communcation II
9\5\2013
Lecture 13

Signal space diagram of M-ary PSK:-

$$s(t) = \sqrt{2P_s} \cos(2\pi f_0 t + \phi_m)$$

$$s(t) = \sqrt{2P_s} \cos(2\pi f_0 t) \cos\phi_m - \sqrt{2P_s} \sin\phi_m \sin(2\pi f_0 t)$$

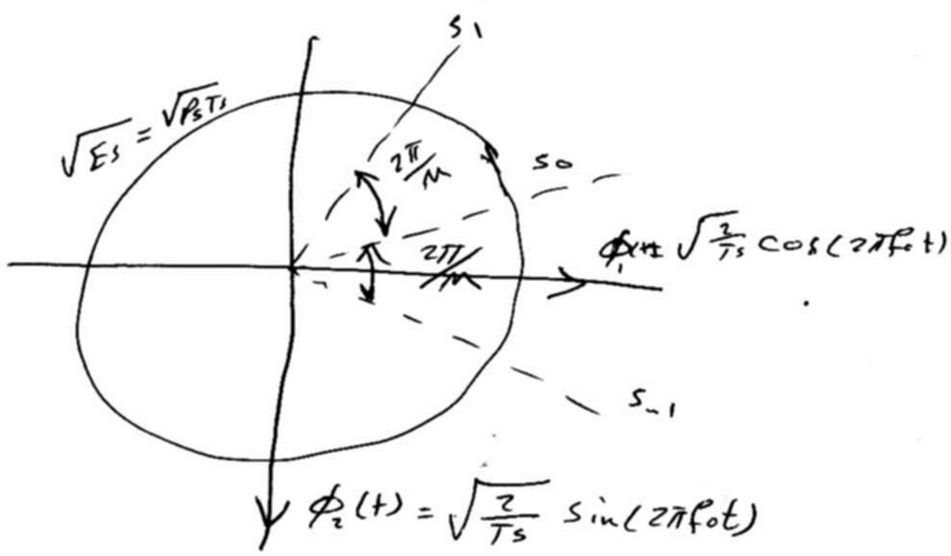
rearrange

$$s(t) = \sqrt{P_s T_s} \cdot \sqrt{\frac{2}{T_s}} \cos\phi_m \cos(2\pi f_0 t) - \sqrt{P_s T_s} \cdot \sqrt{\frac{2}{T_s}} \sin\phi_m \sin(2\pi f_0 t)$$

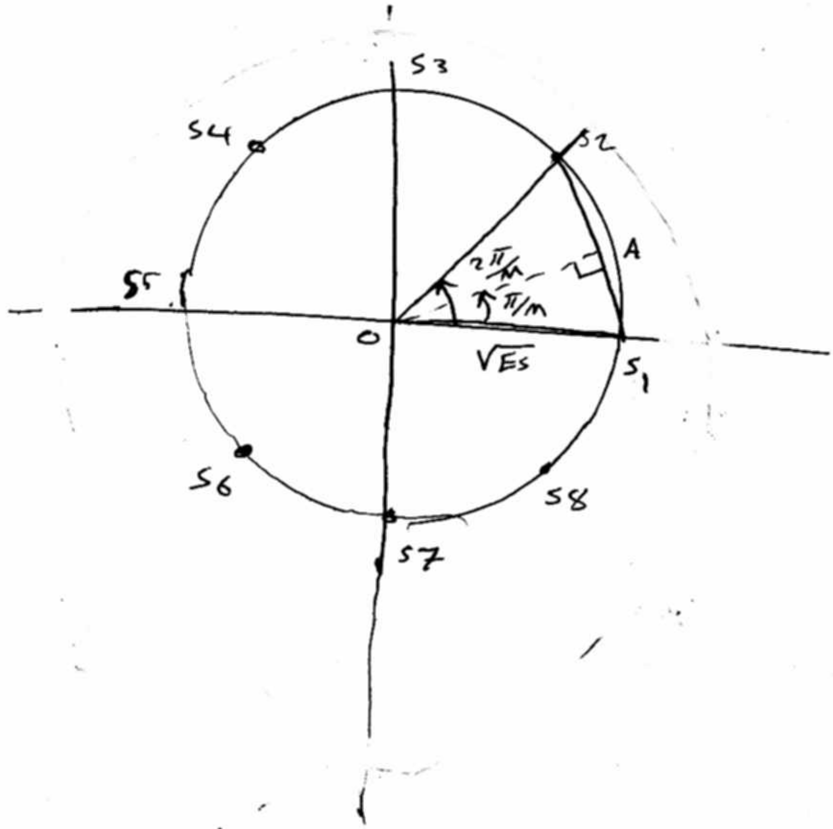
let $\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_0 t)$
$\phi_2(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_0 t)$] orthognal

