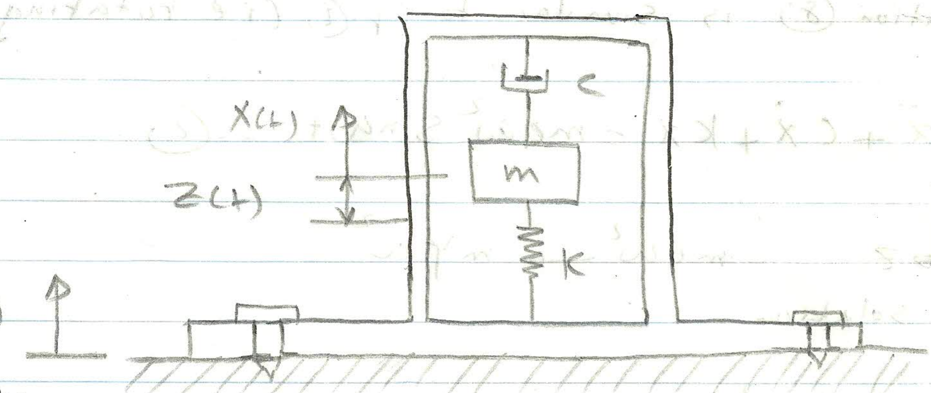
Vibration measuring instruments

There are basically three types of vibration measuring instruments namely those measuring acceleration, velocity and displacement. We shall discuss the first and the third only. These instruments consist of a case containing a spring-damper-mass system and a device measuring the displacement of the mass relative to the case which is generally measured electrically. This relative motion is converted to an electrical voltage by making the mass a magnet moving relative to coils fixed in the case. Since the voltage generated is proportional to the rate of cutting of the magnetic field, the output of the instrument will be proportional to the velocity of the vibrating body. Damping may be provided by a viscous fluid inside the case. The mass is constrained to move along a given axis as shown.

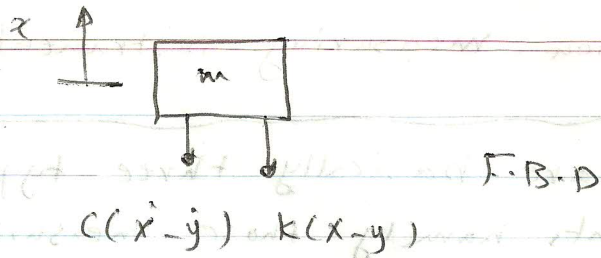


let $y(t)$ - the displacement of the case
$x(t)$ - the absolute displacement of the mass
$z(t)$ - the displacement of the mass relative to the case

$$x(t) = z(t) + y(t)$$

Applying Newton's second law

$\Sigma F = m\ddot{x}$



$$-c(\dot{x}-\dot{y}) - k(x-y) = m\ddot{x}$$

$$m\ddot{x} + c(\dot{x}-\dot{y}) + k(x-y) = 0$$

$$m\ddot{x} + m\ddot{y} - m\ddot{y} + c(\dot{x}-\dot{y}) + k(x-y) = 0$$

$$m(\ddot{x}-\ddot{y}) + c(\dot{x}-\dot{y}) + k(x-y) = -m\ddot{y}$$

$$x - y = z \qquad \dot{x}-\dot{y} = \dot{z} \qquad \ddot{x}-\ddot{y} = \ddot{z}$$

$$\therefore m\ddot{z}(t) + c\dot{z}(t) + kz(t) = -m\ddot{y}(t)$$

Now assuming sinusoidal motion of the vibrating body $y = Y\sin\omega t$

$\ddot{y} = -\omega^2 Y\sin\omega t$

$$m\ddot{z}(t) + c\dot{z}(t) + kz(t) = \omega^2 Y m \sin\omega t \quad \ldots (8)$$

