General Topology

Dr. Reyadh Delphi Ali

Chapter One

1.1Definition: **Let $X \neq \phi$ and T is a collection of subsets of X, then T is called a topology on X if and only if**

1. **a.** $\phi \in T$ **b.** $X \in T$
2. **If** $G_\alpha \in T \quad \forall \alpha$ **, then** $\bigcup_\alpha G_\alpha \in T$ **.**
3. **If** $G_i \in T \quad \forall \quad 1 \leq i \leq n$ **, then** $\bigcap_{i=1}^{n} G_i \in T$ **.**

1.2Notes:

- **If T is a topology on X, then the pair (X, T) is called a topological space.**
- **If T is a topology on X, then the sets $G \in T$ are called open. A subset F of X is called closed, if F^C is open.**

1.3Exmaple: **Let $X \neq \phi$ and $T = \{\phi, X\}$, then T a topology on X and T is called the indiscrete topology.**

1.4Exmaple: **Let $X \neq \phi$ and $T = \{A : A \subseteq X\}$, then T is a topology on X and T is called the discrete topology. Since**

1. a. $\because \phi \subset X \quad \therefore \phi \in T.$ **b.** $\because X \subset X \quad \therefore X \in T$

2. If $G_\alpha \in T \quad \forall \alpha \quad \therefore G_\alpha \subset X \quad \forall \alpha \quad \therefore \bigcup_\alpha G_\alpha \subset X \quad \therefore \bigcup_\alpha G_\alpha \in T.$

3. If $G_i \in T \quad \forall 1 \leq i \leq n \quad \therefore G_i \subset X \quad \forall 1 \leq i \leq n \quad \therefore \bigcap_{i=1}^{n} G_i \subset X \quad \therefore \bigcap_{i=1}^{n} G_i \in T.$

1.5Exmaple: **Let $X = \{1,2,3\}$ .Find all the topologies on X .**

Solution:

$T_1=\{\phi, X\}$, $T_2=\{\phi, X,\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\}\}$, $T_3=\{\phi, X,\{1\}\}$, $T_4=\{\phi, X,\{2\}\}$
$T_5=\{\phi, X,\{3\}\}$, $T_6=\{\phi, X,\{1,2\}\}$, $T_7=\{\phi, X,\{1,3\}\}$, $T_8=\{\phi, X,\{2,3\}\}$,
$T_9=\{\phi, X,\{1\},\{1,2\}\}$, $T_{10}=\{\phi, X,\{2\},\{1,2\}\}$, $T_{11}=\{\phi, X,\{1\},\{1,3\}\}, T_{12}=\{\phi, X,\{3\},\{1,3\}\}$,
$T_{13}=\{\phi, X,\{2\},\{2,3\}\}$, $T_{14}=\{\phi, X,\{3\},\{2,3\}\}$, $T_{15}=\{\phi, X,\{1\},\{2\},\{1,2\}\}$,

$T_{16}=\{\phi, X,\{1\},\{3\},\{1,3\}\}$, $T_{17}=\{\phi, X,\{2\},\{3\},\{2,3\}\}$, $T_{18}=\{\phi, X,\{1\},\{1,2\},\{1,3\}\}$
$T_{19}=\{\phi, X,\{2\},\{1,2\},\{2,3\}\}$, $,T_{20}=\{\phi, X,\{3\},\{1,3\},\{2,3\}\}$, $T_{21}=\{\phi, X,\{1\},\{2\},\{1,2\},\{1,3\}\}$
$T_{22}=\{\phi, X,\{1\},\{3\},\{1,2\},\{1,3\}\}$, $T_{23}=\{\phi, X,\{1\},\{2\},\{1,2\},\{2,3\}\}$, $T_{24}=\{\phi, X,\{2\},\{3\},\{1,2\},\{2,3\}\}$
$T_{25}=\{\phi, X,\{1\},\{3\},\{1,3\},\{2,3\}\}$, $T_{26}=\{\phi, X,\{2\},\{3\},\{1,3\},\{2,3\}\}$, $T_{27}=\{\phi, X,\{1\},\{2,3\}\}$
$T_{28}=\{\phi, X,\{2\},\{1,3\},\}$, $T_{29}=\{\phi, X,\{3\},\{1,2\}\}$.

