

Sheet of Problems in Vibration

Prepared by

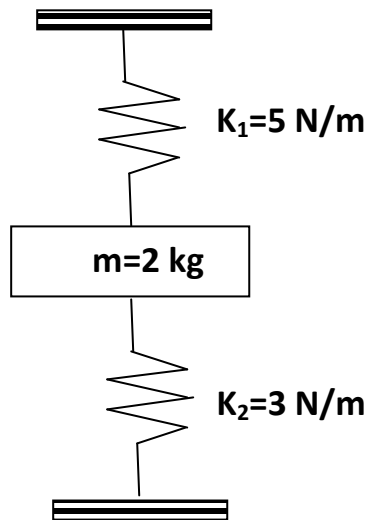
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(a) Find the natural frequency of the system shown in figure (1).

Figure (1)



Ans. 2 rad/sec

(b) What is the displacement of the mass from the equilibrium position at time 3 sec if the initial conditions are $x(0) = 0.003 \text{ m}$ and $\dot{x} = 0$.

Ans. 0.00288 m

(2) Derive an expression for the natural frequency of each of the following systems shown in the figures (2) and (3) assuming massless beams.

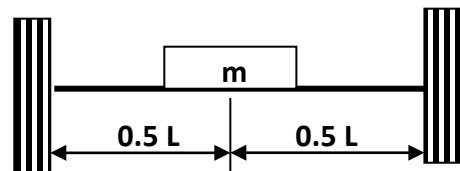
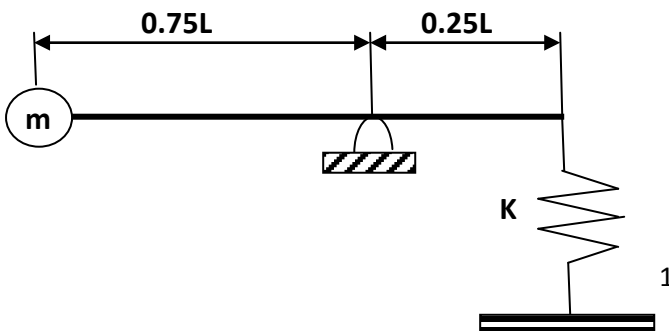
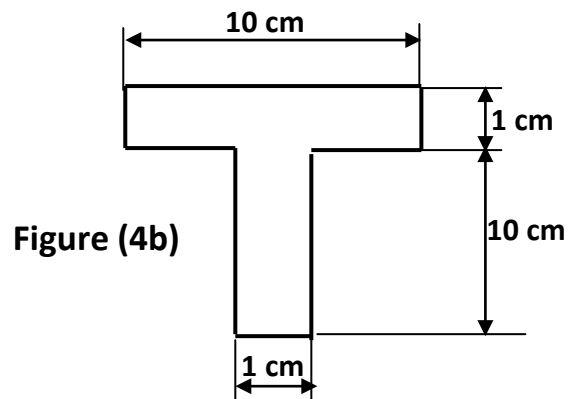
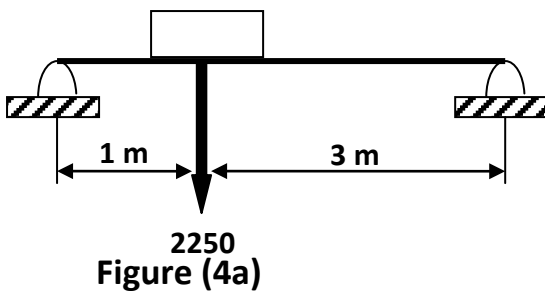


Figure (2) Ans. $\omega_n = \sqrt{\frac{K}{9m}}$

Figure (3) Ans. $\omega_n = \sqrt{\frac{192EI}{mL^3}}$

(3) Calculate the frequency of the free vibration for the system shown in figures (4). Neglect the mass of the beam and take $E_{\text{beam}} = 7 \times 10^{10} \text{ N/m}^2$.

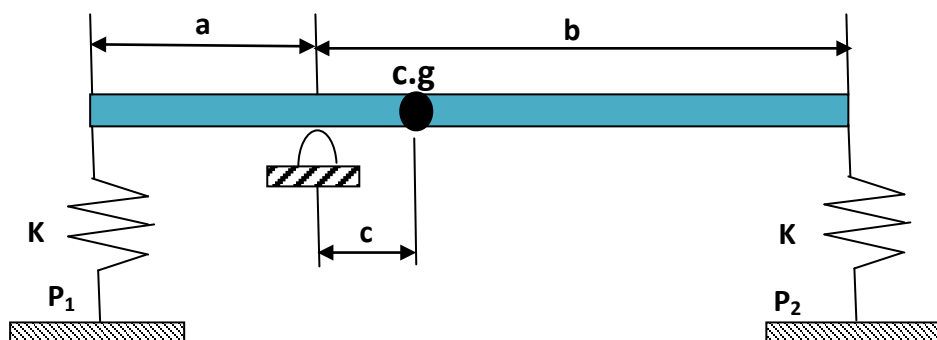
Ans. 4.9235 Hz



(4) The figure (5) shows a uniform bar pivoted about point O with springs of equal stiffness K at each end. The bar is horizontal in equilibrium position with spring forces P1 and P2. Determine the equation of motion and its natural frequency.

Ans. $\omega_n = \sqrt{\frac{K(a^2+b^2)}{J_o}}$

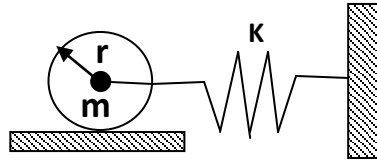
Figure (5)



(5) A circular disc of mass m and radius r is connected by a spring of modulus K as shown in the figure (6). If it is free to roll on a rough horizontal surface without slipping, find its natural frequency.

Ans. $\omega_n = \sqrt{\frac{2K}{3m}}$

Figure (6)



(6) Find the natural frequency of the system shown in the figure (7) in terms of the available variables.

Ans. $\omega_n = \sqrt{\frac{Kr^2}{R^2(m+0.5M)}}$

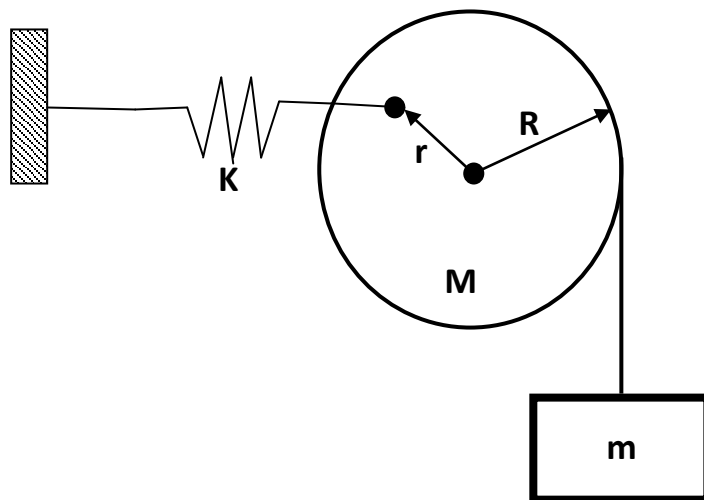


Figure (7)

(7) An amplitude meter consists of a seismic mass suspended as shown in the figure (8). Determine the natural frequency in Hz of the meter in terms of the tension spring K_1 , the compression spring K_2 , the mass m and the mass moment of inertia I of the pointer about O .

Ans. $f_n = \frac{1}{2\pi} \sqrt{\frac{K_1 a^2 + 2K_2 b^2}{I + ma^2}}$

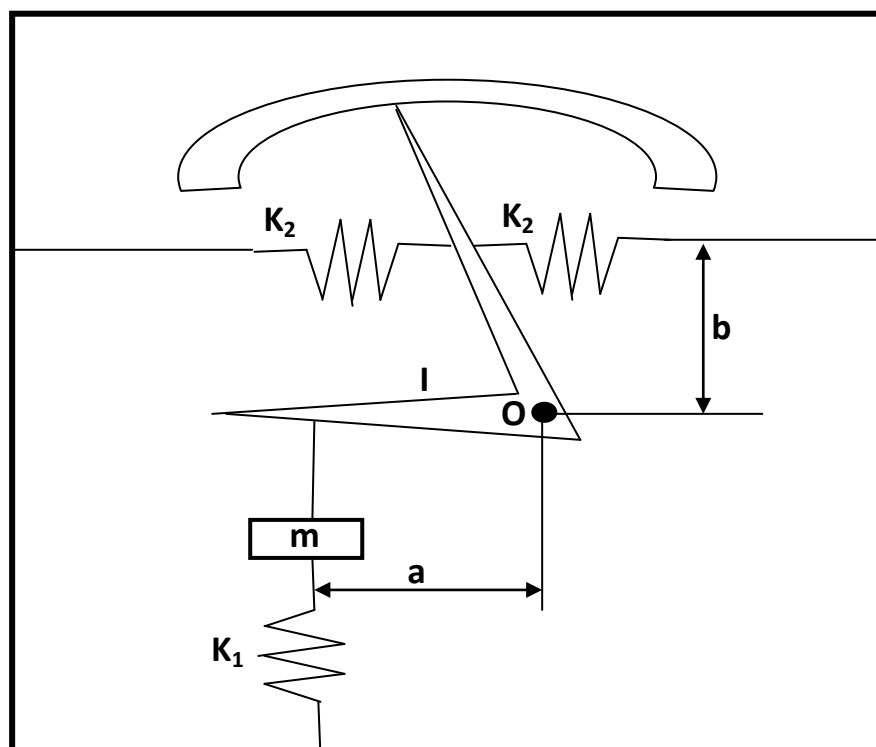
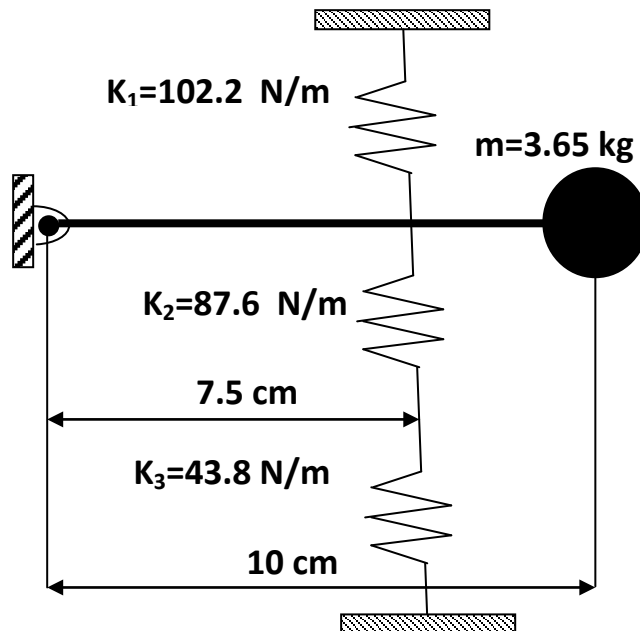


Figure (8)

(8) For the system shown in the figure (9), derive and solve the differential equation of motion with the initial conditions $\theta_o = 0.1 \text{ rad}$ and $\dot{\theta}_o = 0.6 \text{ rad/sec}$. Neglect the weight of the rod.

Ans. $\theta = 0.16667 \cos(4.5t - 0.9273)$

Figure (9)



(9) A semi circular thin shell of radius R is allowed to rock on a rough horizontal surface. Derive the differential equation of small oscillation and calculate the natural frequency. Ans. $\omega_n = \sqrt{\frac{g}{R(\pi-2)}}$

(10) For the system shown in the figure (10), find the natural frequency of the system. Ans. $\omega_n = \sqrt{\frac{2K(a+b)^2}{m(2(a+b)^2+a^2)}}$

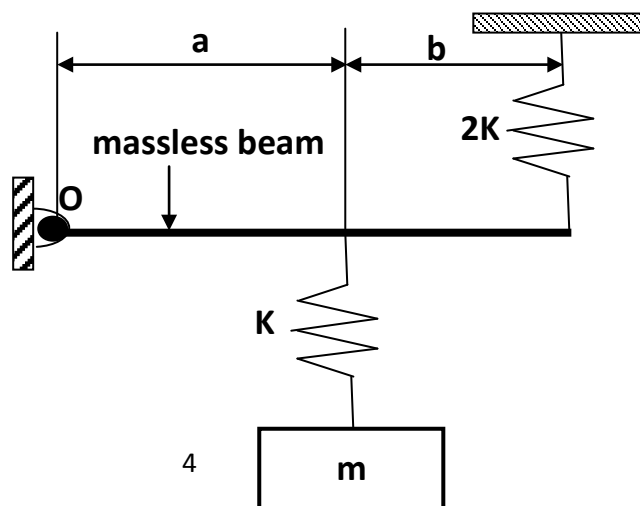
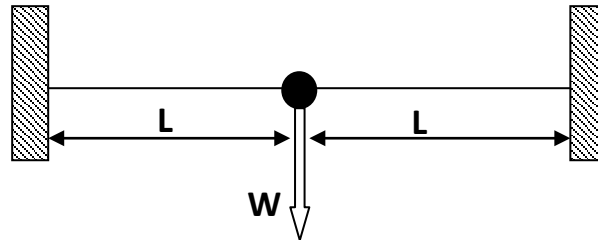