Equation (8) is similar to eq. (2) (i.e rotating unbalance)

$$M\ddot{x} + c\dot{x} + kx = me\omega^2 \sin\omega t \quad \ldots (2)$$

$x \to z \qquad me\omega^2 \to mY\omega^2$

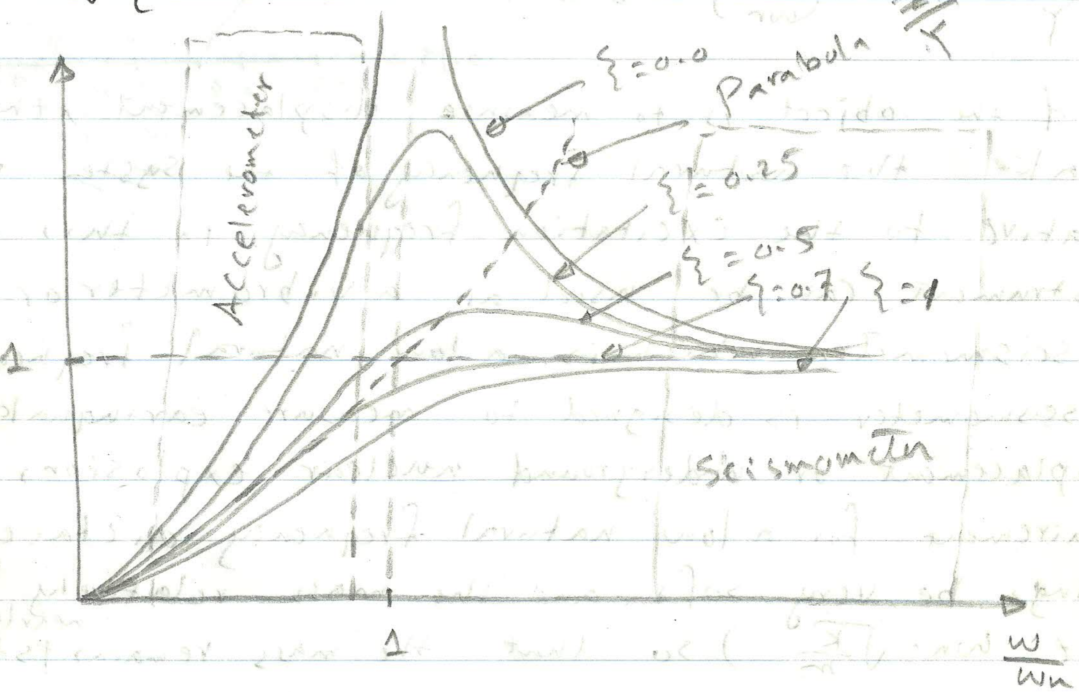
The solution

$$Z = \frac{mY\omega^2}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}} \qquad \tan\phi = \frac{c\omega}{k - m\omega^2}$$

$$Z = \frac{mY\omega^2}{k\sqrt{\left(1-\frac{m}{k}\omega^2\right)^2 + \left(\frac{c\omega}{k}\right)^2}} = \frac{Y\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

$$\tan\phi = \frac{c\omega}{k\left(1 - \frac{m}{k}\omega^2\right)} = \frac{2\zeta\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$\frac{Z}{Y} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$



$$\left|\frac{Z}{A}\right| = \text{magnification factor} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}} = H(\omega)$$

$$\frac{Z}{Y} = \left|\frac{Z}{A}\right|\left(\frac{\omega}{\omega_n}\right)^2 = H(\omega)\left(\frac{\omega}{\omega_n}\right)^2$$

For small values of $\frac{\omega}{\omega_n}$, $H(\omega) = \left|\frac{Z}{A}\right|$ is nearly unity and the amplitude Z can be approximated by

$$Z \simeq Y\left(\frac{\omega}{\omega_n}\right)^2 \rightarrow Z = \frac{Y\omega^2}{\omega_n^2} = \frac{\text{acceleration}}{\omega_n^2}$$

