the resilient modulus of **sands** to the state of stress is given by means of the equation given below [40].

$$M_r = k_1 (\sigma_3 / \sigma_0)^{k2} \cdot (1 - k_3 (\sigma_1 / \sigma_{1,f})^{k4}$$

Where:
σ3 = confining stress [kPa],
σ0 = reference stress = 1 kPa,
σ1 = applied total vertical stress [kPa],
σ1,f = total vertical stress at failure at the given confining stress [kPa],
k1 = model parameter [MPa],
k2 to k4 = model parameters [-].

From this equation it is clear that one needs to have knowledge about the stress conditions as well as on the values for the parameters k1 – k4. Hereafter the models that were developed to predict the values for the different constants in the equation are presented.

The parameter k1 is determined using the following relationship.

$$k1 = k11 \cdot \left(\frac{d50}{1mm}\right)^{k12} \cdot \left(\frac{VVS}{100\%}\right) \cdot qc^{k13}$$

Where:
k11 = model parameter = 24.616 [MPa],
k12 = model parameter = -0.645 [-],
k13 = model parameter = 4.01 [-],
qc = compaction parameter which can be estimated by means of figure 91,
VVS = angularity of the material as determined by means of the outflow test according to the Dutch standards,
d50 = sieve diameter through which 50% of the mass passes [mm].

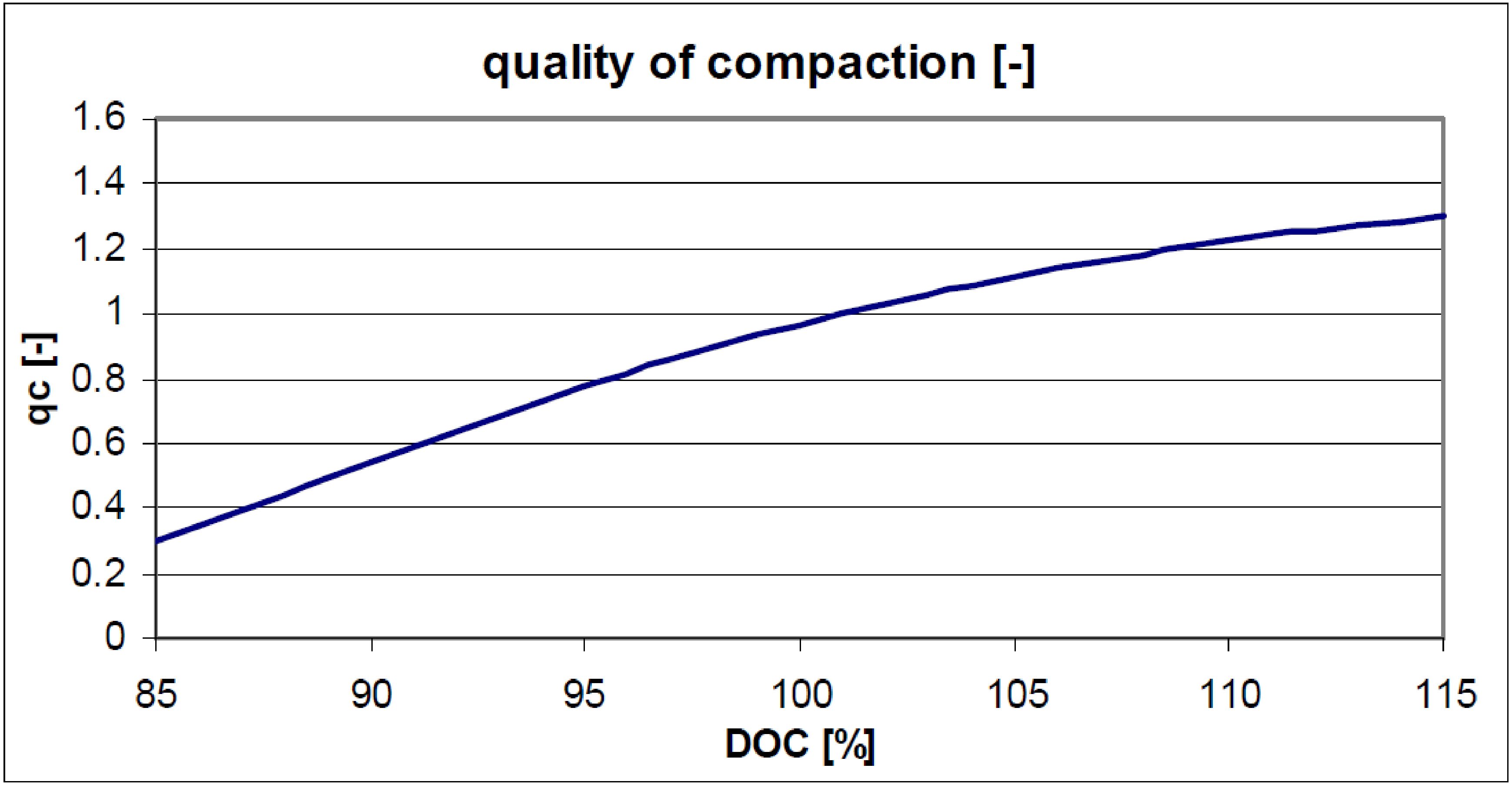


Figure 91: Chart to estimate qc

k2 is is determined using the following relationship.

$$k2 = \frac{\ln\left(\frac{Mr_{1000}}{k1}\right)}{\ln(1000)} \quad \text{with} \quad Mr_{1000} = k21 + k22 \cdot \left(\frac{VVS}{100\%}\right)^{k23} Cu$$

Where:

Mr_{1000} = Mr at 1000 kPa confining pressure combined with small vertical load [MPa],

k21 = model parameter = 1023.25 [MPa],

k22 = model parameter = 30.22 [MPa],

k23 = model parameter = -8.264 [-],

Cu = d60 / d10,

d60 = sieve diameter through which 60% of the mass passes [mm],

d10 = sieve diameter through which 10% of the mass passes [mm].

k3 is determined using:

$$k3 = k31 \cdot \left(\frac{1}{Cu}\right)^{k32}$$

Where:
k31 = model parameter = 2.56 [-],
k32 = model parameter = 0.5511 [-].

k4 is determined using the following relationship.

$$k4 = k41 \cdot \left(\frac{d50}{1mm \cdot Cu}\right)$$

Where:
k41 = model parameter = 46.87 [-]

Although the stress dependency of the resilient modulus of unbound base and sub-base materials can be described by means of the same model as used for sands, the well known Mr - θ model is used for these materials. We recall:

$$M_r = k_1 \theta^{k2}$$

The equations for k1 and k2 for unbound base materials are given below. The degree of compaction and gradation have a large influence on the k1 value while k2 strongly depends on k1.

$$k_1 = \frac{k_{11}}{qg} \cdot qc^{k_{12}} \cdot qp^{k_{13}}$$

$$k_2 = \frac{\ln\left(Mr_{5000} / k_1 \right)}{\ln(5000)}$$

$$Mr_{5000} = k_{21} \,.\, qc^{k22}$$

Where:
k11 = 34.1855 [MPa],
k12 = 1.8183 [-],
k13 = 1.6502 [-],
k21 = 1016.275 [MPa],
k22 = 1.5568 [-],
qg = gradation quality parameter
qc = compaction parameter as described before,
qp = composition parameter = (mass % masonry + mass % concrete) / 100.

