~~Response of 2-DOF system to initial excitation:~~

The equation of motion for mode i must be of the form:-

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}^i = C_i r_i \sin(\omega_{ni} t + \psi_{ii}) \quad i=1,2$$

where the constants C_i, ψ_i are constants can be found after applying the initial conditions.
For initial conditions in general, the free vibration contains both modes simultaneously and the equations of motion are of the form

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = C_1 r_1 \sin(\omega_1 t + \psi_1) + C_2 r_2 \sin(\omega_2 t + \psi_2) \quad -1$$

Constants C_1, C_2 establish the amount of each mode. The phase angles ψ_1, ψ_2 allow the freedom of time origin for each mode.

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = C_1 \omega_1 r_1 \cos(\omega_1 t + \psi_1) + C_2 \omega_2 r_2 \cos(\omega_2 t + \psi_2) \quad --2$$

Using Eqs. 1, 2, the four constants C_1, C_2, ψ_1, ψ_2 can be found page 132

Ex. 5.2.1 Determine the free vibration for the system of Ex. 5.1.1. for the initial conditions

1

$$\begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix} = \begin{Bmatrix} 2 \\ 4 \end{Bmatrix} \text{ and } \begin{Bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

From Ex. 5-1.1 $\omega_1 = 0.796\sqrt{\frac{k}{m}}$ $\omega_2 = 1.538\sqrt{\frac{k}{m}}$

$$r_1 = \begin{Bmatrix} 1 \\ 1.366 \end{Bmatrix} \quad r_2 = \begin{Bmatrix} 1 \\ -0.366 \end{Bmatrix}$$

for $t = 0$

$$\begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix} = C_1 \begin{Bmatrix} 1 \\ 1.366 \end{Bmatrix} \sin\psi_1 + C_2 \begin{Bmatrix} 1 \\ -0.366 \end{Bmatrix} \sin\psi_2$$

$$\begin{Bmatrix} 2 \\ 4 \end{Bmatrix} = C_1 \begin{Bmatrix} 1 \\ 1.366 \end{Bmatrix} \sin\psi_1 + C_2 \begin{Bmatrix} 1 \\ -0.366 \end{Bmatrix} \sin\psi_2 \quad \text{-- } 1$$

$$\begin{Bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \end{Bmatrix} = C_1\omega_1 \begin{Bmatrix} 1 \\ 1.366 \end{Bmatrix} \cos\psi_1 + C_2\omega_2 \begin{Bmatrix} 1 \\ -0.366 \end{Bmatrix} \cos\psi_2$$

$$\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = C_1\omega_1 \begin{Bmatrix} 1 \\ 1.366 \end{Bmatrix} \cos\psi_1 + C_2\omega_2 \begin{Bmatrix} 1 \\ -0.366 \end{Bmatrix} \cos\psi_2 \quad \text{-- } 2$$

Expand Eq. 1

$$[2 = C_1 \sin\psi_1 + C_2 \sin\psi_2 \quad \text{-- } 3] \times 1.366$$

$$4 = 1.366\, \cancel{C_1 \sin\psi_1} - 0.366\, C_2 \sin\psi_2 \quad \text{-- } 4$$

$$2.732 = \mp 1.366\, C_1 \sin\psi_1 \mp 1.366\, C_2 \sin\psi_2 \quad 2\,\text{x}$$

$$-1.268 = -1.732\, C_2 \sin\psi_2 \quad \text{-- } 5$$

Multiply Eq. 3 by 0.366

$$0.732 = 0.366\, C_1 \sin\psi_1 + 0.366\, C_2 \sin\psi_2$$

$$4 = 1.366\, C_1 \sin\psi_1 - 0.366\, C_2 \sin\psi_2$$

$$4.732 = 1.732\, C_1 \sin\psi_1 \quad --- (6)$$

Expand Eq. (2)

$$\left[\, 0 = C_1 \omega_1 \cos\psi_1 + C_2 \omega_2 \cos\psi_2 \quad -- (7) \,\right] \times 1.366$$

$$0 = 1.366\, C_1 \omega_1 \cos\psi_1 - 0.366\, C_2 \omega_2 \cos\psi_2 \quad -- (8)$$

$$0 = \mp 1.366\, C_1 \omega_1 \cos\psi_1 \mp 1.366\, C_2 \omega_2 \cos\psi_2$$

$$0 = 1.732\, C_2 \omega_2 \cos\psi_2 \quad -- (9)$$

Multiply Eq. (7) by 0.366

$$0 = 0.366\, C_1 \omega_1 \cos\psi_1 + 0.366\, C_2 \omega_2 \cos\psi_2$$

$$0 = 1.366\, C_1 \omega_1 \cos\psi_1 - 0.366\, C_2 \omega_2 \cos\psi_2$$

$$1.732\, C_1 \omega_1 \cos\psi_1 = 0 \quad -- (10)$$

From Eqs. (9), (10) $\longrightarrow \cos\psi_1 = \cos\psi_2 = 0$

$$\therefore \psi_1 = \psi_2 = \frac{\pi}{2}$$

From Eq. (5) $\longrightarrow C_2 = \dfrac{-1.268}{1.732} = -0.732$

From Eq. (6) $\quad C_1 = \dfrac{4.732}{1.732} = 2.732$

The response of 2-DOF System will be

(2)

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 2.732 \begin{Bmatrix} 1 \\ 1.366 \end{Bmatrix} \cos \omega_1 t - 0.732 \begin{Bmatrix} 1 \\ -0.366 \end{Bmatrix} \cos \omega_2 t$$

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 2.732 \\ 3.732 \end{Bmatrix} \cos \omega_1 t + \begin{Bmatrix} -0.732 \\ 0.268 \end{Bmatrix} \cos \omega_2 t$$

The above equation shows that for the given initial condition, most of the response is due to the first mode. This is to be expected because the ratio of the initial displacement

$\begin{Bmatrix} 2 \\ 4 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$ is close to the first mode $\begin{Bmatrix} 1 \\ 1.366 \end{Bmatrix}$

and quite different from that of the second mode $\begin{Bmatrix} 1 \\ -0.366 \end{Bmatrix}$