Figure (10)

(11) An elastic string of length $2L$ tightly stretched between two rigid supports as shown in the figure (11). It carries a small ball of weight W at its midpoint, compute the period. Ans. $T = 2\pi \sqrt{\frac{WL}{2Tg}}$

Figure (11)



(12) Obtain the natural frequency of motion of fluid column L with mass density ρ through a manometer of uniform bore of cross sectional area A .

$$\text{Ans. } \omega_n = \sqrt{\frac{2g}{L}}$$

(13) Show that for the damped spring mass system the peak amplitude occurs at a frequency ratio given by the expression $(\frac{\omega}{\omega_n})_p = \sqrt{1 - 2\xi^2}$

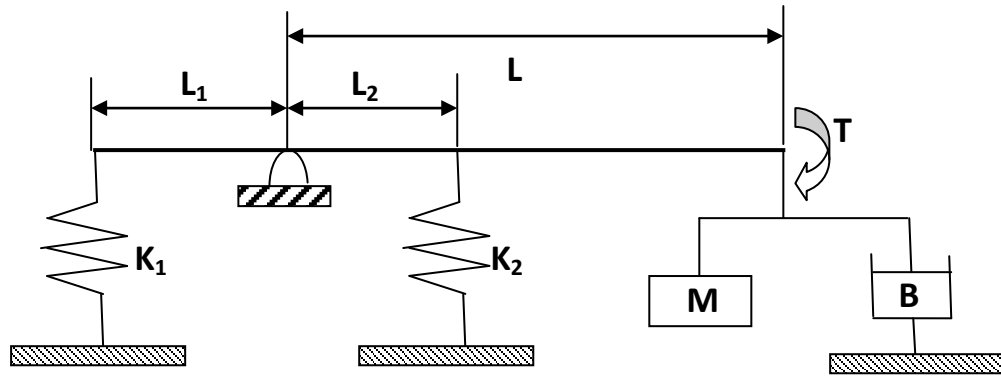
(14) For the system shown in the figure (12), assuming massless beam, it is required the followings:-

(a) the equation of motion (b) the damping ratio ξ and the natural frequency ω_n in terms of parameters given in the figure (c) the value of B such that the system is critically damped and (d) the damped natural frequency.

$$\text{Ans. } \omega_n = \sqrt{\frac{K_1 L_1^2 + K_2 L_2^2}{ML^2}} \quad \xi = \frac{BL}{2\sqrt{M(K_1 L_1^2 + K_2 L_2^2)}} \quad B = \frac{2}{L} \sqrt{M(K_1 L_1^2 + K_2 L_2^2)}$$

$$\omega_d = \frac{1}{2ML} \sqrt{4M(K_1 L_1^2 + K_2 L_2^2) - B^2 L^2}$$

Figure (12)

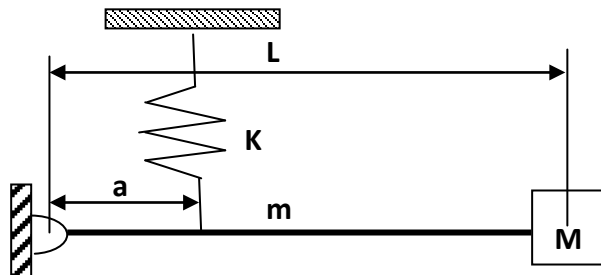


(15) The period of damped linear oscillation for a certain 1 kg mass is 0.32 sec. If the stiffness of the supporting linear spring is 850 N/m, calculate the damping coefficient C and the critical value C_{cr} of the motion.

Ans. $C = 43.1 \text{ N.sec/m}$ $C_{cr} = 58.322 \text{ N.sec/m}$

(16) Find the natural frequency of the system shown in the figure (13) using the energy method, given that the mass of the rod is m and the concentrated mass M and the coefficient of stiffness of the spring is K . Ans. $\omega_n = \sqrt{\frac{Ka^2}{L^2(M + \frac{m}{3})}}$

Figure (13)



(17) The following data are given for a vibrating system with viscous damping:- mass=2 kg, stiffness of the spring=4 kN/mm and damping coefficient=10 N.sec/cm. Determine the logarithmic decrement and the ratio of any consecutive amplitudes. Ans. $\delta = 1.12845$ $\frac{x_1}{x_2} = 3.0908$

(18) A mass less flexible beam is hinged at one end and carries a mass m at the other end as shown in the figure (14). The beam is caused to vibrate by displacing the mass and releasing from rest by 1cm, determine:-

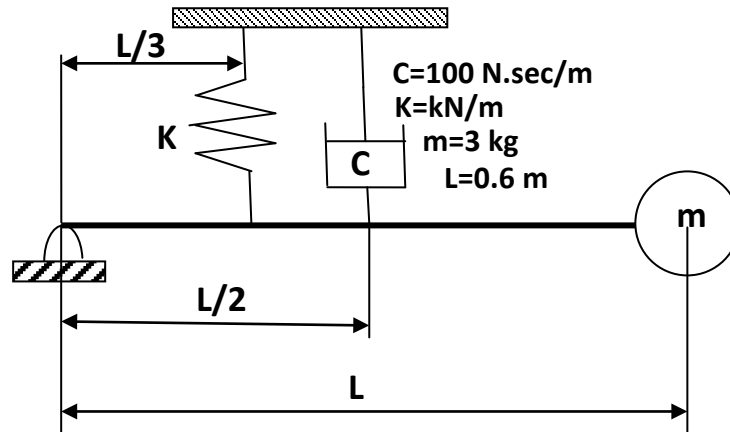
(a) The damped frequency of vibration in Hz. Ans. $\omega_d = 0.7028 \text{ Hz}$

(b) The ratio of any consecutive cycles. Ans. $\frac{x_1}{x_2} = 365.037$

(c) The general solution of the vibrating system.

Ans. $\theta = 0.02287e^{-4.1663t}[\cos(4.4158t - 0.754)]$

Figure (14)



(19) A machine part weighing 4.3 lb vibrates in a viscous medium. Determine the damping coefficient when a harmonic exciting force of 5.5 lb results in resonant amplitude of 0.5 in with a period of 0.2 second.

Ans. $C = 3.82 \text{ lb}\cdot\text{sec}/\text{ft}$

(20) A spring mass system is excited by a force $F_o \sin \omega t$. At resonant the amplitude is measured to be 0.58 in. At 0.8 resonant frequency, the amplitude is measured to be 0.46 in. Determine the viscous damping factor of the system. (Hint:- assume damping term is negligible at 0.8 resonance)

Ans. $\xi = 0.14275$

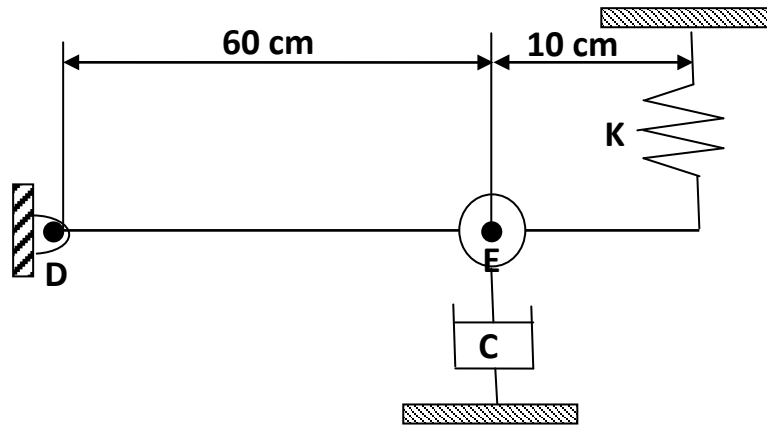
(21) The 4 kg mass E is fastened to the rod DF whose weight can be neglected as shown in the figure (15). The spring has stiffness 1.5 N/m and the coefficient of the dash pot is 0.5 N.sec/m. The system is in equilibrium when DF is horizontal. The rod is displaced 0.1 rad clockwise and release from rest, determine:-

(a) The equation of motion of the rod.

(b) The damped circular frequency of the rod. Ans. $f_d = 0.1136 \text{ Hz}$

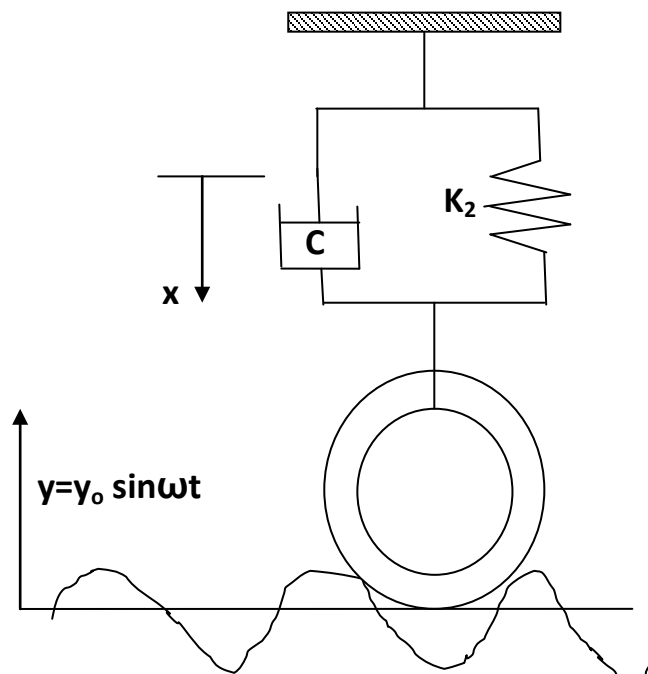
(c) The damping ratio. Ans. $\xi = 0.0875$

Figure (15)



(22) The figure (16) shows the front wheel drive of an automobile with spring and damper. The automobile is running on a sinusoidal road with a function $y = y_0 \sin \omega t$. Consider the tyre as a spring with stiffness K_1 and mass m , find an expression for the critical damping of the system in terms of the given variables. Ans. $C_{cr} = 2\sqrt{m(K_1 + K_2)}$

Figure (16)



(23) An automobile has a body weighing 3000 lb mounted on four equal springs which sag 9 in under the weight of the body. Each one of the shock absorbers has a damping coefficient of 7 lb for a velocity 1 in per second. The car is placed with all its four wheels on a test plate form which is moved up and down at resonant speed of amplitude 1 inch. Find the amplitude of the car body. Assuming that the center of gravity to be in the center of the wheel base. Ans. 2.0733 inch

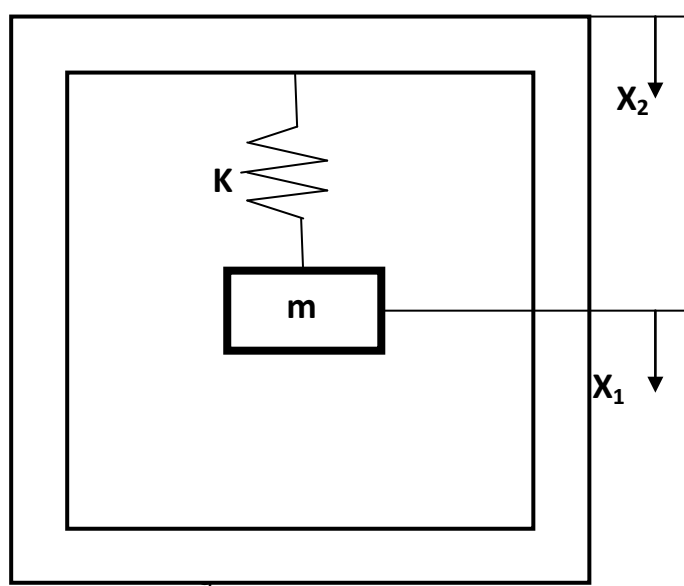
(24) A spring mounted oscillator with a natural undamped frequency of 4 Hz and viscous damping factor of 0.2 is mounted on a frame which vibrates with frequency f which can be varied. If the amplitude of the absolute vibration of the oscillator is not to exceed twice that of the frame specify the permissible range of frequencies of the frame.

$$\text{Ans. } 3.02 \text{ Hz} \leq f \leq 4.5704 \text{ Hz}$$

(25) A simplified spring mass vibration pickup as shown in the figure (17) is used to measure the vertical acceleration of a train which has vertical frequency of 10 rad/sec. The mass is 1.202 kg and the modulus of the spring is 1500 N/m. The amplitude of the relative motion of the mass is 0.12 cm as recorded by the instrument. Find the maximum vertical acceleration of the train. What is the amplitude of vibration of the train?

$$\text{Ans. } A=137.8 \text{ cm/sec}^2 \quad x_2=1.378 \text{ cm}$$

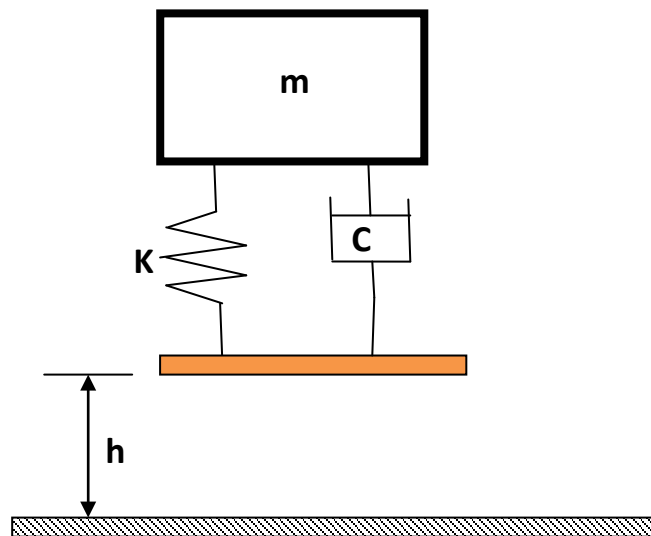
Figure (17)