<u>1.6Notes:</u>

The following table gives the numbers of topologies on X **,** $|X|=n, \quad 1\le n\le 7$ **.**

$\lvert X\rvert=n, \quad 1\le n\le 7$	**the numbers of topologies on** X
n=1	**1**
n=2	**4**
n=3	**29**
n=4	**355**
n=5	**6942**
n=6	**209527**
n=7	**9535241**

<u>1.7Exercise:</u> Let $X=\{1,2,3,4\}$ **.Find 50 topologies on** X **.**

<u>1.8Exmaple:</u> Let T_1 **and** T_1 **are topologies on** X **.**

a. Is $T_1\cup T_2$ **a topology on** X **?**

b. Is $T_1\cap T_2$ **a topology on** X **?**

Solution:

a. $T_1\cup T_2$ **is not necessary a topology on** X **.For example let** $T_1=\{\phi, X,\{1\}\}$ **is a topology on** X **and** $T_2=\{\phi, X,\{2\}\}$ **is a topology on** X **, then** $T_1\cup T_2=\{\phi, X,\{1\},\{2\}\}$ **is not a topology on** X **.**

b. $T_1 \cap T_2$ **is a topology on** X **. Since**

1. a. $\because \phi \in T_1$ **and** $\phi \in T_2$ **,** $\therefore \phi \in T_1 \cap T_2$**. b.** $\because X \in T_1$ **and** $X \in T_2$ **,** $\therefore X \in T_1 \cap T_2$**.**

2. If $G_\alpha \in T_1 \cap T_2 \quad \forall \alpha \quad \therefore G_\alpha \in T_1 \, and \quad G_\alpha \in T_2 \quad \forall \alpha$

$\therefore \bigcup_\alpha G_\alpha \in T_1 \, and \quad \bigcup_\alpha G_\alpha \in T_2$. **(Since** T_1 **and** T_1 **are topologies on** X **)**

$\therefore \bigcup_\alpha G_\alpha \in T_1 \cap T_2$.

3. If $G_i \in T_1 \cap T_2 \quad \forall 1 \le i \le n \quad \therefore G_i \in T_1 \, and \quad G_i \in T_2 \quad \forall 1 \le i \le n$

$\therefore \bigcap_{i=1}^{n} G_i \in T_1 \, and \quad \bigcap_{i=1}^{n} G_i \in T_2$. **(Since** T_1 **and** T_1 **are topologies on** X **)**

$\therefore \bigcap_{i=1}^{n} G_i \in T_1 \cap T_2$.

1.9Exmaple: **Let** $X = \{1,2,3,4\}$ **and** $T = \{\phi, X, \{1,2,3\}, \{2,3\}, \{1,2\}, \{4\}, \{1,2,4\}, \{2,3,4\}\}$**.**

Is T **a topology on** X **?**

Solution:

T **is not a topology on** X **,since** $\{2,3\} \cap \{1,2\} = \{2\} \notin T$ **.**

1.10Exmaple:

Let $X = \{1,2,3,4,5\}$ **and** $T = \{\phi, X, \{1,5\}, \{2,3\}, \{1,2\}, \{2\}, \{1\}, \{1,2,3,5\}, \{1,2,3\}\}$**.**

Is T **a topology on** X **?**

Solution:

T **is not a topology on** X **,since** $\{1,5\} \cup \{1,2\} = \{1,2,5\} \notin T$ **.**

1.11Exmaple:

Let $X = \{1,2,3,4,5\}$ **and** $T = \{\phi, X, \{1,5\}, \{2,3\}, \{1,2\}, \{2\}, \{1\}, \{1,2,3,5\}, \{1,2,3\}\}$**.**

Is T **a topology on** X **?**

Solution:

T **is not a topology on** X **,since** $\{1,5\} \cup \{1,2\} = \{1,2,5\} \notin T$ **.**

1.12Exmaple:

Let $X=\{1,2,3,4\}$ **and** $T=\{\phi, X, \{1,4\}, \{2,3\}, \{1,2\}, \{2\}, \{1\}, \{1,2,3\}, \{1,2,4\}\}$**.**

Is T **a topology on** X **?**

Solution:

T **is a topology on** X **,since the conditions are satisfied.**

1.13Exmaple:

Let $X=\{1,2,3,4,5\}$ **and** $T=\{\phi, X, \{2\}, \{1\}, \{1,,5\}, \{4,5\}\}$**.**

Complete T **to be a topology on** X **?**

Solution:

$T=\{\phi, X, \{2\}, \{1\}, \{1,,5\}, \{4,5\}, \{5\}, \{1,2\}, \{1,2,5\}, \{2,5\}, \{2,4,5\}, \{1,4,5\}, \{1,2,4,5\}\}$

1.14Exercises:

1. Let $X=\{1,2,3,4\}$ **and** $T=\{\phi, X, \{3\}, \{2\}, \{1,4\}, \{2,4\}\}$**.**

Complete T **to be a topology on** X **?**

2. Let $X=\{1,2,3,4,5\}$ **and** $T=\{\phi, X, \{3\}, \{1\}, \{4,,5\}, \{2,4\}\}$**.**

Complete T **to be a topology on** X **?**

1.15Exmaple: Let X **be infinite set and** $T=\{\phi\}\cup\{A : A\subseteq X \; and \; A^C \; finite \; set\}$**, then** T **is a topology on** X **and** T **is called the Co-finite topology. Since**

1. a. $\phi\in T \; by \; definition \; T$ **.** **b.** $\because X^C=\phi \; and \; \phi \; is \; finite \; set \; \therefore X\in T$

2. If $G_\alpha\in T \quad \forall\alpha \quad \therefore G_\alpha{}^C \; is \; finite \; set \; \forall\alpha \quad \therefore \bigcap_\alpha G_\alpha{}^C \; is \; finite \; set.$

$$But \quad \bigcap_\alpha G_\alpha{}^C=\left(\bigcup_\alpha G_\alpha\right)^C$$

$\therefore \left(\bigcup_{\alpha} G_\alpha\right)^C$ **is finite set,** $\quad \therefore \bigcup_{\alpha} G_\alpha \in T$

3.If $G_i \in T \quad \forall 1 \le i \le n \quad \therefore G_i^{\ C}$ $is \quad finite set \quad \forall 1 \le i \le n$

$\therefore \bigcup_{i=1}^{n} G_i^{\ C}$ $is \quad finite \qquad But \quad \bigcup_{i=1}^{n} G_i^{\ C} = \left(\bigcap_{i=1}^{n} G_i\right)^C$

$\therefore \left(\bigcap_{i=1}^{n} G_i\right)^C$ **is finite set,** $\quad \therefore \bigcap_{i=1}^{n} G_i \in T$

<u>1.16Exercise:</u> Let X **be infinite set and** $T = \{\phi\} \cup \{A : A \subseteq X \ and \quad A^C \ countable \quad set\}$**, prove that** T **is a topology on** X **and** T **is called the Co-countable topology.**

<u>1.17Exmaple:</u> Let (R,d) **is the usual metric space and** $T = \{A : A \subseteq R and \quad A is \quad open \quad set \quad in R \quad \}$**, then** T **is a topology on** R **and** T **is called the usual topology. Since**

1. a. $\because \phi is \quad open set and \quad \phi \subset R \quad \therefore \phi \in T.$ **b.** $\because R is \quad open set and \quad R \subset R \quad \therefore R \in T.$

2. If $G_\alpha \in T \quad \forall \alpha \qquad \therefore G_\alpha \ is open \quad set \quad in \quad R \quad \forall \alpha$

$\therefore \bigcup_{\alpha} G_\alpha \ is open \quad set \quad in \quad R$.**(By theorem in mathematical analysis)**

$\because \bigcup_{\alpha} G_\alpha \subset R \qquad \therefore \quad \bigcup_{\alpha} G_\alpha \in T$

3. If $G_i \in T \quad \forall 1 \le i \le n \qquad \therefore G_i \ is open \quad set \quad in \quad R \quad \forall 1 \le i \ge n$

$\therefore \bigcap_{i=1}^{n} G_i \quad is open \quad set \quad in \quad R$ **(By theorem in mathematical analysis)**

$\because \bigcap_{i=1}^{n} G_i \subset R \qquad \therefore \quad \bigcap_{i=1}^{n} G_i \in T$

<u>1.18Exercise:</u> Let $X = N$ **and** $T = \{\phi, N\} \cup \{\{1,2,3,...,n\}, n \in N\}$**,**
i.e. $T = \{\phi, N\} \cup \{\{1\}, \{1,2\}, \{1,2,3\}, \{1,2,3,4\}, \{1,2,3,4,5\}, ...\}$ **. Is** T **a topology on** X **?**

<u>1.19Definition:</u> **Let** (X,T) **be a topological space and** $A \subseteq X$ **, then the point** $x \in X$ **is a limit point to** A **iff** $\forall G \in T \quad , x \in G \ni \quad G - \{x\} \cap A \neq \phi$.