When the parameters that characterize the stress, dependent nature of granular materials have been quantified, the modulus of the granular (and the variation therein over the height and width of the layer) must be determined by means of an iterative procedure. How this is done is discussed further on in the lecture notes.

This procedure of estimating the stress dependent parameters and determining the stiffness modulus of the granular layers by means of an iterative procedure is quite often a cumbersome one. Therefore procedures have been developed to make fair estimates of the stiffness values and tables have been set up to give the designer some guidance about the stiffness values to select. An example of such a table is table 19.

When using values mentioned in table 19 one should be aware of the specifications which are applicable for these materials. These are given in table 20. Special attention is called for the high compaction levels that are required and achieved in South Africa; they might be very difficult to achieve when different materials are used under different climatic conditions.

Material code	Material description	Over cemented layer in slab state	Over granular layer or equivalent	Wet condition (good support)	Wet condition (poor support)
G1	High quality crushed stone	250 – 1000 (450)	150 – 600 (300)	50 – 250	40 – 200
G2	Crushed stone	200 – 800 (400)	100 – 400 (250)	50 – 200	40 – 200
G3	Crushed stone	200 – 800 (350)	100 – 350 (230)	50 -150	40 – 200
G4	Natural gravel (base quality)	100 – 600 (300)	75 – 350 (225)	50 – 150	30 – 200
G5	Natural gravel	50 – 400 (250)	40 – 300 (200)	30 – 200	20 – 150
G6	Natural gravel (sub-base quality)	50 – 200 (150)	30 – 200 (120)	20 – 150	20 – 150

Table 19: Stiffness values for granular bases and sub-bases as recommended in South Africa.

SYMBOL	CODE	MATERIAL	ABBREVIATED SPECIFICATIONS
	G1	Graded crushed stone	Dense - graded unweathered crushed stone; Maximum size 37,5 mm; 86 - 88 % apparent relative density; Soil fines PI < 4
	G2	Graded crushed stone	Dense - graded crushed stone; Maximum size 37,5 mm; 100 - 102 % Mod. AASHTO or 85 % bulk relative density; Soil fines PI < 6
	G3	Graded crushed stone	Dense - graded stone and soil binder; Maximum size 37,5 mm; 98 - 100 % Mod. AASHTO ; Soil fines PI < 6
	G4	Crushed or natural gravel	Minimum CBR = 80 % @ 98 % Mod. AASHTO; Maximum size 37,5 mm; 98 - 100 % Mod. AASHTO; PI < 6; Maximum Swell 0,2 % @ 100 % Mod. AASHTO. For calcrete PI ≤ 8
	G5	Natural gravel	Minimum CBR = 45 % @ 95 % Mod. AASHTO; Maximum size 63 mm or 2/3 of layer thickness; Density as per prescribed layer usage; PI < 10; Maximum swell 0,5 % @ 100 % Mod. AASHTO. *
	G6	Natural gravel	Minimum CBR = 25 % @ 95 % Mod. AASHTO; Maximum size 63 mm or 2/3 of layer thickness; Density as per prescribed layer usage; PI < 12; Maximum swell 1,0 % @ 100 % Mod. AASHTO. *
	G7	Gravel / Soil	Minimum CBR = 15 % @ 93 % Mod. AASHTO; Maximum size 2/3 of layer thickness; Density as per prescribed layer usage; PI < 12 or 3GM** + 10; Maximum swell 1,5 % @ 100 % Mod. AASHTO. ***
	G8	Gravel / Soil	Minimum CBR = 10 % @ 93 % Mod. AASHTO; Maximum size 2/3 of layer thickness; Density as per prescribed layer usage; PI < 12 or 3GM** + 10; Maximum swell 1,5 % @ 100 % Mod. AASHTO. ***
	G9	Gravel / Soil	Minimum CBR = 7 % @ 93 % Mod. AASHTO; Maximum size 2/3 of layer thickness; Density as per prescribed layer usage; PI < 12 or 3GM** + 10; Maximum swell 1,5 % @ 100 % Mod. AASHTO. ***
	G10	Gravel / Soil	Minimum CBR = 3 % @ 93 % Mod. AASHTO; Maximum size 2/3 of layer thickness; Density as per prescribed layer usage;

* For calcrete PI ≤ 15 on condition that the Linear Shrinkage (LS) does not exceed 6 %.

** GM = Grading Modulus (TRH14, 1985) = $\frac{300 - [P_{2,00mm} + P_{0,425mm} + P_{0,075mm}]}{100}$ where $P_{2,00}$ etc., denote the percentage passing through the sieve size.

*** For calcrete PI ≤ 17 on condition that the Linear Shrinkage (LS) does not exceed 7 %.

Table 20: Specifications for granular materials in South Africa.

Stress dependency implies that the stiffness modulus of unbound granular materials varies over the height and width of the granular layer. It is clear that this cannot be analyzed by means of programs like BISAR since such programs assume the layer stiffness to be constant in the horizontal directions. It would therefore be logical to use finite element programs for this purpose (FEM based programs like RUBICON [41] are extremely helpful in this case) but one should keep in mind that the superposition principle that can be used in linear elastic systems to determine the effects of multiple wheel configurations cannot be used anymore for non linear systems. This certainly complicates the analyses.

Such a procedure is adopted in the program KENLAYER [42]. The question however is to what extent an approach as used in KENLAYER is still capable of giving realistic results. An investigation on this was done by Opiyo [43] using the finite element code NOLIP developed by Huurman [44].

He analyzed two pavement structures, one with a 30 mm thick asphalt top layer and one with a 100 mm thick asphalt top layer. In both cases the stiffness modulus of the asphalt was 3000 MPa. The unbound laterite base course had a thickness of 200 mm while the unbound laterite subbase had a thickness of 250 mm. The stress dependency of the stiffness modulus of both laterites was determined in the laboratory by means of repeated load triaxial tests. In order to be able to take the stress dependent nature of the base and subbase into account, Opiyo divided the base into two sublayers with a thickness of 100 mm each. The subbase was divided into two layers as well; the thickness of the top layer was 100 mm and the thickness of the bottom was **150** mm. The stiffness of the subgrade was assumed to be 80 MPa. Some results of this work are shown in figures 92, 93 and 94.