- (26) If the system shown in the figure (18) is dropped from an elevation h to a hard surface, what will be the resulting motion of the mass m ?

$$\text{Ans. } x = \frac{\sqrt{2gh}}{\omega_d} e^{\frac{-ct}{2m}} \sin \omega_d t$$

Figure (18)



- (27) A weight attached to a spring of stiffness 3 lb/in has a viscous damping device. When the weight is displaced and released, the period of vibration is found 1.8 sec and the ratio of consecutive amplitudes is 4.2 to 1. Determine the amplitude and phase when a force $F=2\cos 3t$ lb acts on the system. Ans. $X=1.3964$ inch $\phi=51.41177$ degree
- (28) A piston of mass 4.53 kg is travelling in a tube with a velocity of 15.24 m/sec and engages a spring and damper as shown in the figure (19). Determine the maximum displacement of the piston after engaging the spring-damper. How many seconds does it take?

$$\text{Ans. } X=0.703 \text{ mm } t=0.1665 \text{ sec}$$

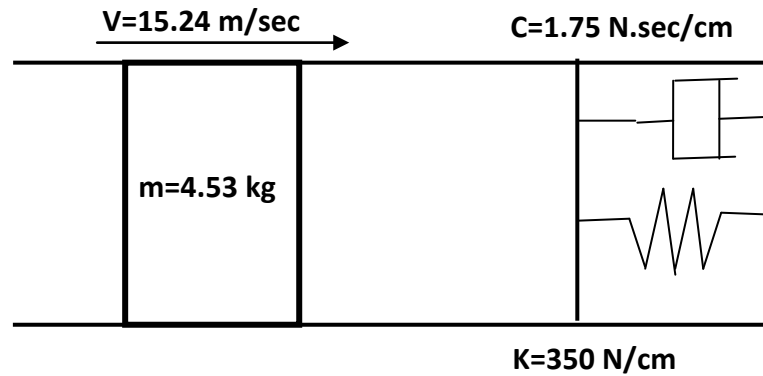


Figure (19)

- (29) An under damped spring-mass system is given a base excitation of $y' = 20(1 - 5t)$ m/sec . If the natural frequency for the system is $\omega_n = 10 \text{ sec}^{-1}$ determine the maximum relative displacement. Ans. $Z_{\max} = -1.236 \text{ m}$
- (30) A given beam was vibrated by an eccentric mass shaker of mass 5.44 kg at the midspan, and resonance was found at 435 cps. With an additional mass of 4.52 kg, the resonance frequency was lowered to 398 cps. Determine the natural frequency of the beam and the stiffness of the shaker in MN/m.
- (31) For a 3-DOF system, the mass and flexibility matrices are:-

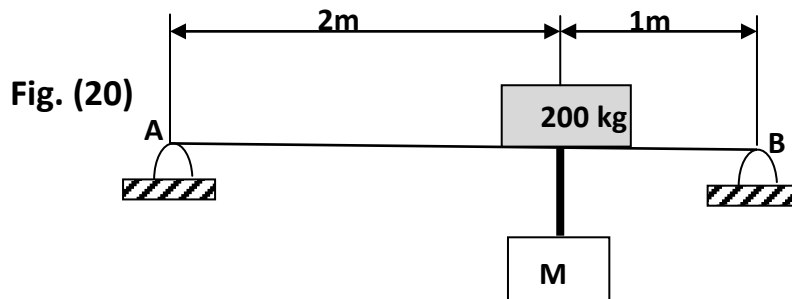
$$[M] = m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [a] = \frac{1}{k} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}. \text{ The natural mode shapes are:-}$$

$$\Theta_1 = \begin{bmatrix} 1 \\ 1.8019 \\ 2.247 \end{bmatrix}, \Theta_2 = \begin{bmatrix} 1 \\ 0.445 \\ -0.802 \end{bmatrix} \text{ and } \Theta_3 = \begin{bmatrix} 1 \\ -1.2468 \\ 0.5544 \end{bmatrix}$$

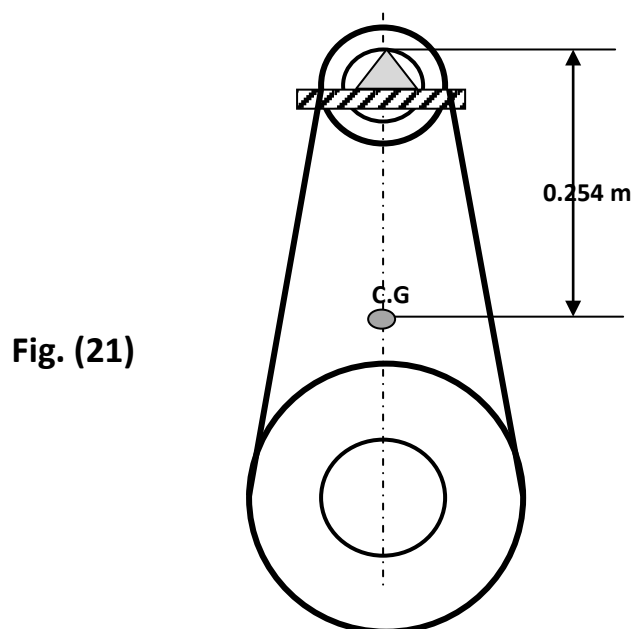
Find the stiffness matrix and the orthonormal modes.

- (32) A 200-kg machine is placed on a mass less simply supported beam as shown in Fig. (20). The machine has a rotating unbalance of 1.41 kg.m and operates at 3000 rpm. The steady state vibrations of the machine are to be absorbed by hanging a mass to 40-cm steel cable from the location on the beam where the machine is

attached. What is the required diameter of the cable such that the machine vibrations are eliminated at 3000 rpm and the amplitude of the absorber is less than 50 mm? Take $E=200\text{GN/m}^2$



- (33) The blade of a fan can be considered as a single degree of freedom system with mass of 0.25 kg, stiffness of 100 N/cm and damping of 1000 Dyn.sec/mm. If the blade is suddenly subjected to a unit step excitation with an initial displacement of 0.25 cm, find the transient response of the blade using Laplace transformation.
- (34) A 5-kg rotor balancing machine is mounted on a table through an elastic foundation of stiffness 3.1×10^4 N/m and damping ratio 0.04. Transducer indicates that the table on which the machine is placed vibrates at a frequency of 110 rad/sec with an amplitude of 0.62 mm. What is the steady state amplitude of acceleration of the balancing machine?
- (35) A connecting rod weighing 21.35 N oscillates 53 times in one minute when suspended as shown in Fig. (21). Determine its moment of inertia about its center of gravity which is located 0.254 m from the point of the support.



(36) The ratio of successive amplitudes of a viscously damped single degree of freedom system is found to be 18:1. Determine the ratio of successive amplitudes if the amount of damping is (a) doubled and (b) halved.

Ans. (a) 14365.0763 (b) 3.83

(37) A uniform slender rod of mass m and length L is hinged at point A and is attached to four linear springs and one torsional spring as shown in Fig. (22). Using the energy method, find the natural frequency of the system if $K=2000$ N/m, $K_t=1000$ N.m/rad, $m=10$ kg and $L=5$ m. Ans. $\omega_n=45.122$ rd/sec

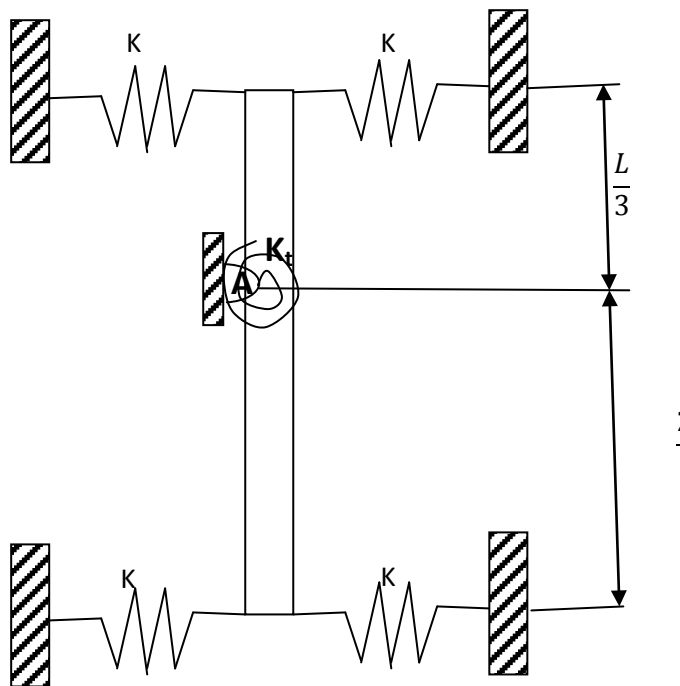


Fig. (22)

- (38) A uniform rigid bar of mass m is pivoted at one end and connected symmetrically by two springs and dampers at the other end as shown in Fig. (23). The bar is stretched initially by an angle of 0.2 rad and released from rest. How many seconds does the bar take to rest again. Take $m=10$ kg, $K=2000$ N/m, $C=13 \times 10^3$ Dyn.sec/cm and $L=5$ m. Ans. 0.0932 sec

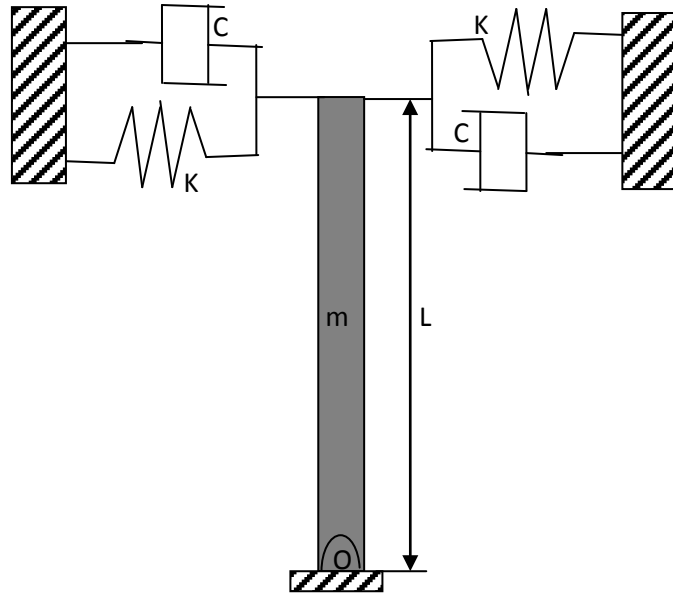


Fig.(23)

- (39) For a 3-DOF system, the mass and flexibility matrices are:-

$$[M]=m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [a]=\frac{1}{k} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}. \text{ The natural mode shapes are:-}$$

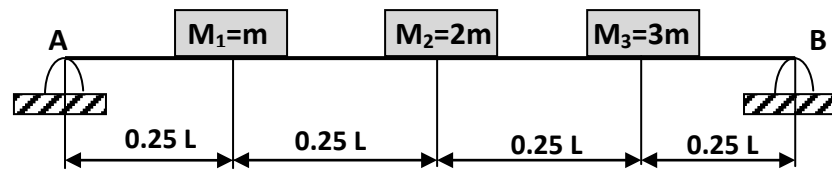
$$\Theta_1 = \begin{bmatrix} 1 \\ 1.8019 \\ 2.247 \end{bmatrix}, \quad \Theta_2 = \begin{bmatrix} 1 \\ 0.445 \\ -0.802 \end{bmatrix} \quad \text{and} \quad \Theta_3 = \begin{bmatrix} 1 \\ -1.2468 \\ 0.5544 \end{bmatrix}$$

Verify the orthogonality properties of these modes with respect to the mass and stiffness matrices.

- (40) For the simply supported beam shown in the Fig. (24), the flexibility influence matrix is :-

$$[a] = \frac{L^3}{768EI} \begin{bmatrix} a_{11} & 11 & a_{13} \\ a_{21} & 16 & 11 \\ a_{31} & a_{32} & 9 \end{bmatrix} \quad \text{Find the upper bound of the fundamental natural frequency.}$$

Fig. (24)



(41) Find the lower bound of the fundamental natural frequency of the system described in Q40.

(42) The two uniform bars shown in Fig. (25) are of equal length and equal mass. Determine the equation of motion, natural frequencies and mode shapes.