$$\therefore s(t) = \sqrt{P_s T_s} \cos\phi_m \, \phi_1(t) - \sqrt{P_s T_s} \sin\phi_m \, \phi_2(t)$$

The signal points $s_0, s_1, s_2, \ldots s_{m-1}$ are placed on the circumference of the circle. The signal points are equispaced with phase shift of $\frac{2\pi}{M}$



(67)



$\Delta S_1 O A$:-

signal space diagram for M-ary PSK to find the euclidean distance. Here $M=8$

d = distance between S_1 and S_2

$$S_1 A = S_2 A = \frac{d_{12}}{2}$$

$$\frac{\text{Distance } S_1 A}{\text{Distance } OS_1} = \sin\frac{\pi}{M}$$

$$\frac{d_{12}/2}{\sqrt{E_s}} = \sin\pi/M$$

$$\boxed{d_{12} = 2\sqrt{E_s}\sin\frac{\pi}{M}}$$

if $M=8$ $N=3$

$$d = 2\sqrt{E_s}\sin\frac{\pi}{8}$$ [8-ary PSK]

if $M=4$

$$d = 2\sqrt{E_s}\sin\pi/4$$ [QPSK ≡ 4-ary PSK]

$$d = 2\sqrt{E_s} * \frac{1}{\sqrt{2}}$$

$$\boxed{d = \sqrt{2E_s}}$$ for QPSK