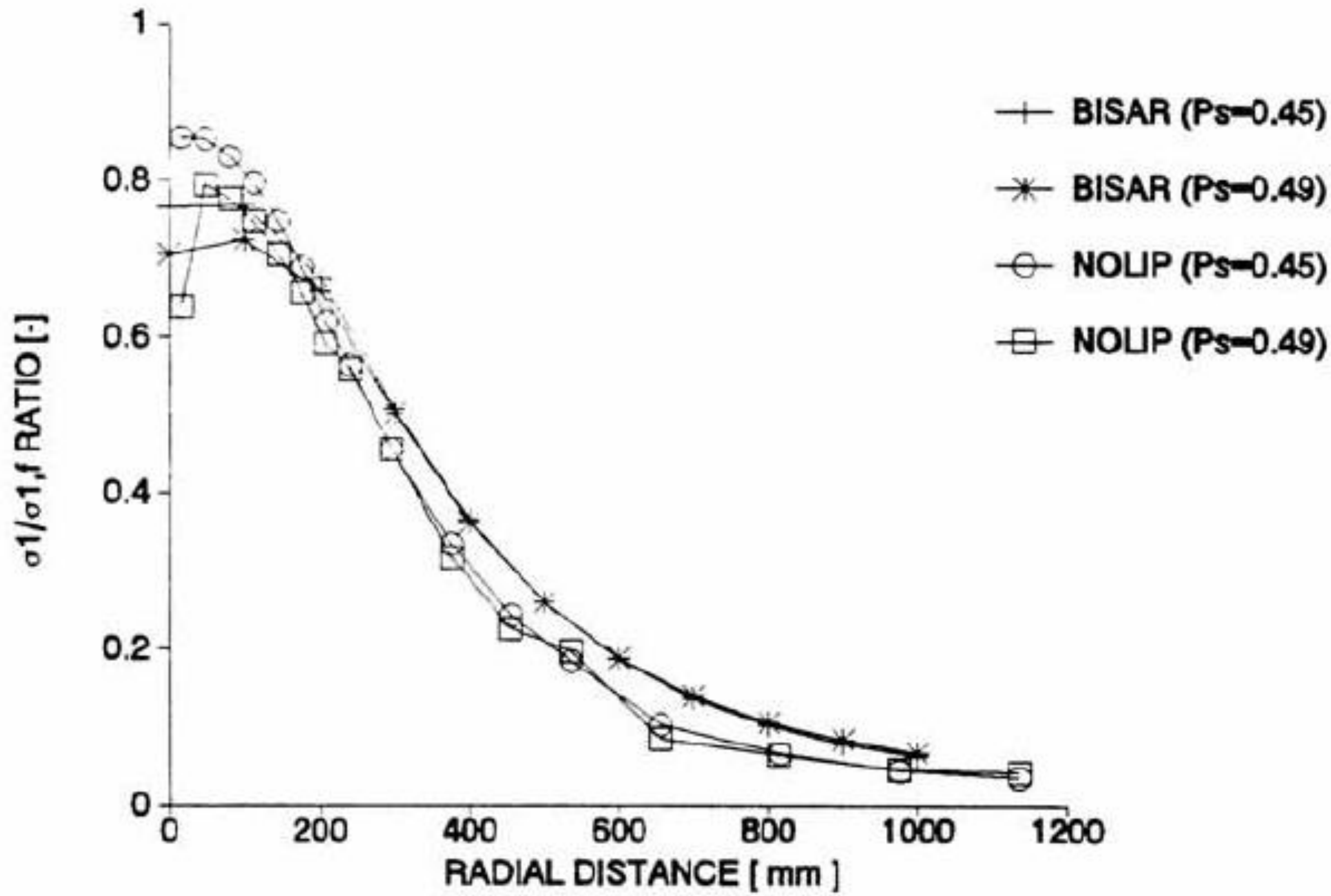


Figure 92: Variation of the failure ratio in the top of the base course for the 100 mm asphalt pavement at a depth of 137.5 mm from the pavement surface.

Note: Ps = Poisson's ratio.

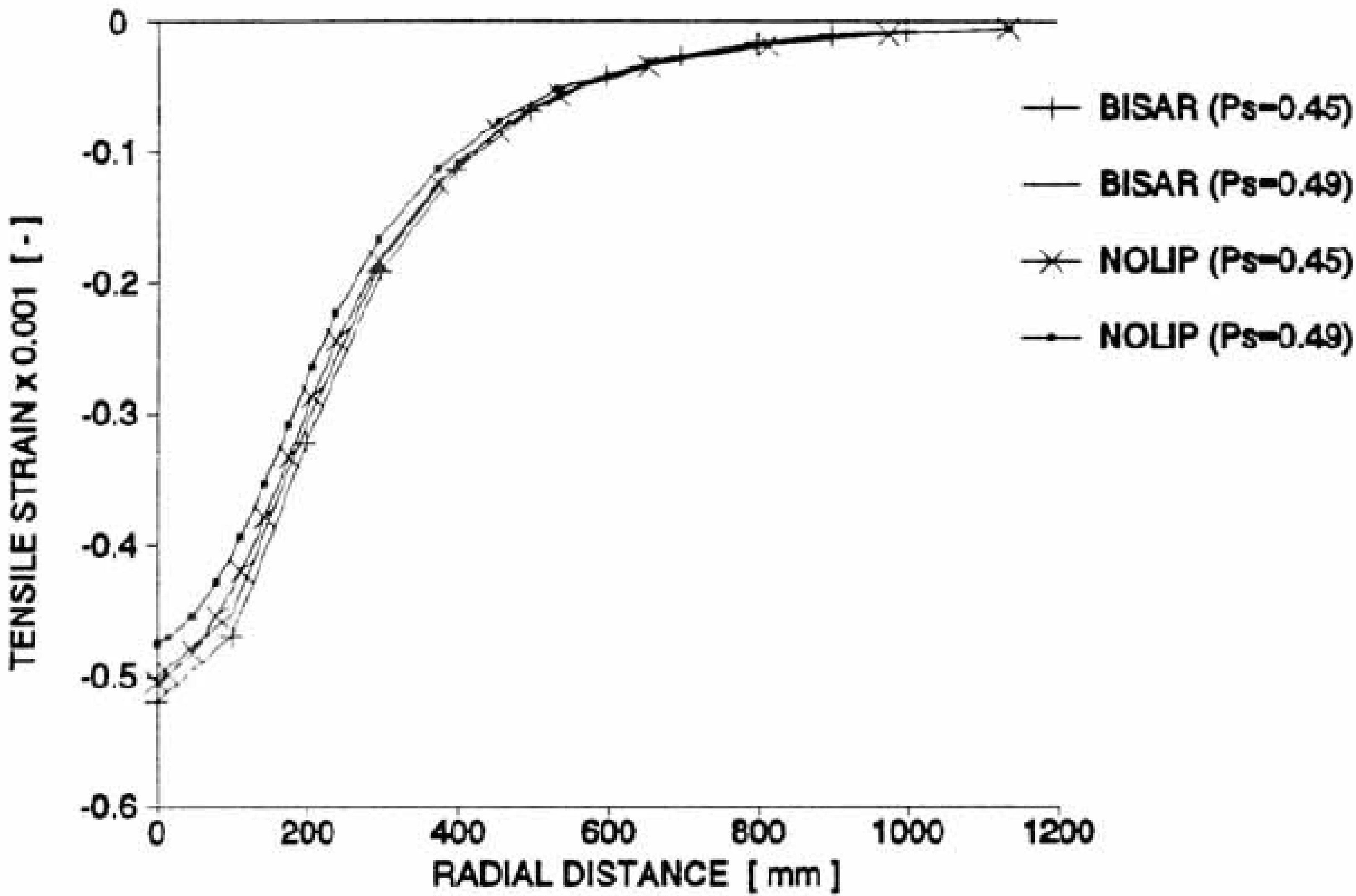


Figure 93: Tensile strains at the bottom of the 100 mm thick asphalt layer.