<u>1.20Notes:</u> **The set of limit points of** A **is denoted by** A' **.** A' **is called the derived set of** A **.**

<u>1.21Exmaple:</u>

Let $X = \{1,2,3,4\}$ **and** $T = \{\phi, X, \{1,2,3\}, \{1,3\}, \{1,2\}, \{1\}, \{3\}\}$ **is a topology on** X **.Find**

1) A' **If** $A = \{1,3\}$.
2) B' **If** $B = \{4\}$.
3) C' **If** $C = \{1,2,3\}$.

Solution:1)

a. Take the point $x = 1$**. Open sets contains** $x = 1$ **are**

$$X \quad \{1\} \quad \{1,2\} \quad \{1,3\} \quad \{1,2,3\}$$

$$X - \{1\} \cap A \neq \phi$$
$$\{1\} - \{1\} \cap A = \phi$$

$\therefore x = 1$ **is not limit point to** A **.**

b. Take the point $x = 2$**. Open sets contains** $x = 2$ **are**

$$X \quad \{1,2\} \quad \{1,2,3\}$$

$$X - \{2\} \cap A \neq \phi$$
$$\{1,2\} - \{2\} \cap A \neq \phi$$
$$\{1,2,3\} - \{2\} \cap A \neq \phi$$

$\therefore x = 2$ **is a limit point to** A **.**

c. Take the point $x = 3$**. Open sets contains** $x = 3$ **are**

$X \quad \{3\} \qquad \{1,3\} \quad \{1,2,3\}$

$X - \{3\} \cap A \neq \phi$

$\{3\} - \{3\} \cap A = \phi$

$\therefore x = 3$ **is not limit point to** A **.**

d. Take the point $x = 4$**. Open sets contains** $x = 4$ **are**

X

$X - \{4\} \cap A \neq \phi$

$\therefore x = 4$ **is a limit point to** A **.**

$\therefore A' = \{2,4\}$.

2)

a. Take the point $x = 1$**. Open sets contains** $x = 1$ **are**

$X \quad \{1\} \quad \{1,2\} \quad \{1,3\} \quad \{1,2,3\}$

$X - \{1\} \cap B \neq \phi$

$\{1\} - \{1\} \cap B = \phi$

$\therefore x = 1$ **is not limit point to** B **.**

b. Take the point $x = 2$**. Open sets contains** $x = 2$ **are**

$X \quad \{1,2\} \quad \{1,2,3\}$

$X - \{2\} \cap B \neq \phi$

$\{1,2\} - \{2\} \cap B = \phi$

$\therefore x = 2$ **is not limit point to** B **.**

c. Take the point $x = 3$**. Open sets contains** $x = 3$ **are**

$X \quad \{3\} \qquad \{1,3\} \quad \{1,2,3\}$

$X-\{3\}\bigcap B \neq \phi$

$\{3\}-\{3\}\bigcap B = \phi$

$\therefore x=3$ **is not limit point to** B **.**

d. Take the point $x=4$**. Open sets contains** $x=4$ **are only**

X

$X-\{4\}\bigcap B = \phi$

$\therefore x=4$ **is not limit point to** B **.**

$\therefore B' = \phi$ **.**

3) Exercise.

<u>1.22Theorem:</u> Let (X,T) **be a topological space and** $A, B \subseteq X$ **.**

1) $\phi' = \phi$ **.**
2) **If** $A \subset B$ **, then** $A' \subset B'$ **.**
3) **If** $x \in A'$ **, then** $x \in (A-\{x\})'$ **.**
4) $(A \bigcup B)' = A' \bigcup B'$ **.**

Proof:

1) $\forall x \in X \quad and \quad \forall \quad G \in T \quad , \quad x \in G \quad \ni \quad G-\{x\}\bigcap \phi = \phi$ **.**

$\therefore x$ **is not limit point to** ϕ

$\therefore \phi' = \phi$

2) **Let** $x \in A'$

$\therefore \quad x$ **is a limit point to** A

$\therefore \quad \forall G \in T \quad , x \in G \ni \quad G - \{x\} \bigcap A \neq \phi$

$\because \quad A \subset B \quad by \quad \text{hypothesis}$

$\therefore \quad \forall G \in T \quad , x \in G \ni \quad G - \{x\} \bigcap B \neq \phi$