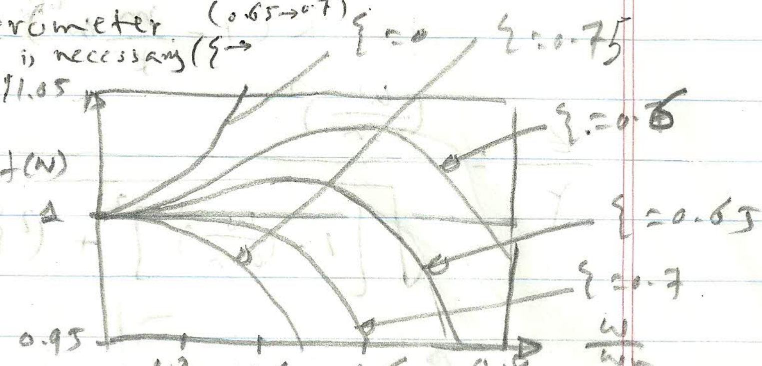
∴ Z is proportional to the acceleration of the car. Hence the amplitude ratio $\frac{Z}{Y}$ can be approximated to the parabola

$$\frac{Z}{Y} = \left(\frac{\omega}{\omega_n}\right)^2 \longleftarrow \text{Parabola equation}$$

So for this range (small values of $\frac{\omega}{\omega_n}$), the instrument Can be used as an accelerometer (0.65→0.7) accelerometer is necessary ($\xi \to$

The figure above shows larger damping ξ1.05

For large value of $\frac{\omega}{\omega_n}$



$$\frac{Z}{Y} = H(\omega)\left(\frac{\omega}{\omega_n}\right)^2 \longrightarrow 1$$

So if the object is to measure displacement, then we should make the natural frequency of the instrument very low relative to the excitation frequency. in this case the instrument Can be used as a vibrometer or seismograph or seismometer which is a low natural frequency instrument. A seismometer is designed to measure earthquake displacement or underground nuclear explosions. The requirement for a low natural frequency dictates that the springs be very soft and the mass relatively heavy (i.e $\omega_n = \sqrt{\frac{K}{m}}$) so that the mass remains nearly stationary in inertial space while the case being attached to the ground moves relative to the mass. Seismometers are generally undamped.

Seismometer are considerably larger in size than the accelerometers.

In case of using accelerometers and the interest lies in the displacement, the acceleration measured must be integrated twice with respect to time to obtain the displacement.

Prob. 3.29, page 85 (Thomson) A sensetive instrument with mass 113 kg is to be installed at a location where the acceleration is 15.24 cm/s² at a frequency of 20 HZ. It is proposed to mount the instrument on a rubber pad with the following properties: k = 2802 N/cm and $\zeta = 0.1$, what acceleration is transmitted to the instrument?

$a = Y\omega^2$ $\qquad \omega = 2\pi f = 2\pi * 20 = 40\pi$

$$Y = \frac{a}{\omega^2} = \frac{15.24}{(40\pi)^2} = 0.000965\ cm$$

$$\frac{X}{Y} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}} \qquad r = \frac{\omega}{\omega_n}$$

$$\omega_n = \sqrt{\frac{K}{m}} \qquad K = 2802\frac{N}{cm} * \frac{100\ cm}{m} = 280200\ N/m$$

$$\therefore \omega_n = \sqrt{\frac{280200}{113}} = 49.8\ rad/sec$$

$$\therefore r = \frac{\omega}{\omega_n} = \frac{40\pi}{49.8} = 2.524$$

$$\therefore X = Y\sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

$$X = 0.000965\sqrt{\frac{1+(2*0.1*2.524)^2}{(1-2.524^2)^2+(2*0.1*2.524)^2}}$$

$X = 0.0002$ cm

$\therefore$ The acceleration transmitted to the instrument $= X\omega^2$

$= 0.0002 \times (40\pi)^2 = 3.1644$ cm/sec^2

Prob. 3.43 page 87 (Thomson): An undamped vibration pickup having a natural frequency of 1 cps is used to measure a harmonic vibration of 4 cps. If the amplitude indicated by the pickup (relative amplitude between pickup mass and frame) is 0.052 cm what is the correct amplitude?