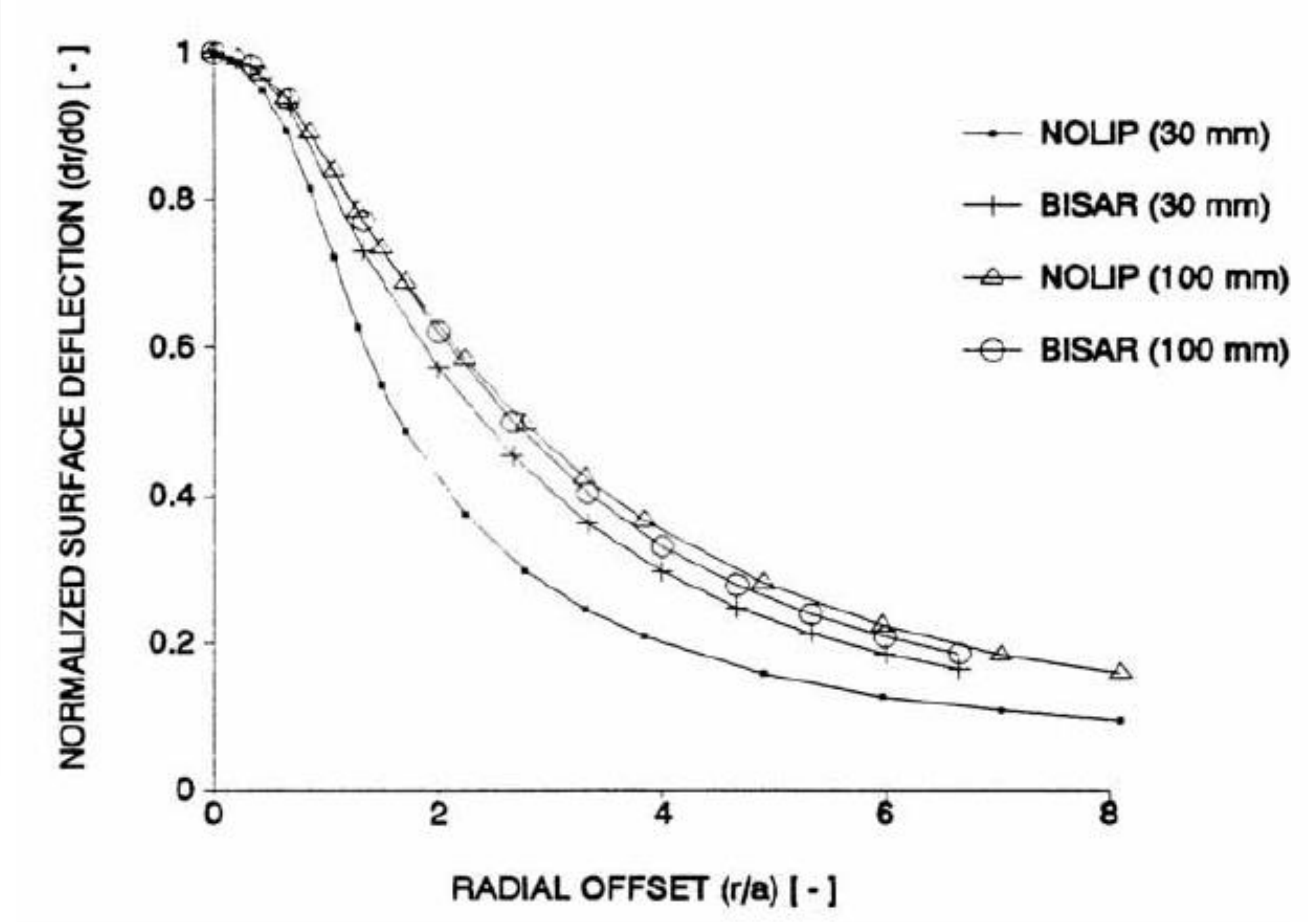


Figure 94: Normalized surface deflections for both pavements.

- From these figures one can conclude that the influence of using BISAR, in combination with subdividing the base and subbase, on the calculated stresses, deflections and tensile strains is only marginal. However this was only true for the 100 mm asphalt pavement. Significant differences and even unrealistic results were obtained when using BISAR for the 30 mm asphalt pavement. The conclusion therefore is that the stress dependent behaviour of granular materials can be successfully simulated using BISAR and subdividing the base and subbase layer, provided that the top layer is not too thin. It is estimated that realistic results will already be obtained when the asphalt thickness is 70 mm.

- In taking into account the stress dependent nature of unbound granular materials one should not forget to take into account the stresses due to the dead weight of the material. The vertical dead weight stresses can simply be calculated following:

$$\sigma_{v,dw} = \gamma * z$$

Where:
$\sigma_{v,dw}$ = vertical stress due to the dead weight of the material,
γ = volume weight of the material,
z = depth below the surface.

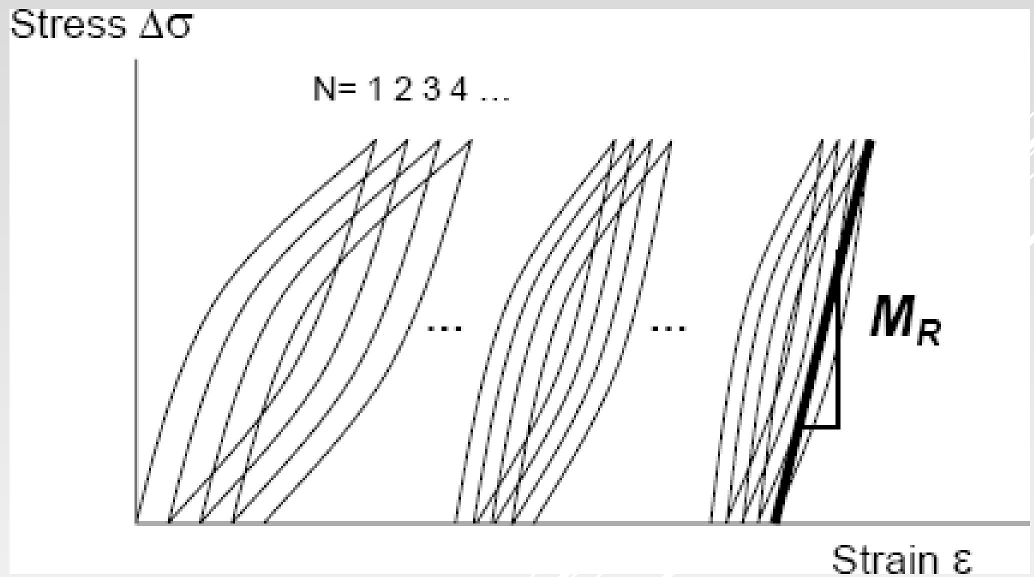
In principle, the horizontal dead weight stresses are not equal to the vertical stresses as is the case in fluids. We can write:

$\sigma_{h,dw} = K * \sigma_{v,dw}$

Where:
$\sigma_{h,dw}$ = horizontal stress due to the dead weight of the material,
K = constant depending on a large number of factors.

The constant K depends a.o. on the degree of compaction, the tendency of the aggregate skeleton to dilate when loaded etc. K can easily take a value of 2, but because very little information can be found on this issue, a value of 1 is recommended for design purposes.

As has been shown in [39], the repeated load CBR test can be used to determine the resilient modulus of fine grained materials. For the type of sand used as subgrade for most road projects in the western part of the Netherlands it has been found that the resilient modulus as determined by means of the repeated load CBR test is the same as the resilient modulus determined from a repeated load triaxial test performed with at 20 kPa confinement stress [45]. For other confinement levels one could write:



$$M_r = 0.211\ \sigma_3^{\,0.563}\ M_{rep\ CBR}$$

Where:
Mr = resilient modulus
σ_3 = confining stress [kPa],
$M_{rep\ CBR}$ = resilient modulus obtained from the repeated load CBR test.

If none of the above mentioned information is available then the modulus of unbound materials can be estimated using “rules of the thumb”. Some well known rules which can be applied to estimated the stiffness modulus of fine grained soils are given below.