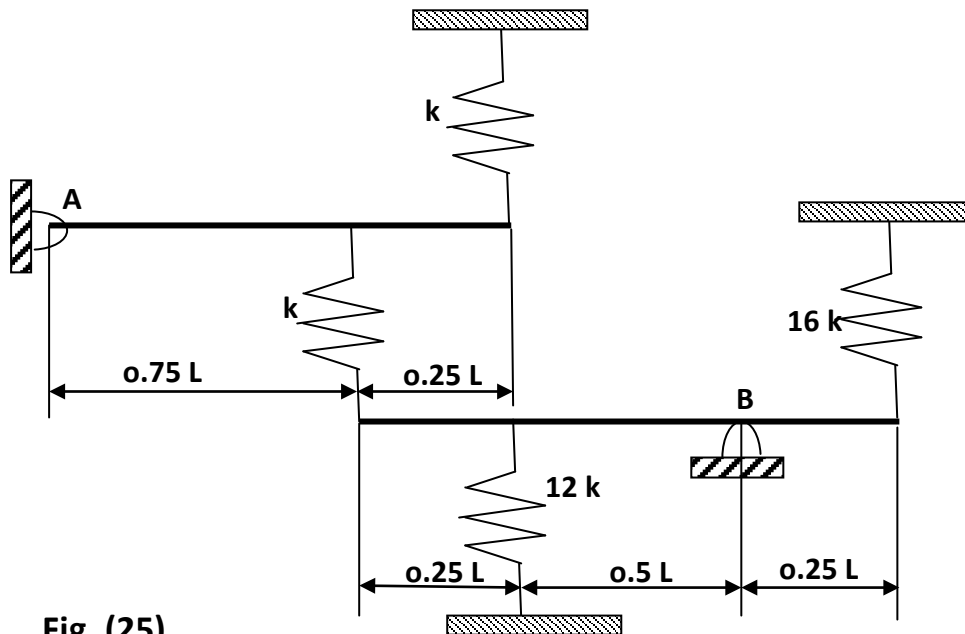


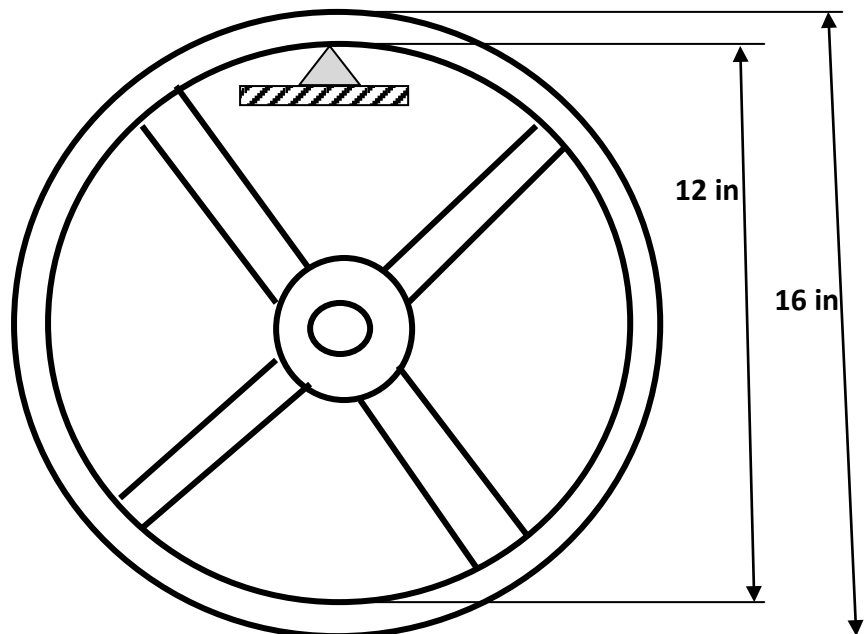
Fig. (25)

(43) A counter rotating eccentric mass exciter is used to determine the vibrational characteristics of a structure of mass 181.4 kg. At a speed of 900 rpm stroboscope shows that the eccentric mass to be at the top at

the instant that the structure is moving upwards through its static equilibrium position (i.e. the resonance condition) and the corresponding amplitude is 21.6 mm. If the unbalance of wheels re 0.1842 kg.m, determine:- (a) the natural frequency of the structure, (b) the damping factor of the structure, (c) the amplitude t 1200 rpm and (d) the phase angle.

- (44) A flywheel weighing 70 lb was allowed to swing as s pendulum about knife edge t the inner side of the rim as shown in Fig. (26). If the measured period of oscillation was 1.22 sec, determine the moment of inertia of the flywheel about its geometrical axis.

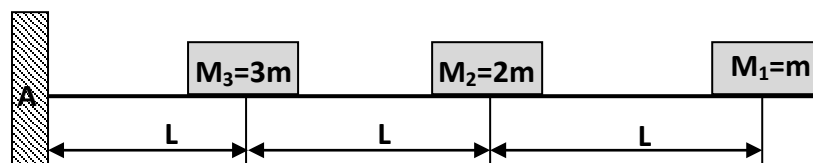
Fig. (26)



- (45) For the cantilever beam shown in Fig. (27), the flexibility influence matrix is:-

$$[a] = \frac{L^3}{3EI} \begin{bmatrix} a_{11} & a_{12} & 4 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \quad \text{Find the upper bound of the fundamental natural frequency.}$$

Fig. (27)



- (46) Find the lower bound of the fundamental natural frequency of the system described in Q45.
- (47) A 35-kg machine is attached to the end of a cantilever beam of length 2 m, elastic modulus 210 GN/m^2 and second moment of area $1.3 \times 10^{-7} \text{ m}^4$. The machine operates at 180 rpm and has a rotating unbalance of 0.3 kg.m. It is required the followings:- (a) what is the required stiffness of an undamped absorber of mass 5 kg such that the steady state vibrations are eliminated at 180 rpm? and (b) with the absorber in place what are the natural frequencies of the system?
- (48) An electric train made up of two cars each weighing 50 000 lb is connected by coupling of stiffness equal to 16 000 lb/in. Determine the natural frequency of the system.
- (49) A vehicle is traveling over a rough road at a constant horizontal speed equal to 40 m/sec. The road contour is approximated by a sine wave of amplitude 10 mm and wave length of 1.6 m. The mass of the vehicle is 900 kg, the stiffness of the suspension system is $8 \times 10^4 \text{ N/m}$ and the system has a damping ratio of 0.2. Determine the acceleration and displacement amplitude of the body of the vehicle.
- (50) The ratio $\frac{k}{m}$ of a spring mass system is given as 4. If the mass is deflected 2 cm down measured from its equilibrium position and given an upward velocity of 8 cm/sec, determine its amplitude and maximum acceleration.
- (51) The springs of an automobile trailer are compressed 10.16 cm under its weight. Find the critical speed when the trailer is traveling over a road

with a profile approximated by a sine wave of amplitude 7.62 cm and wavelength of 14.63 m. What will be the amplitude of vibration at 64.4 km/hr?
 Ans. 82 km/hr , 19.63 cm

(52) A 100 kg lathe is mounted at the midspan of 1.8 m simply supported beam ($E=200$ GPa, $I=4.3 \times 10^{-4} \text{ m}^4$). The lathe has a rotating unbalance of 0.43 kg.m and operates at 200 rpm. When a free vibration test is performed on the system, it is found that the ratio of amplitude of successive cycles is 1.8 to 1. Determine the steady state amplitude of vibration induced by the rotating unbalance. Ans. 6.526 mm

(53) A 150 kg electric motor has a rotating unbalance of 0.5 kg situated at 0.2 m from the center of rotation. The motor is to be mounted at the end of steel cantilever beam of length 1 m. The operating range of the rotor is from 500 to 1200 rpm. For what values of I , the beam's cross sectional moment of inertia will the steady state amplitude of vibration be less than 1 mm? Assume that the damping ratio is 0.1.

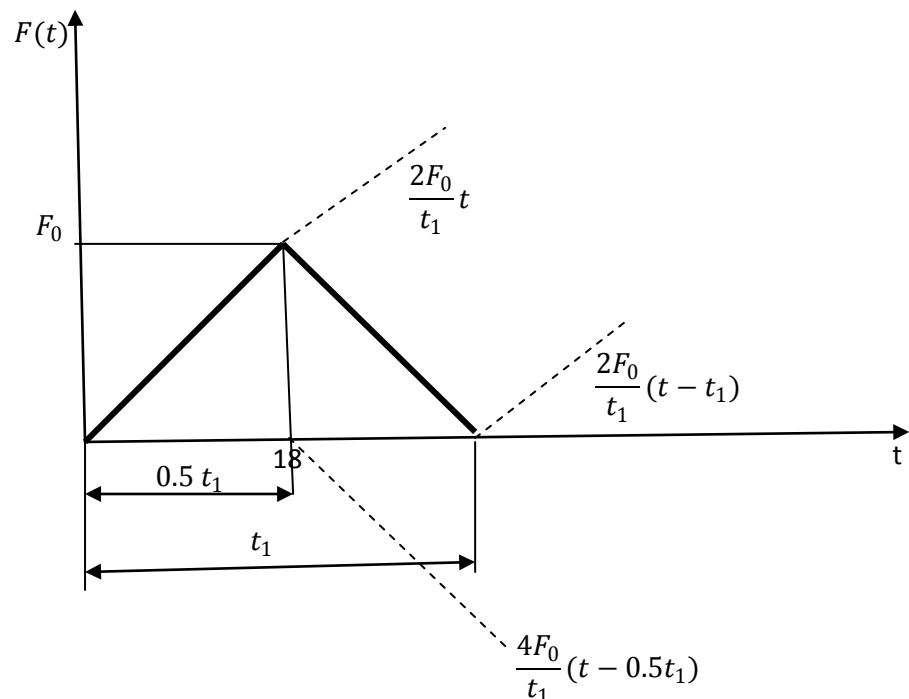
$$\text{Ans. } 6.079 \times 10^{-6} \text{ m}^4 \leq I \leq 0.224 \times 10^{-6} \text{ m}^4$$

(54):- For the triangular pulse shown in Fig. (1), show that the response for the time interval $0.5t_1 < t < t_1$ is

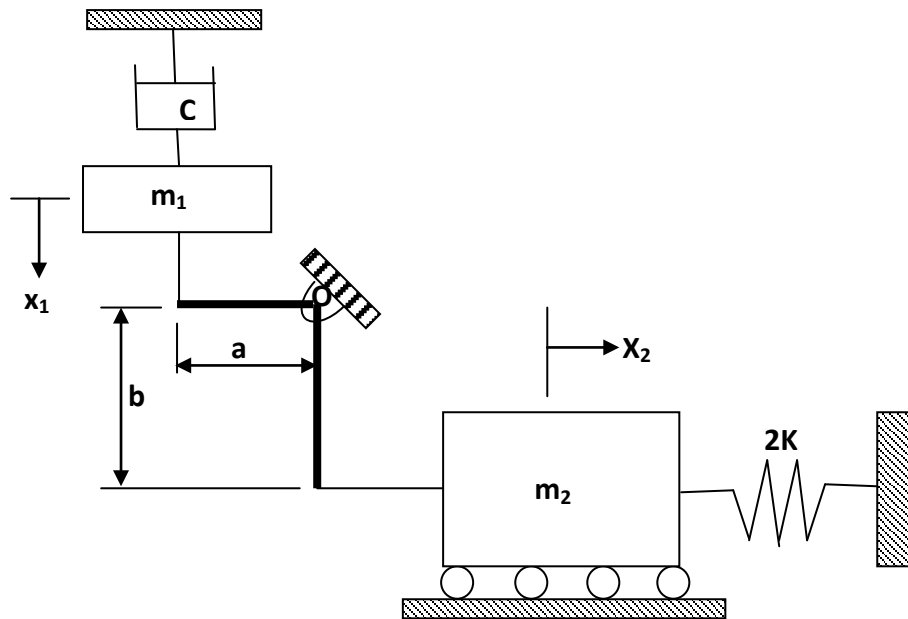
$x = \frac{2F_0}{K} \left[1 - \frac{t}{t_1} + \frac{\tau}{2\pi t_1} \left\{ \sin \frac{2\pi}{\tau} (t - 0.5t_1) - \sin \frac{2\pi t}{\tau} \right\} \right]$. Knowing that the response for the time interval $0.0 < t < 0.5 t_1$ is

$$x = \frac{2F_0}{K} \left(\frac{t}{t_1} - \frac{\tau}{2\pi t_1} \sin 2\pi \frac{t}{\tau} \right)$$

Fig. (1)

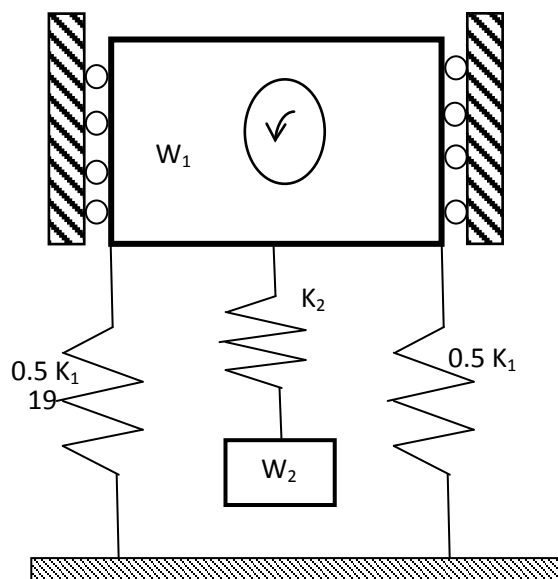


- (55) For the system shown in the figure below:- (a) find the value of the damping coefficient C when the damping ratio $\xi=0.007$ and mass moment of inertia for the link about point O is 0.3 kg.m^2 , and (b) if the mass m_2 is displaced by 20 mm and released from rest, how many cycles will be exerted when the amplitude is reduced to 0.85 mm . Given that $m_1=10 \text{ kg}$, $m_2=25 \text{ kg}$, $K=k\text{N/m}$, $a=10 \text{ cm}$ and $b=20 \text{ cm}$.



