Systems with two or more degrees of freedom

when a system requires more than one coordinate to describe its motion it is called a multi-degree of freedom system or an N-DOF.

N-DOF has N-natural frequencies and N-normal modes

$$\left.\begin{array}{l}\text{Natural frequencies} \rightarrow \text{eigenvalues} \\ \text{Normal modes} \rightarrow \text{eigenvectors}\end{array}\right\} \text{normal mode analysis}$$

Normal mode vibrations are free undamped vibrations that depend only on the mass and stiffness of the system and how they are distributed.

Generalized coordinates:

They are any set of independent coordinates or parameters such as length, angle or some other physical parameters (q)

Generally generalized coordinates = No. of degrees of freedom

Normal mode analysis: (modal analysis)

The are two methods for obtaining the equations of motion necessary for normal mode analysis

(1)

(a) Newton's Second law

for translation $\Sigma F = m\ddot{x}$
(+ve) forces are in the direction of motion and vise versa
for rotation $\Sigma M = J\ddot{\theta}$
(+ve) Moments are in the direction of rotation

(b) Lagrange's equation

Generally

$$\frac{d}{dt}\left(\frac{\partial KE}{\partial \dot{q}}\right) - \frac{\partial KE}{\partial q_i} + \frac{\partial PE}{\partial q_i} + \frac{\partial DE}{\partial q_i} = Q_i$$

where
KE - Kinetic energy of the System $= \frac{1}{2} m\dot{x}^2$
PE - Potential energy of the System $= \frac{1}{2} kx^2$
DE - Dissipation energy of the system $= \frac{1}{2} c\dot{x}^2$
For conservative system DE = zero
Q_i - Generalized external force acting on the system.
for free vibration Q_i = zero
q - generalized coordinates