Organisation	Equation
Shell	$E = 10\ CBR$
US Army Corps of Engineers	$E = 37.3\ CBR^{0.711}$
CSIR South Africa	$E = 20.7\ CBR^{0.65}$
Transport and Road Research Laboratory UK	$E = 17.25\ CBR^{0.64}$
Delft University, Ghanaian laterite	$E = 4\ CBR^{1.12}$

Table 21: Equations to estimate the subgrade modulus [E] = [MPa], [CBR] = [%].

- It is clear that there is no unique relationship to predict the stiffness modulus of fine grained materials from the CBR. Therefore one should be very cautious in adopting these equations.
- Also other procedures are available for the estimation of the stiffness modulus of the base and sub-base material. Barker e.a. [46] e.g. have presented the chart given in figure 95.

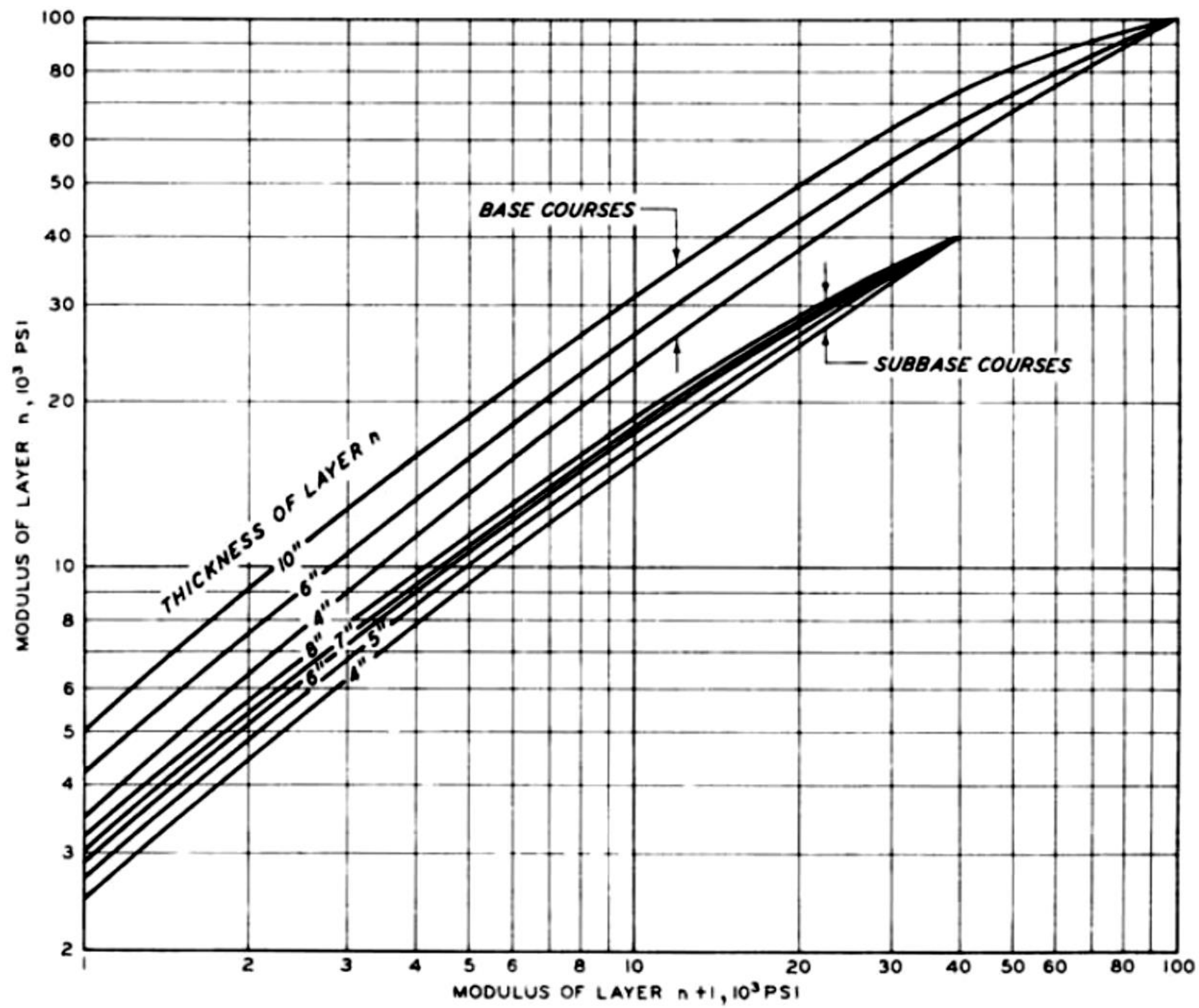


Figure 95: Relationship between modulus of layer n and modulus of layer n + 1 for various thicknesses of unbound base and subbase layers.

Please note that in figure 95, the maximum value for the stiffness modulus of the subbase layer is set at 40,000 psi (280 MPa), while the maximum stiffness for the base layer is set at 100,000 psi (700 MPa). The use of the chart will be illustrated by means of an example.

- Let us assume that the stiffness modulus of the subgrade equals 4,000 psi.
- If we place an 8 inch subbase on top of the subgrade, the stiffness of that subbase will be 10,000 psi (enter the horizontal axis at 4,000 psi and determine the subbase stiffness at the point where the vertical line through the 4,000 value crosses the 8 inch subbase line).
- To know the stiffness of a 6 inch base placed on top of the subbase, we have to enter 10,000 on the horizontal axis and determine where the vertical line through the 10,000 value crosses the 6 inch base line. In this way we determine that the base stiffness equals 27,000 psi.

Barker e.a. [46] also presented the equations which are the background for figure 95. For the sake of completeness they are given here as well because they shown that some assumptions had to be made to derive figure 95.

$$E_n = E_{n+1}(R + S \log t - T \log t \log E_{n+1} + W \log E_{n+1})$$

Where:

E_n = stiffness modulus of the upper layer [psi],

E_{n+1} = stiffness modulus of the lower layer [psi],

$R = a - X \log b + \{(a - 1) / Y\} \log c$,

$S = X + T \log c$,

$T = X / Y$,

$X = (a - 1) / \log (b / e)$,

$Y = \log (d / e)$

$W = T \log b - (a - 1) / Y$

t = thickness of the upper layer [inch],

a = ratio En / En+1 for a layer with thickness b over a material having modulus of c, this means one have to set a certain thickness b (e.g. 4” or 6” for which a certain modulus ration (e.g. 1.5 or 2) is obtained,

d = maximum limiting modulus value for the particular material,

e = layer thickness [inch] for which the modulus ratio equals 1.

Summarizing, it means that assumptions have to be made for the parameters a, b, d and e. Furthermore the stiffness modulus of the lower layer (c) should be known. In case of figure **95**, Barker e.a. assumed the following values:

For the subbase course: a = 2, b = 6”, d = 40,000 psi, e = 1″.

For the base course: a = 3, b = 6″, d = 100,000 psi, e = 1″.

One could argue whether or not the selected a values are a bit on the high side (this author would have used a = 1.5) and whether the selected e values are a bit on the low side (this author would have selected e = 2″ since it is impossible that a thin layer produces any appreciable stiffness).

A very simple relationship to estimate the stiffness modulus of the base course has been developed by Shell [27]. This relationship is written as:

$E_b = k * E_{sg}$

Where:
E_b = stiffness modulus of the base course [MPa],
E_{sg} = stiffness modulus of the subgrade [MPa] ,
$k = 0.2 * h_b^{0.45}$, $2 \le k \le 4$,
h = thickness of the base course [mm].