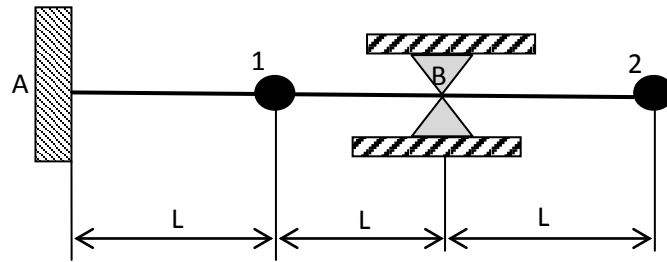
- (56) (Prob. 5.40 page 158) For the system shown in Fig. (1), $W_1=200 \text{ lb}$ and the absorber weight $W_2=50 \text{ lb}$. If W_1 is excited by a 2 lb.in unbalance rotating at 1800 rpm , determine the proper value of the absorber spring K_2 . What will be the amplitude of W_2 ?

Fig. (1)



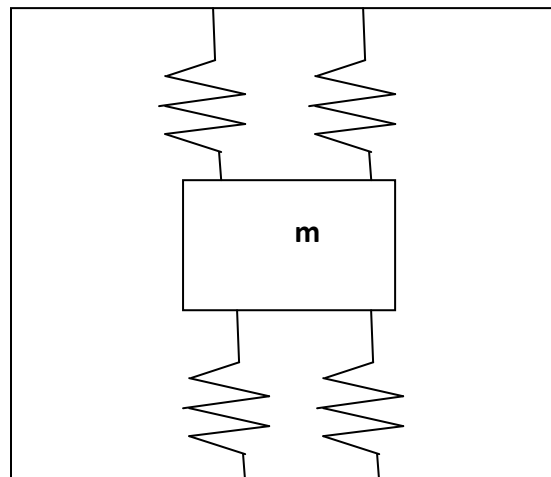
(57) (Prob. 6.7 page 191) Determine the flexibility matrix for the uniform beam shown in Fig. (2) by using the area moment method.

Fig. (2)



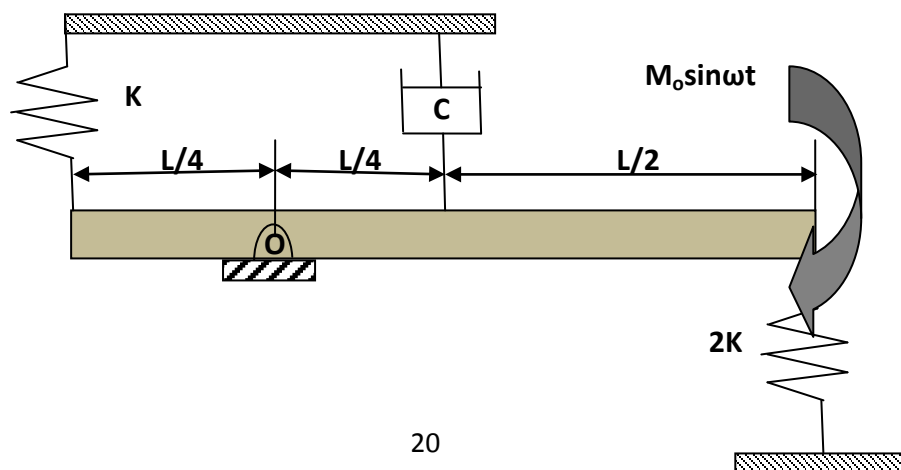
(58) A delicate instrument is supported in a box by springs as shown in Fig. (1) where its natural frequency is 10 cps. In the supported position the clearance space between the instrument and the box is 1 inch. If the box is suddenly dropped through a height of 20 inches, will the instrument hit the wall?

Fig. (1)



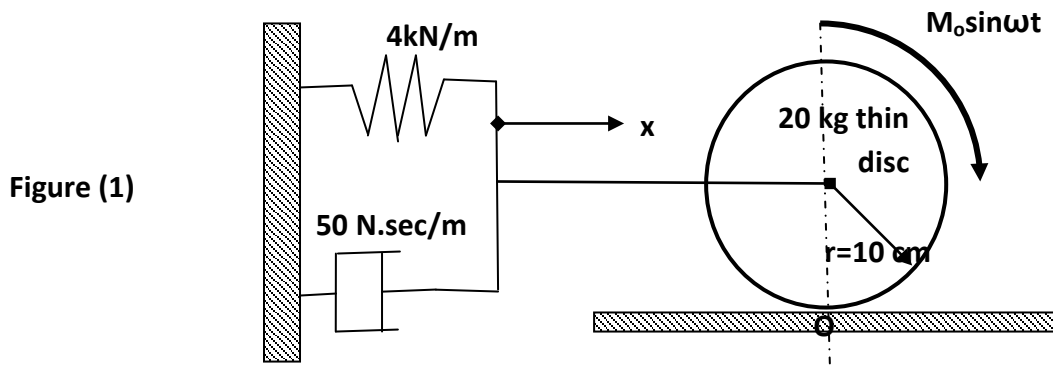
(59) A moment is applied to the end of the bar shown in Fig. (2). Determine the maximum value of M_o such that the steady state amplitude of angular oscillation does not exceed 10° . If $N=500$ rpm, $K=7000$ N/m, $C=650$ N.sec/m, $L=1.2$ m and the mass of the bar is 15 kg.

Fig. (2)



(60) An accelerometer has a natural frequency of 80 Hz and a damping coefficient of 8 N.sec/m. When attached to a vibrating structure, it measures an amplitude of 8 m/sec² and a frequency of 50 Hz. The true acceleration of the structure is 7.5 m/sec². Determine the mass and stiffness of the accelerometer.

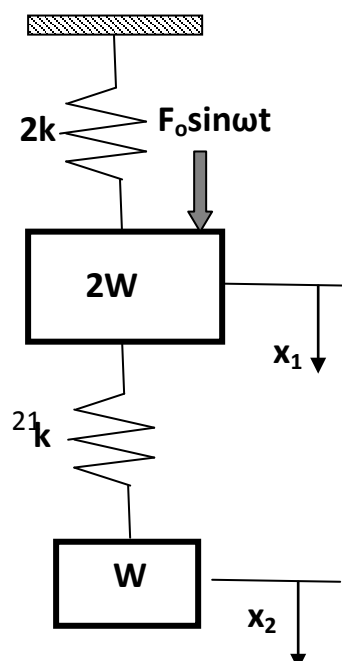
(61) If $\omega=16.5$ rad/sec what is the maximum value M_o such that the disc shown in figure (1) rolls without slip a distance equal to 5 cm?



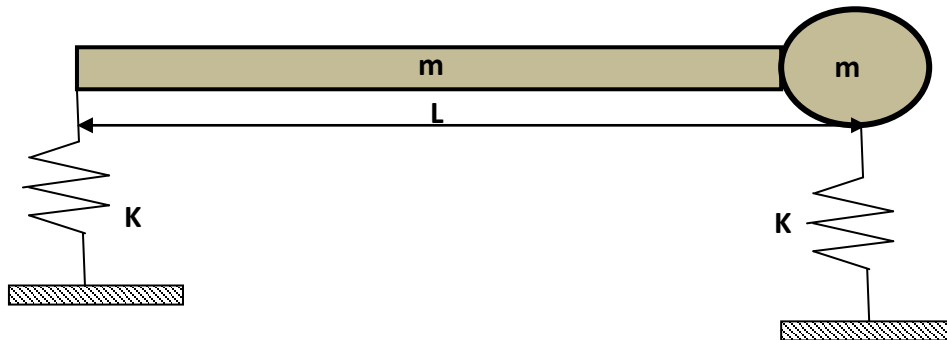
(62) A 20 kg block is connected to a movable support through a spring of stiffness 100 kN/m in parallel with a viscous damper of damping coefficient 600 N.sec/m. The support is given a harmonic displacement of amplitude 25 mm and frequency 40 rad/sec. An accelerometer of natural frequency 25 Hz and damping ratio 0.2 is attached to the block. What is the output of the accelerometer in mm/sec²?

(63) For the system shown in the figure below, determine the natural frequencies and normal modes when $K=10$ lb/in and $W=1.93$ lb. Also determine the equations of the amplitudes X_1 and X_2 and plot the later against

$$\frac{\omega}{\omega_1}$$

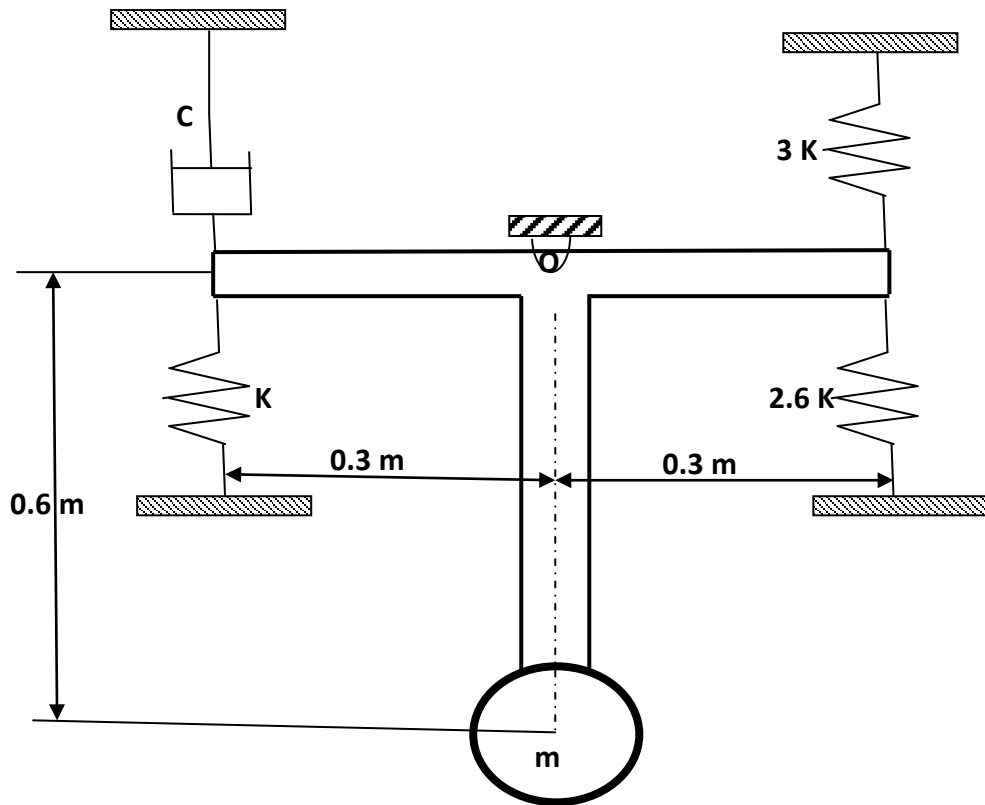


(64) A uniform bar of length L and mass m carries at its right end an additional mass also m . It is mounted on two equal springs K at the extremities as shown in the figure below. Find the two natural frequencies and the location of the node of the bar in each of the natural frequency.

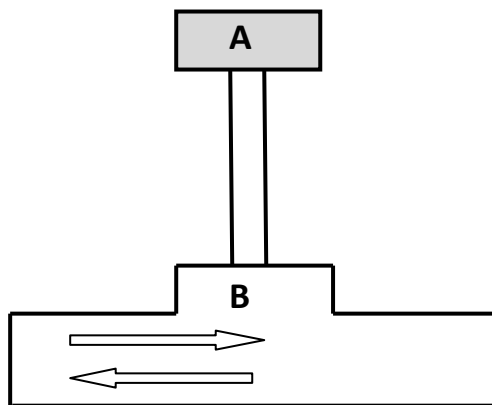


(65) A 50 kg lathe machine mounted on an elastic foundation of stiffness 4×10^5 N/m has a vibration amplitude of 35 cm when the motor speed is 95 rad/sec. Design an undamped dynamic vibration absorber such that the steady state vibrations are completely eliminated at 95 rad/sec and the maximum displacement of the absorber mass at this speed is 5 cm.

(65) The rigid T-bar of negligible weight as shown in the Fig. below rotates in a vertical plane about a horizontal axis at O. The bar is caused to vibrate by displacing the weight (m) and releasing it from rest by 0.5 cm. Determine:- (a) the ratio of the positive amplitudes of third and fourth cycles, (b) the damped frequency of vibration in Hz and (c) the general solution of the vibrating system. Given data $K=5$ N/m, $C=11$ N.sec/m and $m=2$ kg.