$Z = 0.052$ cm

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + \cancel{(2\zeta r)^2}}} \qquad \zeta = 0$$

$$\frac{Z}{Y} = \frac{r^2}{1-r^2} \qquad r = \frac{\omega}{\omega_n} = \frac{4}{1} = 4$$

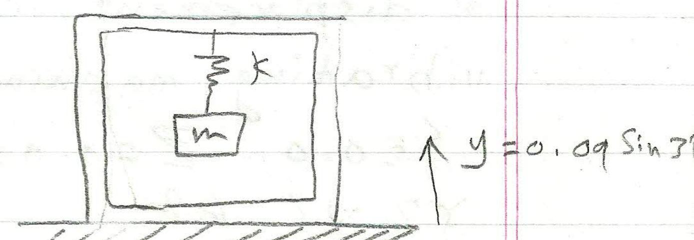
$$\frac{Z}{Y} = \frac{4^2}{1-4^2} = 1.067$$

$$\frac{0.052}{Y} = 1.067 \longrightarrow Y = 0.048734 \text{ cm}$$

Prob. A mass weighing 3.86 lb is suspended in a box by a vertical spring whose constant $K = 50\,lb/in.$ The box is placed on top of a shake table producing a vibration $y = 0.09 \sin 3t$ in. Find the absolute amplitude of the mass.

$$m = \frac{3.86}{32.2 \times 12} = 0.01 \frac{lb \cdot sec^2}{in} \qquad K = 50\,lb/in$$

$$Y = 0.09 \qquad \omega = 3\,rad/sec$$



$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{50}{0.01}}$$

$\omega_n = 70.71\,rad/sec$

$$r = \frac{\omega}{\omega_n} = \frac{3}{70.71} = 0.0424$$

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + \cancel{(2\zeta r)^2}}} \qquad \zeta = zero$$

$$\frac{Z}{Y} = \frac{r^2}{1-r^2} = \frac{0.0424^2}{1-0.0424^2} = 0.0018$$

$\therefore Z = 0.0018 \times 0.09 = 0.000162$ in

The absolute amplitude of the mass $X = Z + Y$

$X = 0.000162 + 0.09 = 0.090162$ in.

④

Prob. A vibrometer whose damping is negligible is employed to find the magnitude of vibration of a machine structure. It gives a reading of the relative displacement of 0.002 in. The natural frequency of the vibrometer is given as 300 cpm and the machine is running at 100 rpm. What will be the magnitude of displacement, velocity and acceleration of the vibrating machine part?

$\zeta = 0.0 \quad Z = 0.002$ in $\quad \omega_n = 300$ cpm $\quad \omega = 100$ rpm

$Y? \; V? \; A?$

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + \cancel{(2\zeta r)^2}}} \quad \text{zero} \qquad \frac{Z}{Y} = \frac{r^2}{1-r^2}$$

$$r = \frac{\omega}{\omega_n} = \frac{100}{300} = \frac{1}{3}$$

$$\frac{0.002}{Y} = \frac{\left(\frac{1}{3}\right)^2}{1-\left(\frac{1}{3}\right)^2} = 0.125$$

$$\therefore Y = \frac{0.002}{0.125} = 0.016 \text{ inch.}$$

$$V = \omega Y = \left(\frac{2\pi * 100}{60}\right) * 0.016 = 10.472 * 0.016$$

$V = 0.167552$ in/sec

$$A = Y\omega^2 = 0.016 * (10.472)^2 = 1.7546 \text{ in/sec}^2$$