$\therefore \quad x$ **is a limit point to** B

$\therefore \quad x \in B'$

$\therefore \quad A' \subset B'$

3) Let $x \in A'$

$\therefore \quad x$ **is a limit point to** A

$\therefore \quad \forall G \in T \quad , x \in G \ni \quad G - \{x\} \bigcap A \neq \phi$

$\therefore \quad \forall G \in T \quad , x \in G \ni \quad G - \{x\} \bigcap A - \{x\} \neq \phi$

$\therefore \quad x$ **is a limit point to** $A - \{x\}$

$\therefore \quad x \in \left(A - \{x\}\right)'$

4) We must prove that

a. $A' \bigcup B' \subset \left(A \bigcup B\right)'$ **b.** $\left(A \bigcup B\right)' \subset A' \bigcup B'$

Proof a: $\because \quad A \subset A \bigcup B$

$\therefore \quad A' \subset \left(A \bigcup B\right)' \, by \quad part 2$ **…(1)**

$\because \quad B \subset A \bigcup B$

$\therefore \quad B' \subset \left(A \bigcup B\right)' \, by \quad part 2$ **…(2)**

From (1) and (2) we get

$$A' \bigcup B' \subset (A \bigcup B)'$$

Proof b: **Suppose** $x \notin A' \bigcup B'$

$$\therefore \quad x \notin A' \wedge \quad x \notin B'$$

$\because x \notin \quad A'$ $\therefore$ x **is not limit point to** A

$$\therefore \quad \exists O \in T \quad , x \in O \ni \quad O - \{x\} \bigcap A = \phi$$

$\because x \notin \quad B'$ $\therefore$ x **is not limit point to** B

$$\therefore \quad \exists V \in T \quad , x \in V \ni \quad V - \{x\} \bigcap B = \phi$$

$$Let \quad G = O \bigcap V$$

$$\therefore x \in G \quad and\, G \in T \quad \ni G - \{x\} \bigcap (A \bigcup B) = \phi$$

$\therefore$ x **is not limit point to** $A \bigcup B$

$$\therefore \quad x \notin (A \bigcup B)'$$

$$\therefore \quad (A \bigcup B)' \subset A' \bigcup B'$$

From (a) and (b) we get

$$(A \bigcup B)' = A' \bigcup B'$$

<u>1.23Exercise:</u> Let (X, T) **be a topological space and** $A, B \subseteq X$ **.**

Is $(A \bigcap B)' = A' \bigcap B'$ **?**

<u>1.24Definition:</u> **Let** (X,T) **be a topological space and** $F \subseteq X$ **, then** F **is closed iff** $F' \subset F$ **.**

<u>1.25Theorem:</u> **Let** (X,T) **be a topological space and** $F \subseteq X$ **,** F **is closed.**

If $x \notin F$ **, then** $\exists G \in T \quad \ni \quad x \in G \subset F^C$ **.**

Proof: $\because$ F **is closed,** $\therefore$ $F' \subset F$

$\because$ $x \notin F$ $\therefore$ $x \notin F'$

$\therefore$ x **is not limit point to** F **.**

$\therefore$ $\exists G \in T \quad , x \in G \ni \quad G - \{x\} \bigcap F = \phi$

$\because$ $x \notin F$

$\therefore$ $G \bigcap F = \phi$

$\therefore$ $x \in G \subset F^C$

<u>1.26Corollary:</u> **Let** (X,T) **be a topological space and** $F \subseteq X$ **, then** F **is closed iff** F^C **is open.**

Proof: $\Longrightarrow$ **If** F **is closed. To prove** F^C **is open**

Let $x \in F^C$ **,** $\therefore$ $x \notin F$

$\because$ F **is closed**

$\exists G_x \in T \quad \ni \quad x \in G_x \subset F^C$ **(By1.25Theorem)**

$$\therefore \quad F^C = \bigcup_{x \in F^C} G_x$$

$$\because \quad \bigcup_{x \in F^C} G_x \; is \quad open$$

$\therefore \quad F^C \; is \quad open$

$\Leftarrow$ **If F^C is open. To prove F is closed**

Suppose F is not closed

$\therefore \; \exists x$ **is limit point to F , $x \notin F$**

$\therefore \quad x \in F^C$ **,** $\because \quad F^C$ **is open and** $x \in F^C$