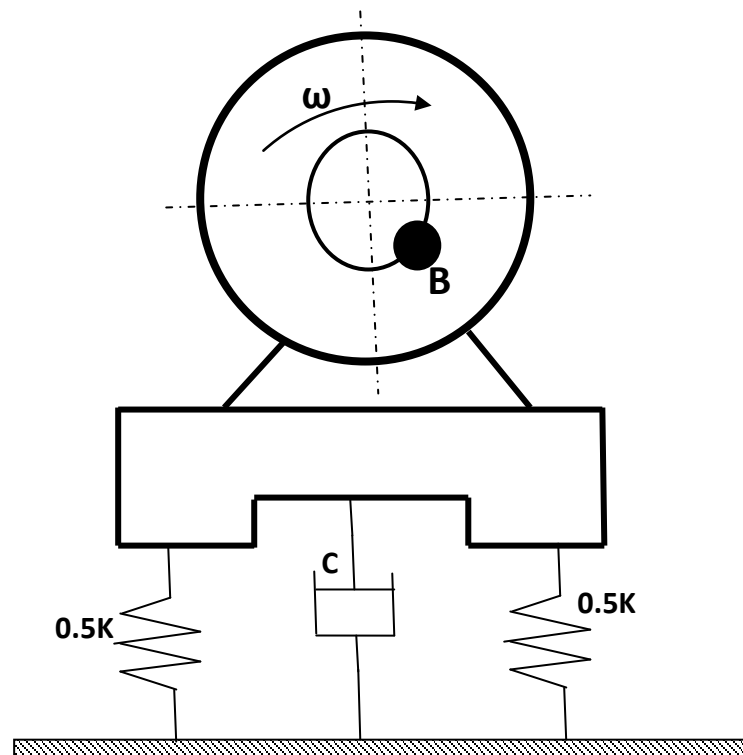


(66) A static horizontal force of 10 N gives the 1.5 kg mass (A) a deflection of 12 mm against the elasticity of the light cantilever spring to which is attached. If the bar (B) shown in the Fig. below of the spring is given a horizontal harmonic oscillation with a frequency of 2 Hz and an amplitude of 6 mm. Calculate the amplitude of the resulting vibration of (A). Assume negligible damping.

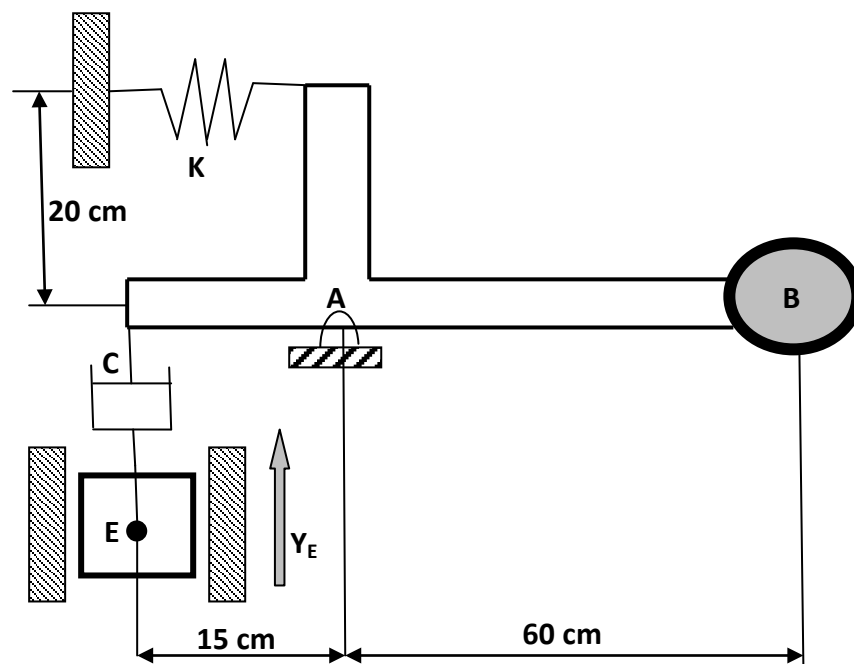


(67) A vibrometer whose damping is negligible is employed to find the magnitude of vibration of a machine structure. It gives a reading of the relative displacement of 0.005 cm. The natural frequency of the vibrometer is 300 cpm and the machine is running at 100 rpm. What will be the magnitude of displacement, velocity and acceleration of the vibrating machine part?

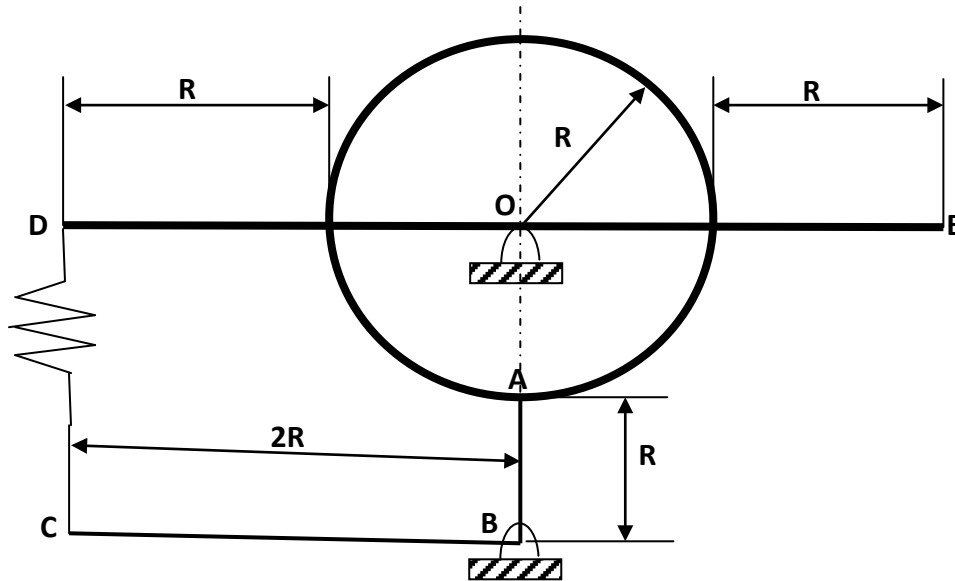
(68) The motor shown in the Fig. below is mounted on two springs each with a modulus of $0.5K = 10.5 \times 10^3$ N/cm. The dashpot has a coefficient of 140 N.sec/m. The motor including the unbalanced mass B weighs 17.5 kg and the unbalanced body B mass is 0.45 kg located at 7.62 cm from the center of the shaft. It is required the following:- (a) If the motor rotates at 3000 rpm, determine the amplitude and the phase angle (relative to the position of B) of the resulting motion. (b) Determine the maximum force on the rotor exerted by the springs, by the damper and by both the springs and the damper when the motor rotates at 3000 rpm. (c) Determine the resonant speed in rpm and the resulting amplitude of motion.



(68) The spring in the Fig. shown below is adjusted to equilibrium when the part AB of the \perp shaped mass less body is horizontal. Body E has a harmonic motion given by the equation $Y_E = 1.524 \sin 12t$ where Y_E in cm and t in seconds. The mass of body B is 3.5 kg, the damping coefficient is 0.7 kN.sec/m and the spring modulus is 3.153 kN/m. Determine (a) the amplitude of motion of B, (b) it's phase relationship to the motion of E and (c) the amplitude of the force acting on the dashpot.

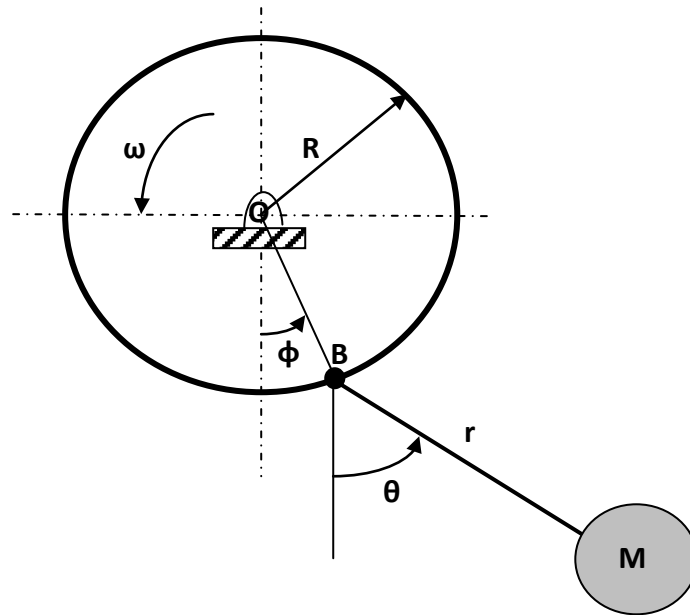


(69) Find the natural frequency for the system shown in the Fig. below. Given that the ring mass is m and its radius R and the mass of the bar DOE is also m . Neglect the mass of the arm CBA ($CB \perp AB$)

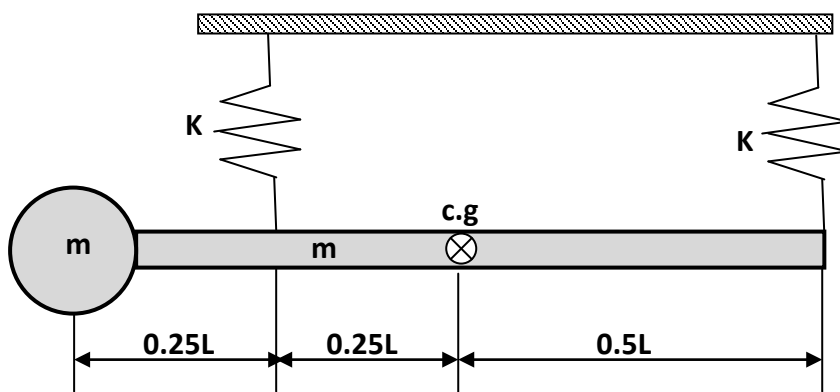


(70) A gas turbine rotates at 2000 rpm. It is found that the turbine rotor suffers from the vibration of unbalance of 0.2 kg at a radius of 15 cm of sinusoidal form. The response of the rotor is transmitted to the bearing and then to the supporting structure of the system. If the turbine is placed on foundation of four props, each prop is considered as a spring of 4 kN/mm stiffness coefficient and the mass of the turbine is 1500 kg. It is required to find:- (a) the natural frequency of the turbine, (b) the general solution of the vibrating system and (c) if the displacement of the turbine mass measured from the equilibrium position is 5 mm at 5 second and the velocity of the turbine mass is 2 mm/sec, What is the displacement of the turbine mass at 10 seconds?

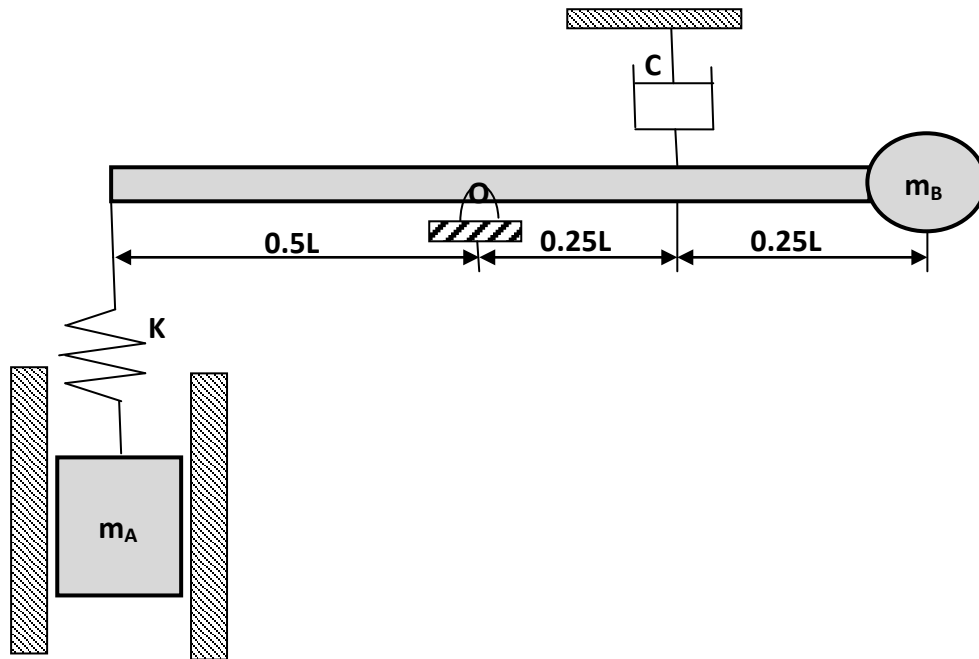
(71) A disc of radius R is rotating with constant angular velocity ω about its center O . A pendulum of mass M is connected by a string of length r at point B as shown in the Fig. below. Find the natural frequency of the pendulum using Lagrange's equation.