The question now is, to what extend realistic stiffness modulus values are predicted using e.g. the Shell equation. In order to determine this, a comparison was made between the base layer stiffness as estimated by means of the Shell equation and the stiffness as estimated by means of an analysis in which the stress dependency of the base material was taken into account. Figure 96 shows the variation of the stiffness modulus of a base course of different thicknesses when placed on subgrades with different stiffness values. KENLAYER was used for the analysis.

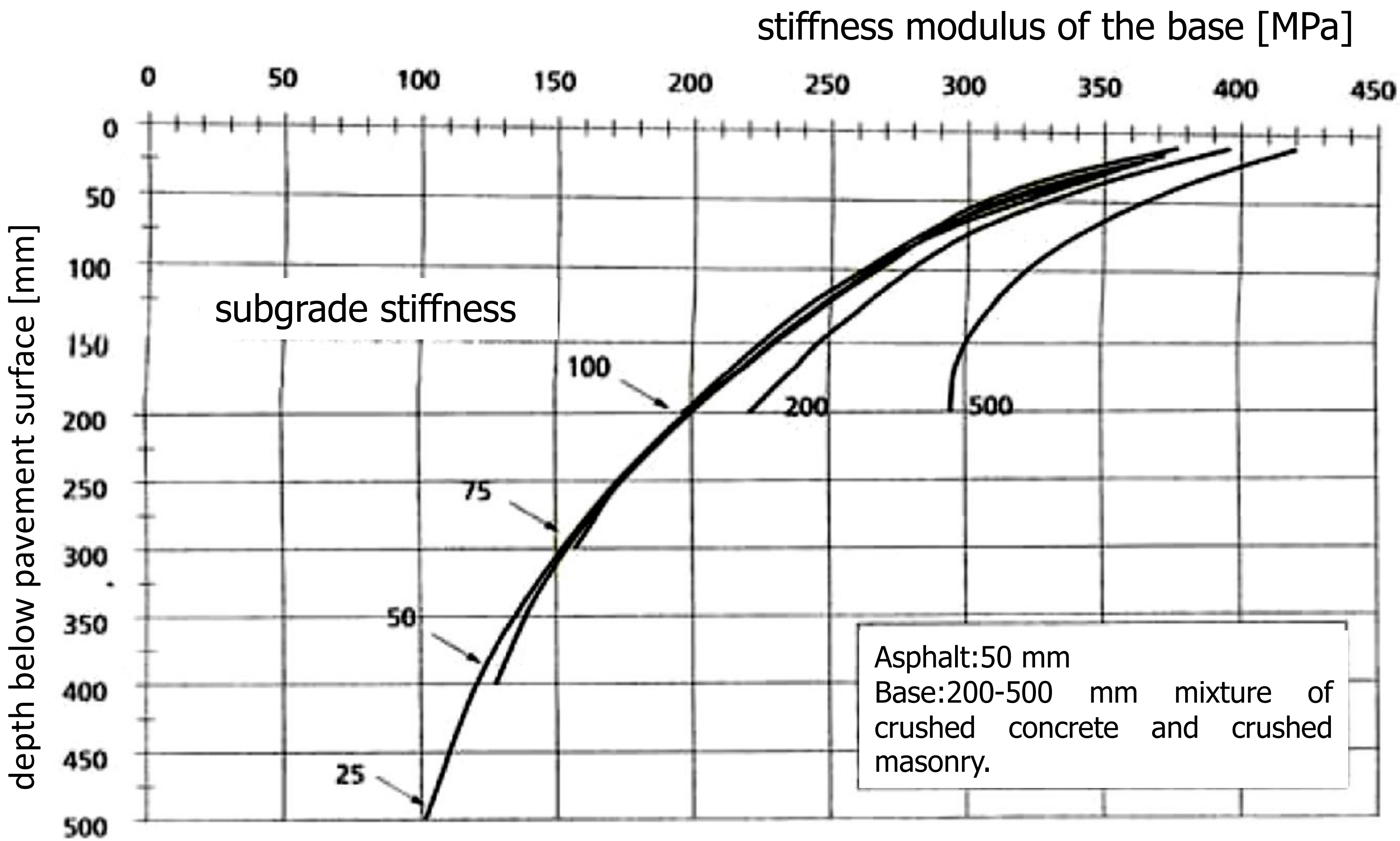


Figure 96: Variation of the stiffness modulus over the thickness of a granular base course.

- One can easily derive from this figure that for this particular case, the Shell equation produces a stiffness value for the base course which seems to be on the safe side. For a 200 mm thick base course k equals 2.17 giving the base course a stiffness of 217 MPa if the subgrade stiffness is 100 MPa. According figure 96, the mean stiffness would be approximately 250 Mpa (at 50 is 325, at 250 is 175).
- For a 400 mm thick base course k equals 2.96 giving the base course a stiffness of 148 MPa if the subgrade stiffness is 50 MPa. According to figure 96 the mean base stiffness would be approximately 225 MPa.

It is recalled once more that the observations made here are only valid for the material under consideration. If weaker materials are used which are compacted to a lesser degree of compaction, the Shell rule might very well overpredict the value of the stiffness modulus of the base course.

9.3 Estimation of the failure characteristics of unbound materials

We recall that the vertical stress at which shear failure occurs in a granular material depends on the amount of confinement as well as the cohesion and angle of internal friction of the material considered. We can write:

$$\sigma_{1,f} = [(1 + \sin\phi) \,.\, \sigma_3 + 2c \,.\, \cos\phi] / (1 - \sin\phi)$$

Where:

$\sigma_{1,f}$ = vertical stress at which failure occurs [kPa],
σ_3 = confining pressure [kPa],
ϕ = angle of internal friction,
c = cohesion [kPa].

As was the case for the resilient characteristics, procedures have also been developed to estimate the failure characteristics of sands and unbound base course materials made of mixtures from crushed concrete and crushed masonry [40]. These equations will be presented hereafter.

Based on the triaxial test results obtained on the **sands**, the following equation could be developed to predict the cohesion (c) and the angle of internal friction (ϕ) of the sands.

$$c = c1 \cdot qc^{c2} \cdot \left(\frac{d50}{1mm}\right)^{c3} \cdot Cu^{c4} \cdot \left(\frac{VVS}{100\%}\right)^{c5}$$

$$\phi = \phi 1 \cdot qc^{\phi 2} \cdot \left(\frac{VVS}{100\%}\right)^{\phi 3}$$

Where:

c1:	model parameter = 0.1375 [kPa]
c2:	model parameter = 2.553 [-]
c3:	model parameter = -1.698 [-]
c4:	model parameter = 2.959 [-]
c5:	model parameter = 0.384 [-]

ϕ1:	model parameter = 45.71 [degr.]
ϕ2:	model parameter = 0.833 [-]
ϕ3:	model parameter = 0.091 [-]
d50	sieve diameter through which 50% of the mass passes [mm]
VVS	angularity of the material as determined by means of an outflow test according to the Dutch standards [%]
Cu	d60 / d10 [-]

The strength characteristic of **unbound granular base materials** in relation to their gradation, compaction quality index and ratio amount of crushed masonry to amount of crushed concrete has been determined in a similar way.

$$c = c6 \cdot qg \cdot qp \cdot qc^{c7}$$

$$\phi = \phi 4 + \phi 5 \cdot qc \cdot qg$$

Where:

c6	model parameter = 134.506 [kPa]
c7	model parameter = 2.2495 [-]
ϕ4	model parameter = 30.27 [degr.]
ϕ5	model parameter = 18.86 [degr.]