$\therefore \quad F^C - \{x\} \bigcap F = \phi$

$\therefore \quad x$ **is not limit point to F and which is contradiction**

$\therefore \quad F$ **is closed.**

<u>1.27Corollary:</u> Let(X,T) be a topological space and
$\Psi = \{F \;:\; F \subseteq X \quad and \quad F \quad is \quad closed\}$**, then**

1. $\bigcap_{\alpha} F_\alpha \in \Psi \qquad \forall \alpha \quad , F_\alpha \in \Psi$

2. $\bigcup_{i=1}^{n} F_i \in \Psi \quad \forall 1 \le i \le n, \quad F_i \in \Psi$

Proof: 1.

Suppose $F_1 \;,\; F_2 \;,\; F_3 \;,\; \ldots$ **closed sets in** (X,T) **.**

$\therefore \quad {F_1}^C \;,\; {F_2}^C \;,\; {F_3}^C \;,\; \ldots$ **Open sets by corollary1.26**

$\therefore \quad \bigcup_{\alpha} {F_\alpha}^C$ **Open set by definition 1.1**

But $\bigcup_{\alpha} {F_\alpha}^C = \left(\bigcap_{\alpha} F_\alpha \right)^C$

$\therefore \quad \left(\bigcap_{\alpha} F_{\alpha}\right)^{C}$ **Open set**

$\therefore \quad \bigcap_{\alpha} F_{\alpha}$ **Closed set by corollary 1.26**

2.

Suppose $F_1 \quad , \quad F_2 \quad , \quad F_3 \quad , \quad ...F_n$ **closed sets in** (X,T) **.**

$\therefore \quad F_1^{\,C} \quad , \quad F_2^{\,C} \quad , \quad F_3^{\,C} \quad , \quad ...F_n^{\,C}$ **Open sets by corollary1.26**

$\therefore \quad \bigcap_{i=1}^{n} F_i^{\,C}$ **Open set by definition 1.1**

But $\bigcap_{i=1}^{n} F_i^{\,C} = \left(\bigcup_{i=1}^{n} F_i\right)^{C}$

$\therefore \quad \left(\bigcup_{i=1}^{n} F_i\right)^{C}$ **Open set**

$\therefore \quad \bigcup_{i=1}^{n} F_i$ **Closed set by corollary 1.26**

<u>1.28Definition:</u> **Let** (X,T) **be a topological space and** $A \subseteq X$ **, then the closure of** A **denoted by** $\overline{A}$ **or** $C(A)$ **and we define the closure of** A **by** $\overline{A} = \bigcap_{i} F_i$

$\ni \quad F_i \quad closed \quad set \quad and \quad A \subseteq F_i \quad \forall \quad i$

$\overline{A}$ **Closed set by corollary 1.27.**

<u>1.29Exmaple:</u> **Let** $X = \{1,2,3,4\}$ **and** $T = \{\phi, X, \{1,2,3\}, \{1,3\}, \{1,2\}, \{1\}, \{2\}\}$ **is a topology on** X **.Find**

1) $\overline{A}$ **If** $A=\{4\}$.

2) $\overline{B}$ **If** $B=\{2,3\}$.

3) $\overline{C}$ **If** $C=\{1,2,4\}$.

4) $\overline{D}$ **If** $D=\{1,2,3\}$.

5) $\overline{E}$ **If** $E=\{1,2,3\}$.

Solution: Closed sets are

$$X,\phi,\quad \{4\},\quad \{2,4\},\quad \{3,4\},\quad \{2,3,4\},\quad \{1,3,4\}$$

1) **Closed sets which contained** $A=\{4\}$ **are**

$$X,\quad \{4\},\quad \{2,4\},\{3,4\}\quad ,\quad \{2,3,4\},\quad \{1,34\}$$

$$\therefore\quad \overline{A}=X\bigcap\{4\}\bigcap\{2,4\}\bigcap\{3,4\}\bigcap\{2,3,4\}\bigcap\{1,3,4\}=\{4\}$$

2) **Closed sets which contained** $B=\{2,3\}$ **are**

$$X,\{2,3,4\}$$

$$\therefore\quad \overline{B}=X\bigcap\{2,3,4\}=\{2,3,4\}$$

3) **Closed sets which contained** $C=\{1,2,4\}$ **are**

$$X$$

$$\therefore\quad \overline{C}=X$$

4) **Exercise**

5) **Exercise**