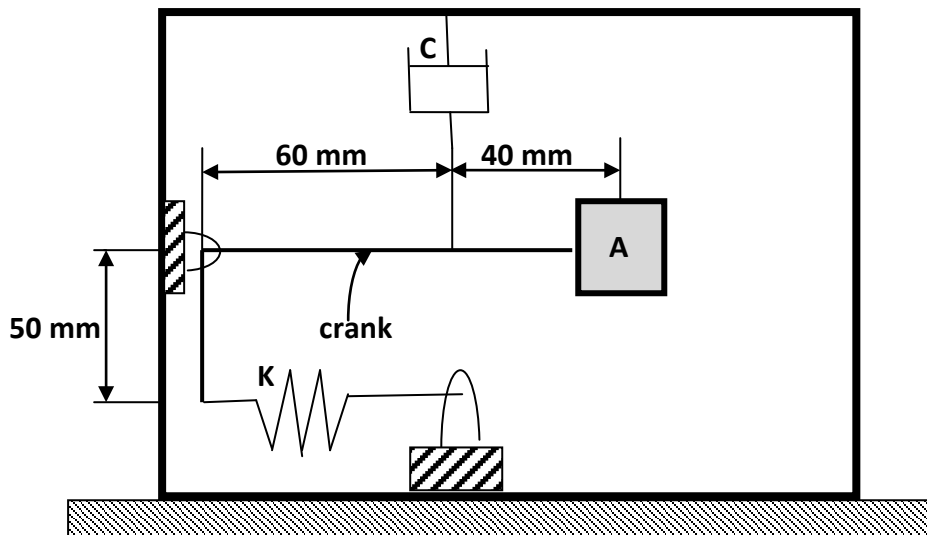
(72) For the system shown in the Fig. below, the uniform beam of mass m with an end mass m is supported by two springs of stiffness K . Choose x and θ coordinates at the center of gravity of the beam and find the natural frequencies and the corresponding mode shapes.



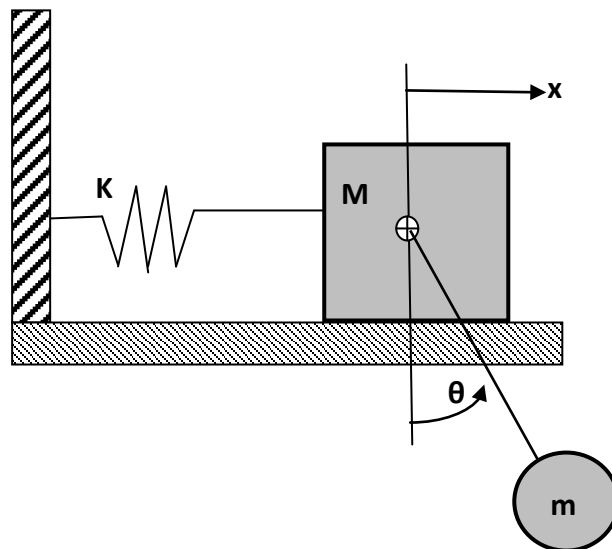
(73) A uniform slender bar of mass 3 kg is hinged at O and has a horizontal position under the action of masses $m_A=1$ kg, $m_B=4$ kg, the spring whose stiffness is 320 N/cm and the dashpot whose damping coefficient is 20.8 N.sec/cm as shown in the Fig. below. The bar is excited by giving the mass A a harmonic motion $Y_A = 3.5 \sin 50t$ where Y_A in cm and t in seconds. Determine:- (a) the damped natural frequency, (b) the amplitude of motion and its phase angle and (c) the ratio between the first and third cycles of motion.



(74) An instrument for measuring small amplitudes of displacement is shown in the Fig. below. The crank can be considered weightless while the mass of the body A is 0.5 kg. Knowing that the damping ratio is 0.6 and the undamped natural frequency is 2.5 Hz, determine the damping coefficient C and the spring stiffness K.



(75) A simple pendulum of length L and mass m is pivoted to the mass M which slides without friction on a horizontal plane as shown in the fig. below. Use Lagrange's equation to determine the natural frequencies of the system.

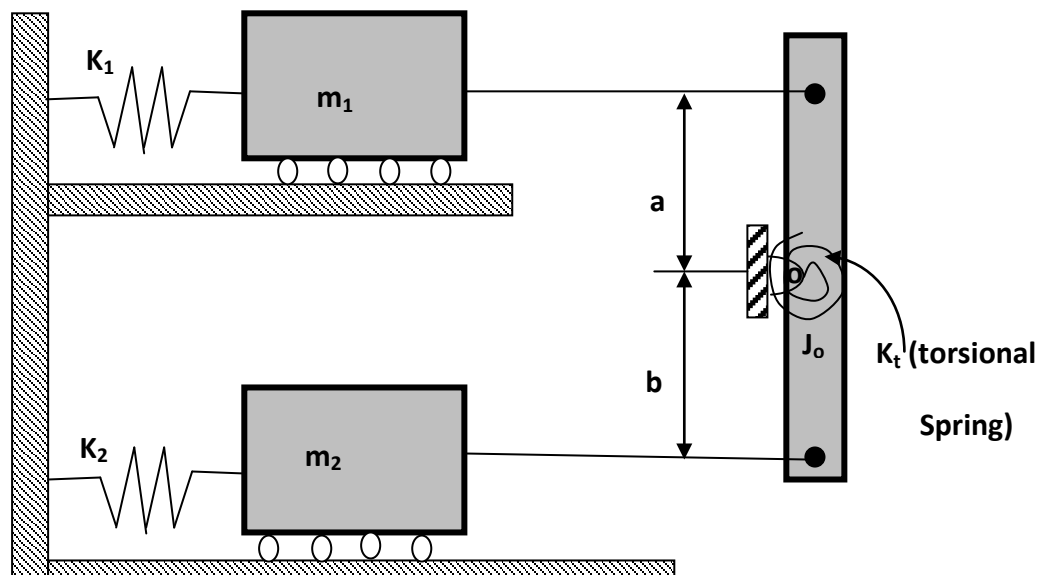


(76) An under damped shock absorber is to be designed for a motor cycle of mass 200 kg. When the shock absorber is subjected to an initial velocity due to a road bump, the resulting displacement-time curve is to be such that the amplitude X_1 is to be reduced to one fourth in one half cycle (i.e. $\frac{X_{1.5}}{X_1} = \frac{1}{4}$). Find the necessary stiffness and damping constants of the shock absorber if the

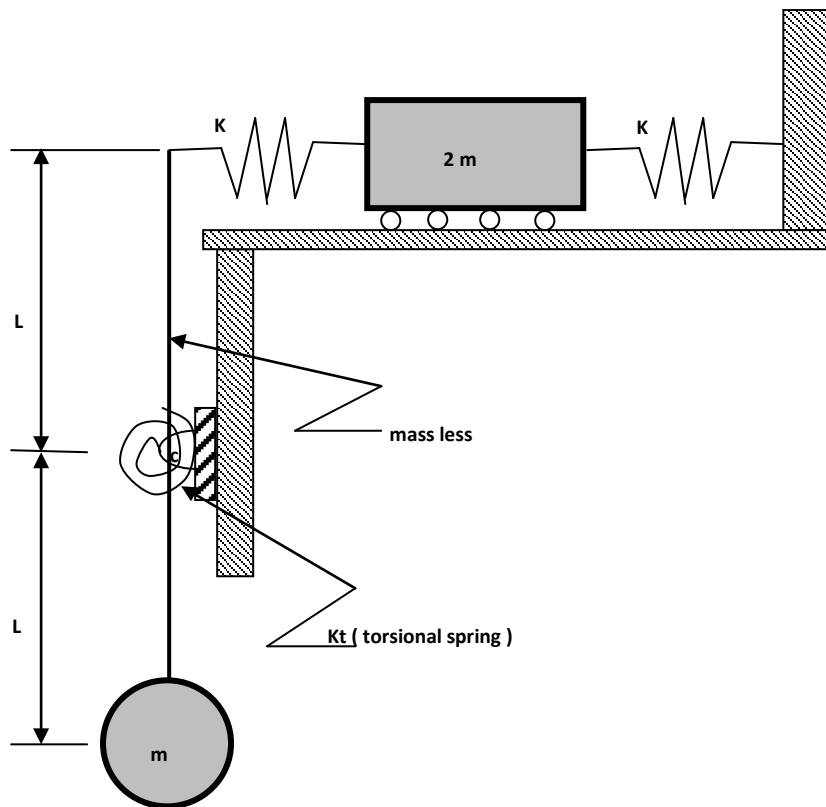
damped period of vibration is to be 2 sec. Also find the minimum initial velocity that leads to a maximum displacement of 250 mm.

(76) The peak amplitude of a single-degree-of freedom system under harmonic excitation is found to be 5 cm. If the undamped natural frequency of the system is 10π rad/sec and the amplitude of the harmonic force is five times the spring constant, find (a) damping ratio, (b) the amplitude of the system at frequency 20π rad/sec.

(77) For the system shown in the fig. below, find the equation of motion and determine the natural frequency.

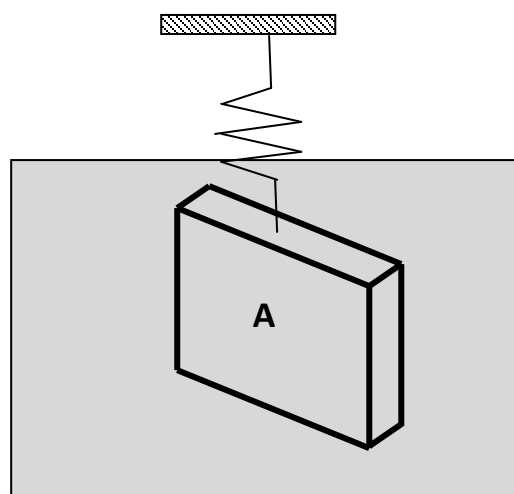


(78) Using Lagrange's equation, find the characteristics equation of the system shown in the Fig. below and specify the type of coupling.

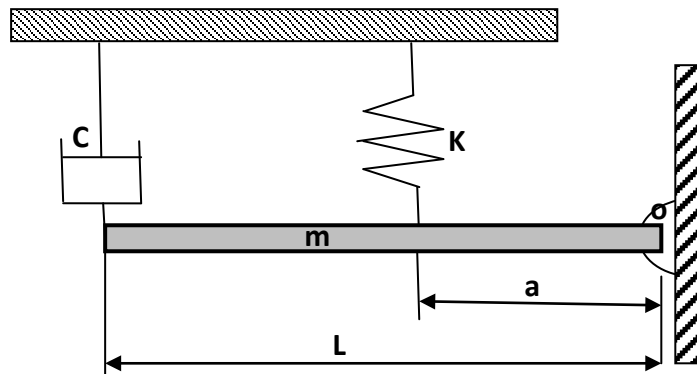


(79) A thin plate of area A and weight W is attached to the end of a spring and is allowed to oscillate in a viscous fluid as shown in the Fig. below. If τ_1 is the natural period of undamped oscillation (i.e. with the system oscillating in air) and τ_2 is the damped period with the plate immersed in the fluid, show that:-

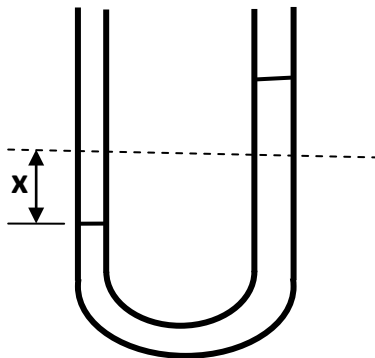
$$\mu = \frac{2\pi W}{gA\tau_1\tau_2} \sqrt{(\tau_2^2 - \tau_1^2)}$$
 where the damping force of the plate is $F_d = \mu(2A)v$, $2A$ is the total surface area of the plate and v is its velocity.



(80) A rigid uniform bar of mass m and length L is pinned at o and suspended by a spring and viscous damper as shown in the Fig. below. Determine an expression for:- (a) undamped natural frequency and (b) critical damping.

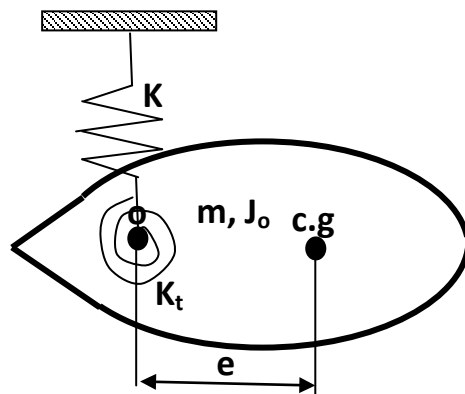


(81) Using the energy method show that the natural period of oscillation of the fluid in the U-tube manometer shown in the Fig. below is $\tau = 2\pi \sqrt{\frac{L}{2g}}$ where L is the length of the fluid column.



(82) An airfoil section is to be tested in a wind tunnel as shown in the Fig. below. If the center of gravity has a distance e ahead of the point of support. Determine the characteristic equation and specify the type of coupling.

Take $m=12$ kg, $e=70$ cm, $J_{c.g.}=6.5$ kg.cm², $K=70$ kN/m and $K_t=65$ kN/m and find the natural frequencies and the corresponding mode shapes.



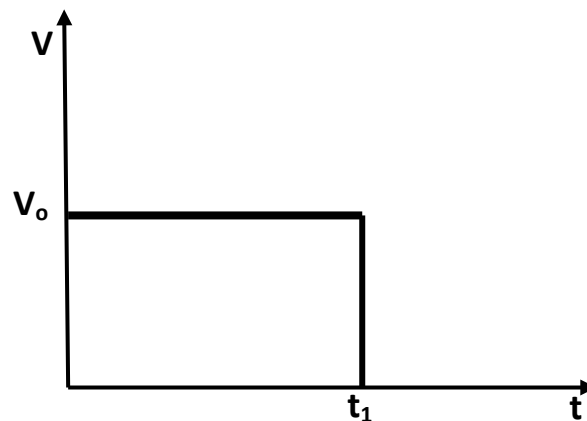
(83) A solid disc of diameter 30 cm and mass 10 kg is attached to the end of a solid 3-cm diameter, 1-m long steel shaft ($E=200$ GPas and $G=80$ GPas). A torsional vibration absorber consist of a disc attached to a shaft that is then attached to the primary system. If the absorber disc has a mass of 3 kg and a diameter of 10 cm, what is the required diameter of a 50-cm long absorber shaft to eliminate steady state vibrations of the original system excited at 500 rad/sec?