qp:	(percentage masonry + percentage concrete rubble)/100 [-]
qg:	grading quality [-], (UL=1 / FL=1 / CO=0,9 / AL=0,89 / LL= 0,75 / UN=0,63, see also figure 97)

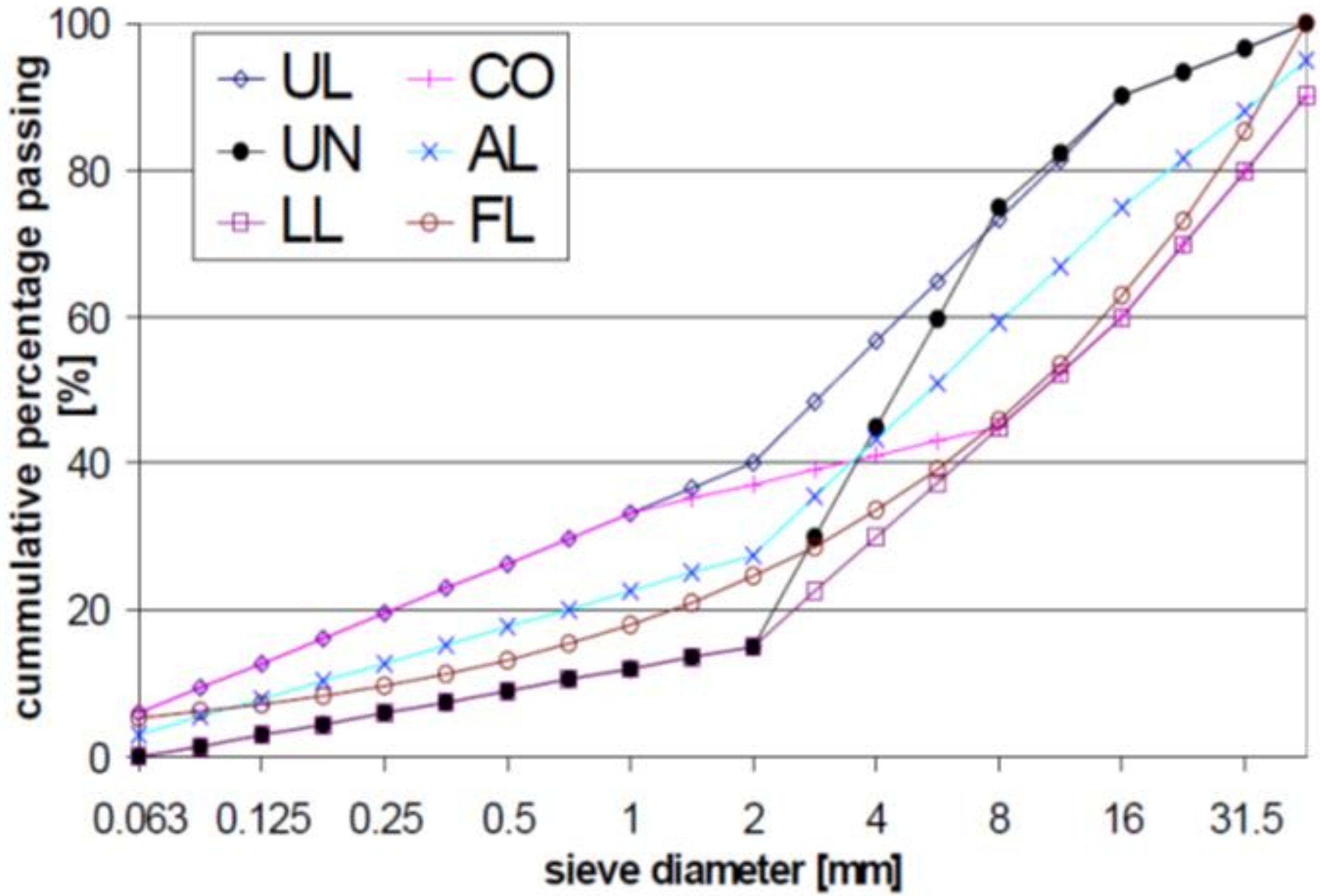


Figure 97: Gradations of the base course materials for which the equations have been developed.

The South-African pavement design procedure [47] also contains a method to evaluate the resistance of granular material to the applied stress levels and, in other words, to determine whether excessive permanent deformation occurs due to the applied stresses. In order to do so, a safety factor has been derived which is calculated using the following equation.

$$F = \{\sigma_3 [K (\tan^2 (45 + \varphi/2) -1] + 2 K c (\tan (45 + \varphi/2)\} / (\sigma_1 - \sigma_3)$$

This equation can be rewritten as:

$$F = (\sigma_3 \varphi_{term} + c_{term}) / (\sigma_1 - \sigma_3)$$

Where:

F = safety factor,

c = cohesion [kPa],

K = constant = 0.65 for saturated soils, 0.8 for moderate moisture conditions and 0.95 for normal conditions,

φ = angle of internal friction,

$\sigma 1$, $\sigma 3$ = major and minor principle stress in the layer [kPa].

It should be noted that the F factor is in fact the inverse of the σ1/σ1f ratio which was used earlier in the description of the chance on failure and excessive permanent deformation. The only difference is that in the F equation, the factor K is introduced which takes care for the effect of the moisture conditions in the layer.

Values for the c_{term} and φ_{term} are given in table 22.

	Dry conditions	Dry conditions	Moderate conditions	Moderate conditions	Wet conditions	Wet conditions
Material code	ϕ_{term}	C_{term}	ϕ_{term}	C_{term}	ϕ_{term}	C_{term}
G1	8.61	392	7.03	282	5.44	171
G2	7.06	303	5.76	221	4.46	139
G3	6.22	261	5.08	188	3.93	115
G4	5.50	223	4.40	160	3.47	109
G5	3.60	143	3.30	115	3.17	83
G6	2.88	103	2.32	84	1.76	64

Table 22: Values for the ϕ_{term} and c_{term}.

9.4 Allowable stress and strain conditions in granular materials

If the stress conditions in the granular base or sub-base are becoming too high, permanent deformation or even shear failure will occur. Figure 98 is a nice example of excessive deformation in a pavement due to excessive deformation in the unbound layers.

Figure 98: Excessive pavement deformation due to deformation of the unbound base and/or sub-base layer.

The deformation shown in figure 98 is clearly due to deformation in the base or sub-base because the permanent deformation bowl is rather wide. In case of asphalt rutting a much narrower deformation bowl would have appeared.

There are two options to analyze the resistance to permanent deformation in the unbound layers.

- **The first one** is making predictions of the development of the permanent deformation as a function of the number of load repetitions, the stress conditions and the material characteristics.
- **The second one** is based on keeping the stress conditions in the unbound layers below a certain level such that excessive deformation will not occur. It is obvious that the latter procedure is a more straightforward one.

Work by van Niekerk [48] has shown that if the stress ratio σ1 / σ1,f stays below 0.4, no excessive deformation will occur. This ratio is valid for gradations UL and AL and compaction levels of 97 – 103%. For the coarser LL gradation the stress ratio could go up to 0.45 if the degree of compaction is 100% and even to 0.62 at a degree of compaction of 103% (see figure 97 for gradation codes).