Transmitter and receiver of M-ary PSK:-

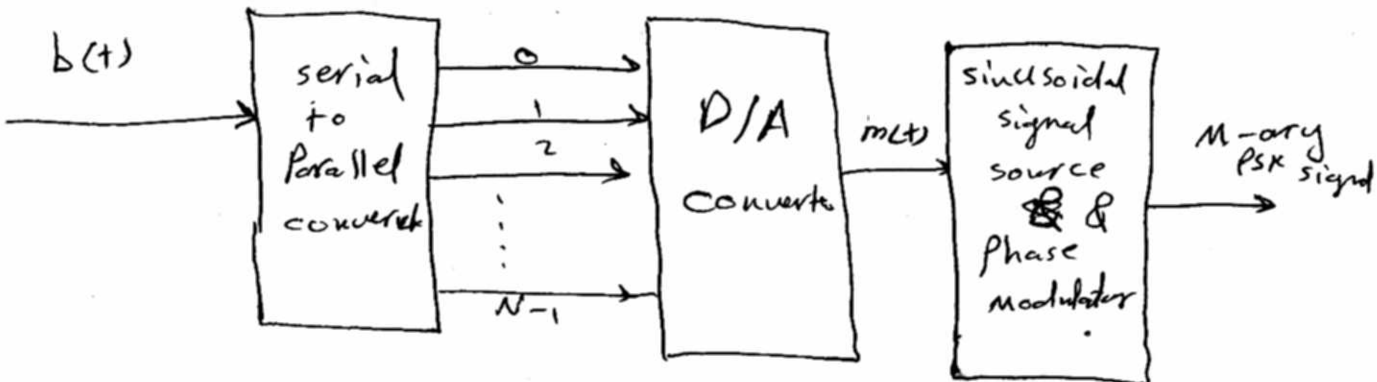
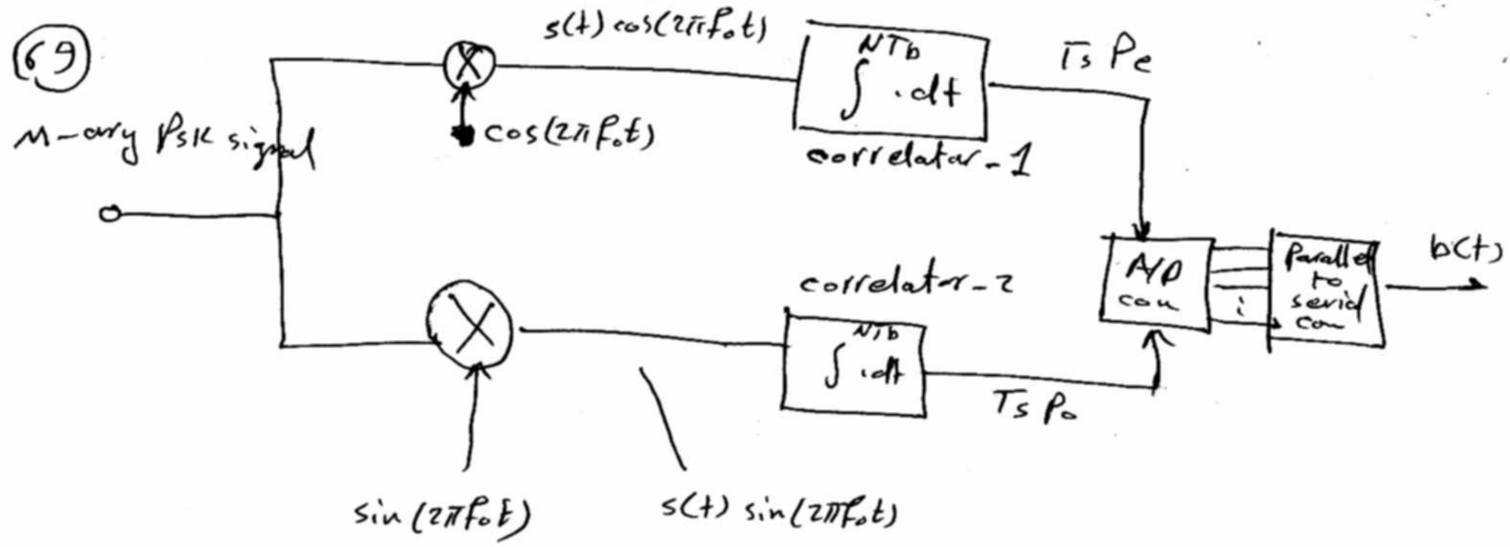


Fig : Tx of M-ary

The D/A converter output remains unchanged till last N^{th} bit is received. Then there are $2^N = M$ different values depending upon the input bits. The output of D/A converter $m(t)$ is applied to modulator which modulates the Phase of sinusoidal carrier depending upon the amplitude of the $m(t)$.

M-ary PSK receiver:- The reciever is simillar to QPSK receiver. The signal is multiplied by coherent carriers. The output of the multipliers are given to the integrators which integrate over a period cycle and applied to the A/D converter. The integrator o/p Proportional to $T_s P_s$

(69)



"M-ory PSK receiver"

A/D converter, reconstructs "N" bit symbol. which is converted to b(t) sequence.