(84) A 35-kg machine is attached to the end of a cantilever beam of length 2 m, elastic modulus 210 GN/m and moment of inertia 1.3×10^{-7} m⁴. The machine operates at 180 rpm and has a rotating unbalance of 0.3 kg.m. It is required the following:- (a) what is the required stiffness of an undamped absorber of mass 5 kg such that the steady state vibrations are eliminated at 180 rpm?, (b) with the absorber in place what are the natural frequencies of the system? and (c) for what range of operating speeds will the steady state amplitude of the machine be less than 8 mm?

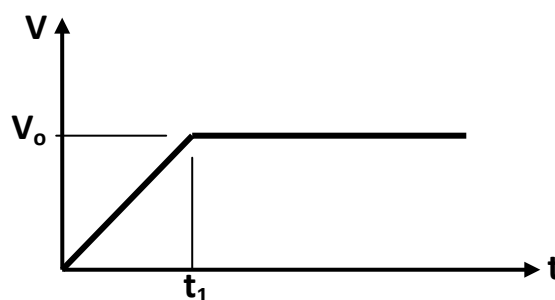
(85) What is the required stiffness of an undamped dynamic vibration absorber whose mass is 5 kg to eliminate vibrations of 25-kg machine of natural frequency 125 rad/sec when the machine operates at 110 rad/sec?

(86) A 50 kg lathe machine mounted on an elastic foundation of stiffness 4×10^5 N/m has a vibration amplitude of 35 cm when the motor speed is 95 rad/sec. It is proposed to attach an undamped dynamic absorber to eliminate vibrations. With absorber in place (a) what are the natural frequencies of the system? and what is the range around 95 rad/sec such that the steady state amplitude of the lathe is less than 1cm?

(87) The base of an undamped spring-mass system (m & K) is given a velocity pulse as shown in the Fig. below. If the peak response occurs at $t < t_1$, find;- (a) the maximum response and (b) the response of the system for the time ranges $t < t_1$ and $t > t_1$.



(88) The base of an undamped spring-mass system (m & K) is given a velocity pulse as shown in the Fig. below. If the peak response occurs at $t < t_1$, find;- (a) the maximum response and (b) the response of the system for the time ranges $t < t_1$ and $t > t_1$.



(89) For a 3-DOF system, the mass and stiffness matrices are:-

$$[M]=m \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 2 & 2 \end{bmatrix} \quad [K]=k \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \text{ The natural mode shapes are:-}$$

$$\Theta_1 = \begin{bmatrix} 1 \\ 1.7614 \\ 0.5513 \end{bmatrix}, \Theta_2 = \begin{bmatrix} 2.7518 \\ 1 \\ -2.57 \end{bmatrix} \text{ and } \Theta_3 = \begin{bmatrix} 0.2579 \\ -0.8049 \\ 1 \end{bmatrix}$$

Verify the orthogonality relations with respect to mass and stiffness matrices.

(90) For a 3-DOF system, the mass and stiffness matrices are:-

$$[M]=m \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 2 & 2 \end{bmatrix} \quad [K]=k \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \text{ The natural mode shapes are:-}$$

$$\Theta_1 = \begin{bmatrix} 1 \\ 1.7614 \\ 0.5513 \end{bmatrix}, \Theta_2 = \begin{bmatrix} 2.7518 \\ 1 \\ -2.57 \end{bmatrix} \text{ and } \Theta_3 = \begin{bmatrix} 0.2579 \\ -0.8049 \\ 1 \end{bmatrix}$$

Find the orthonormal modes.

(91) For a 2-DOF system, the mass and stiffness matrices are:-

$$[M]=m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad [K]=k \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}. \text{ The natural mode shapes are:-}$$

$$\Theta_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \Theta_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}. \text{ Find the natural frequencies using the orthonormal modes concept.}$$

(92) Determine the characteristic equation of free vibration of a uniform beam of length L fixed at one end and pinned at the other end.

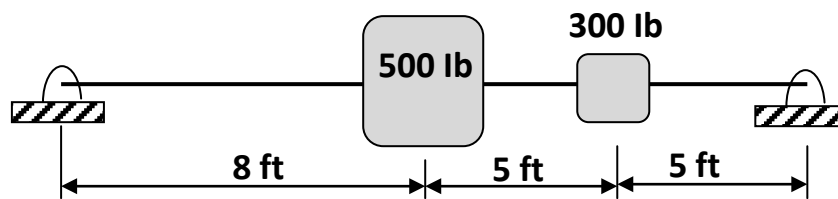
(93) For a 2-DOF system, the mass and stiffness matrices are:-

$$[M]=m \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \quad [K]=k \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}. \text{ The natural mode shapes are:-}$$

$$\Theta_1 = \begin{bmatrix} 1 \\ 0.562 \end{bmatrix} \text{ and } \Theta_2 = \begin{bmatrix} 1 \\ -3.562 \end{bmatrix}. \text{ Find the natural frequencies using the orthonormal modes concept.}$$

(94) A simply supported beam carrying two masses as shown in the Fig. below. Knowing that the second moment of area of the beam section I and the

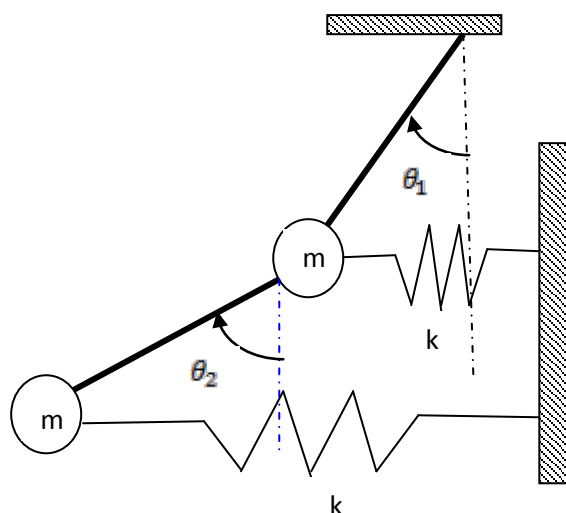
modulus of elasticity of the beam material is E . Estimate the fundamental natural frequency using Rayleigh and Dunkerley method.



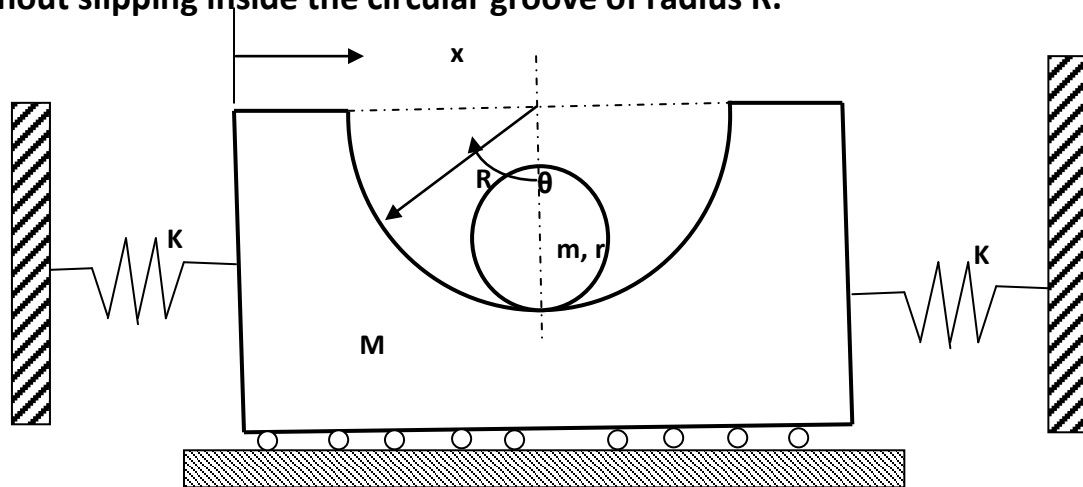
(95) A thin disc is supported on spring-mounted bearing with vibration pickup and stroboscope. Running at 600 rpm ccw, the original disc indicates a maximum amplitude of 2.8 mm at 45° from a reference mark on the disc. Next a trial mass of 2 ounce is added at the rim in a position of 135° cw from the reference mark, the new unbalance is found to be 4.3 mm at 111° cw from the reference mark and run at the same speed. Determine the position and weight necessary to balance the original disc.

(96) A double pendulum consists of two equal masses m , hanging on weightless string of length L each. In addition to gravity, there are two mechanical springs of stiffness k . The equilibrium position is a vertical line s shown in the figure below. Using Lagrange's equations set up the differential equation of motion carefully and calculate the two natural frequencies and the corresponding mode shapes.

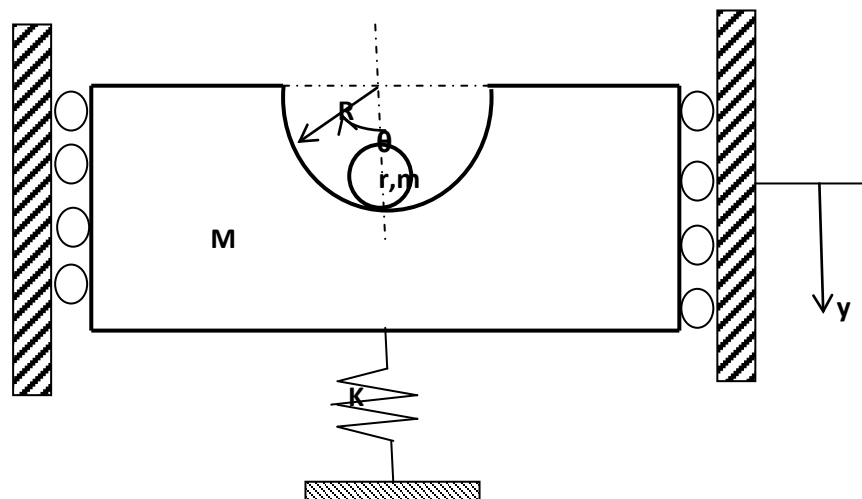
Fig. (20)



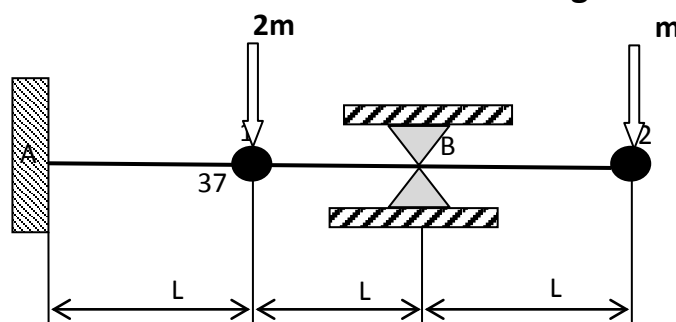
(97) Derive the equations of motion of the system shown in the figure below using Lagrange's equation. The circular cylinder has a mass m and radius r rolls without slipping inside the circular groove of radius R .



(98) A circular cylinder of radius r and mass m rolls without slipping inside a semi-circular groove of radius R . The block of mass M is supported by a spring of stiffness K and constrained to move without friction in the vertical guide as shown in the figure below. Using Lagrange's equation, find the equations of motion of the system.



(99) The flexibility matrix for the uniform beam shown in figure below is



$$[a] = \frac{L^3}{EI} \begin{bmatrix} \frac{7}{96} & a_{12} \\ a_{21} & \frac{5}{6} \end{bmatrix}$$

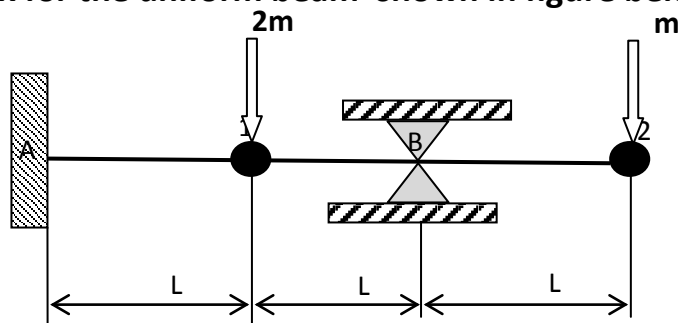
Estimate the lower limit of the natural frequency.

Hint :- Use the double integration method to find the unknown coefficients in the flexibility matrix. _

(100) The flexibility matrix for the uniform beam shown in figure below is

$$[a] = \frac{L^3}{EI} \begin{bmatrix} \frac{7}{96} & a_{12} \\ a_{21} & \frac{5}{6} \end{bmatrix}$$

Estimate the upper limit of the natural frequency.



Hint :- Use the double integration method to find the unknown coefficients in the flexibility matrix. _