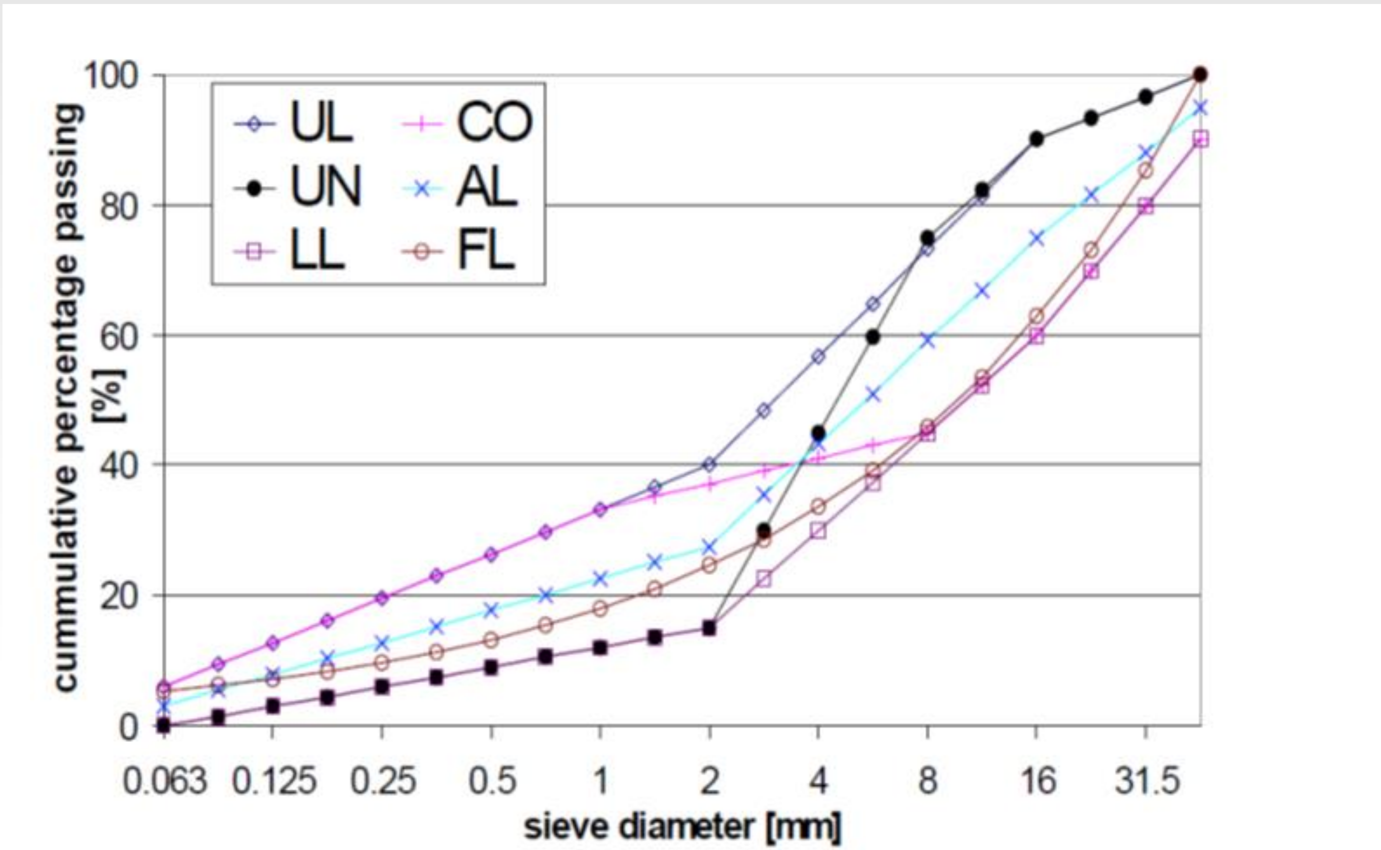


Figure 97: Gradations of the base course materials for which the equations have been developed.

According to the South-Africans however, F values of smaller than one can still be allowed for a significant number of load repetitions. From the results presented above it is clear that these South-African findings should be treated with great care. Ratios not higher than 0.6 for σ1/σ1f or 1.66 for F are strongly recommended to avoid excessive deformation in unbound granular layers to take place.

Van Niekerk's work has also been used to develop relations for the **allowable vertical strain at the top of the unbound base or subbase** [49]. These relationships are shown in figure 99.

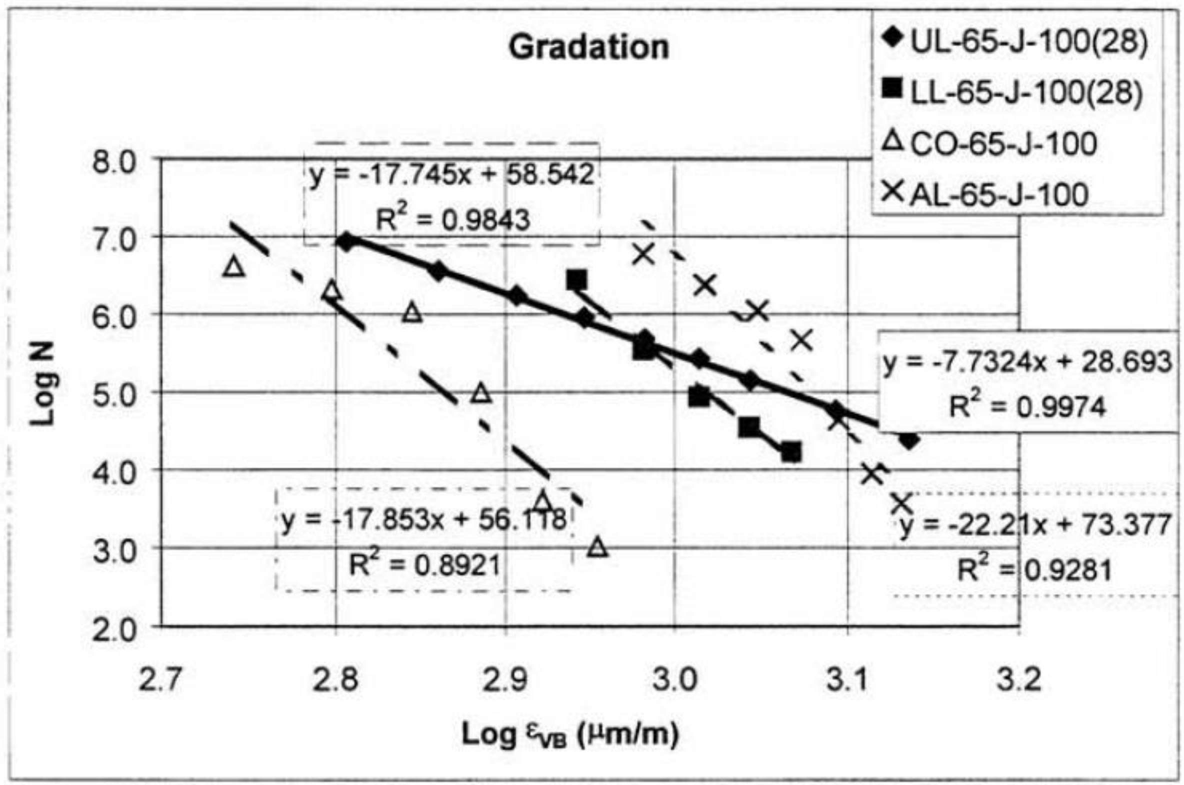


Figure 99: Allowable vertical strain levels in unbound base materials.

In the work by Huurman and Van Niekerk on **sands** that for these materials much higher σ1/σ1f ratios can be allowed before permanent deformation occurs. A typical ratio value is 0.9.