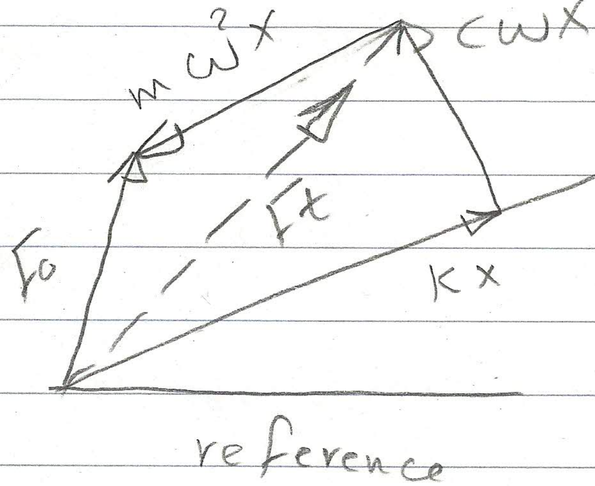
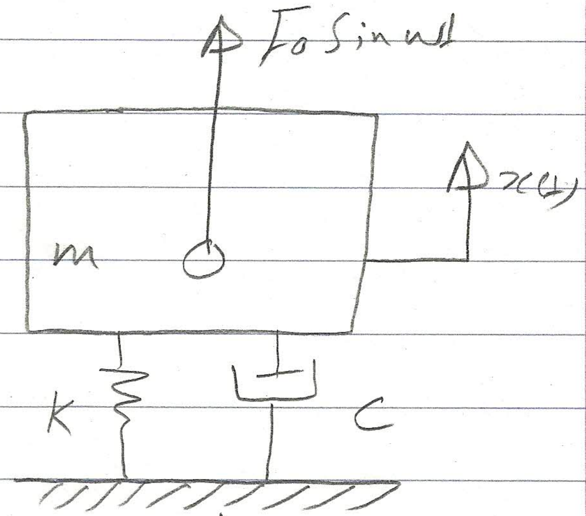
Vibration Isolation :-

It is required to transmit as little vibration as possible to the base. Clearly the force transmitted to the base is through springs and dampers, which are referred to as isolators.





let F_t - transmitted force through the spring and damper.

$$F_t = \sqrt{(kX)^2 + (c\omega X)^2} = kX\sqrt{1 + \left(\frac{c\omega}{k}\right)^2}$$

But X = Amplitude $= \dfrac{F_o}{K\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$

which represents the response to harmonic excitation. knowing that $\frac{c\omega}{k} = 2\zeta\frac{\omega}{\omega_n}$

$$\therefore F_t = \frac{F_o}{K} K \sqrt{\frac{1 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

1

$$\therefore \frac{F_t}{F_o} = \sqrt{\frac{1+[2\zeta \frac{\omega}{\omega_r}]^2}{[1-(\frac{\omega}{\omega_r})^2]^2+[2\zeta\frac{\omega}{\omega_r}]^2}} = \left|\frac{X}{Y}\right|$$

$$\therefore \frac{F_t}{F_o} = \left|\frac{X}{K}\right|$$

Thus the problem of exciting a mass by the motion of the support point is identical to the problem of isolating disturbing force.

let $TR = \frac{F_t}{F_o} = \left|\frac{X}{Y}\right|$ is called the transmissibility of force or displacement

when damping is negligible (i.e. $\zeta = 0$) and for $TR < 1$ (i.e $\frac{\omega}{\omega_r} > \sqrt{2}$)

$$TR = \frac{1}{(\frac{\omega}{\omega_r})^2 - 1}$$

page 67 (Thomson)

2

Ex. 3.6.1 / A machine of 100 kg mass is supported on springs of total stiffness 700 kN/m and has an unbalanced rotating element, which results in a disturbing force of 350 N at a speed of 3000 rpm. Assuming a damping ratio of $\zeta = 0.2$ determine (a) its amplitude of motion due to the unbalance, (b) the transmissibility and (c) the transmitted force.

$m = 100\,kg \quad k = 700\,kN/m \quad F_0 = 350\,N \quad \omega = 3000\,rpm$

$\zeta = 0.2 \quad X? \quad TR? \quad F_t?$

$$\delta_{st} = \frac{mg}{k} = \frac{100 * 9.8}{700 * 10^3} = 1.4 * 10^{-3}\,m$$

$$\omega_n = \sqrt{\frac{g}{\delta_{st}}} = \sqrt{\frac{9.8}{1.4 * 10^{-3}}} = 83.67\,rad/sec$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi * \cancel{3000}^{50}}{\cancel{60}_{1}} = 314.16\,rad/sec$$

$$r = \frac{\omega}{\omega_n} = \frac{314.16}{83.67} = 3.75$$

$$X = \frac{F_0}{k\sqrt{[1-r^2]^2 + [2\zeta r]^2}} = \frac{\cancel{350}}{\cancel{700}_{2} * 10^{+3}\sqrt{[1-3.75^2]^2 + [2*0.2*3.75]^2}}$$

$$X = \frac{1}{2 * 13.148 * 10^{+3}} = 0.038 * 10^{-3}\,m = 0.038\,mm$$

$$TR = \frac{\sqrt{1 + (2r\zeta)^2}}{\sqrt{[1-r^2]^2 + [2r\zeta]^2}} = \frac{\sqrt{1 + (2*0.2*3.75)^2}}{\sqrt{[1-3.75^2]^2 + [2*0.2*3.75]^2}} = \frac{1.803}{13.148}$$

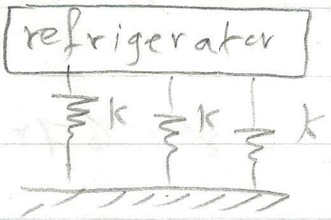
$\therefore TR = 0.137$

$F_t = F_o * TR = 0.137 * 350 = 47.95 N$

Prob. 3.25 page 85 (Thomson). A refrigerator unit weighing 65 lb is to be supported by three springs of stiffness k lb/in. each. If the unit operates at 580 rpm, what should be the value of the spring constant k if only 10% of the shaking force of the unit is to be transmitted to the supporting structure?

$TR = 0.1$

$m = \frac{65}{32.2 * 12}$



$\omega = 580 \ rpm$

The springs are connected in parallel manner

$K_{eq} = 3k$. Since $TR < 1$

$TR = \frac{1}{r^2 - 1}$ $\quad r = \frac{\omega}{\omega_n}$ $\quad \omega = \frac{2\pi N}{60}$

$\omega = \frac{2\pi * 580}{60} = 60.74 \ rad/sec$

$0.1 = \frac{1}{r^2 - 1} \longrightarrow r^2 - 1 = \frac{1}{0.1} = 10 \longrightarrow r^2 = 11$

$r = 3.3166 = \frac{\omega}{\omega_n} \longrightarrow \omega_n = \frac{60.74}{3.3166} = 18.313 \ rad/sec$

$\therefore K_{eq} = m\omega_n^2 = \frac{65}{32.2 * 12} * 18.313^2 = 56.4126 \ lb/in$

$\therefore k = \frac{K_{eq}}{3} = \frac{56.4126}{3} = 18.8 \ lb/in$