Ex: Draw the 4-ory PSK which has the following wave form

$$s_i(t) = 4\cos\left(2\pi f_c t + \frac{i\pi}{2}\right)$$

where $i = 0, 1, 2, 3$ and $0 \le t \le T_s$ or $0 \le t \le 2T_c$

Sol

$$s(t) = \sqrt{2P_s}\cos(2\pi f_0 t + \phi_m)$$

$$\phi_m = (2m+1)\frac{\pi}{M}$$

comparing with

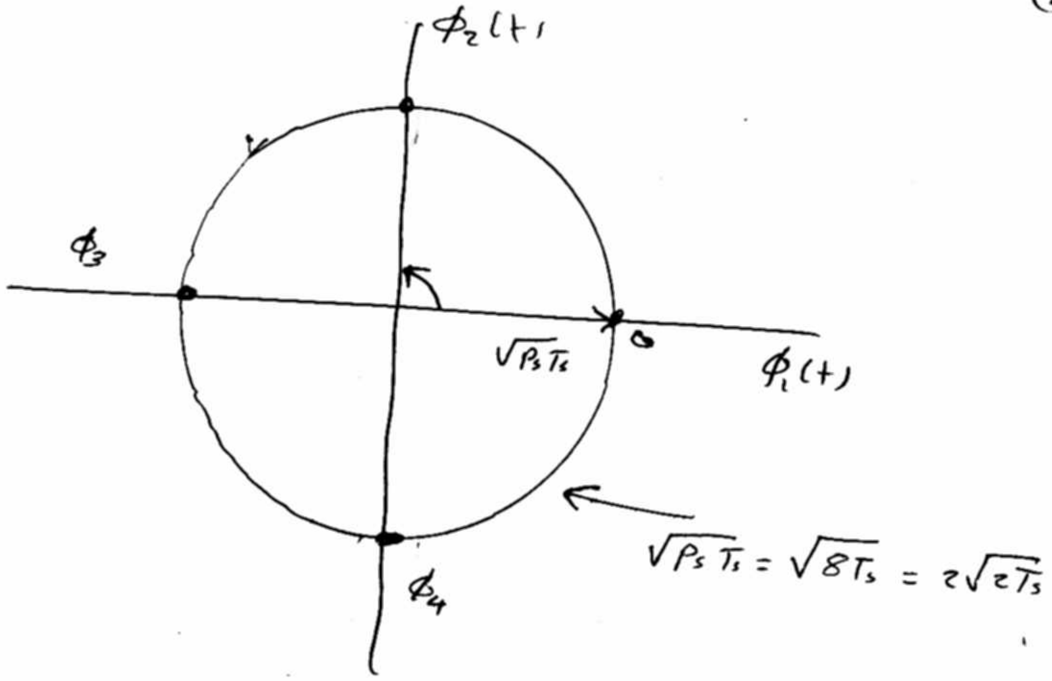
$$s_i(t) = 4\cos\left(2\pi f_c t + \frac{i\pi}{2}\right)$$

$$\sqrt{2P_s} = 4$$

$$2P_s = 16 \qquad P_s = 8$$

$$\phi_i = \phi_m = \frac{i\pi}{2} \quad : \quad i = 0, 1, 2, 3$$

$$\phi_0 = 0, \quad \phi_1 = \pi/2, \quad \phi_2 = \pi, \quad \phi_3 = \frac{3\pi}{2}$$



BW for M-ary PSK

$$BW = \frac{2f_b}{N}$$ — QPSK

where N is the Number of bit used for one symbol

if $M = 16$ and bit rate is 1 MHZ

$$N = \log_2 M$$

$$= \log_2 16 = \frac{\log_{10} 16}{\log_{10} 2} = 4$$

$$BW = \frac{2 * 1 * 10^6}{4} = 500\,KHZ$$

while if QPSK is used

$$BW = f_b$$

$$= 1\,MHZ$$

reduction in BW

(21) $S_B(f) = 2P_s N T_b \left[\frac{\sin(\pi f N T_b)}{\pi f N T_b} \right]^2$ Power spetral density

↑ general

For $M = 16$

$N = \log_2 M = \log_2 16 = 4$

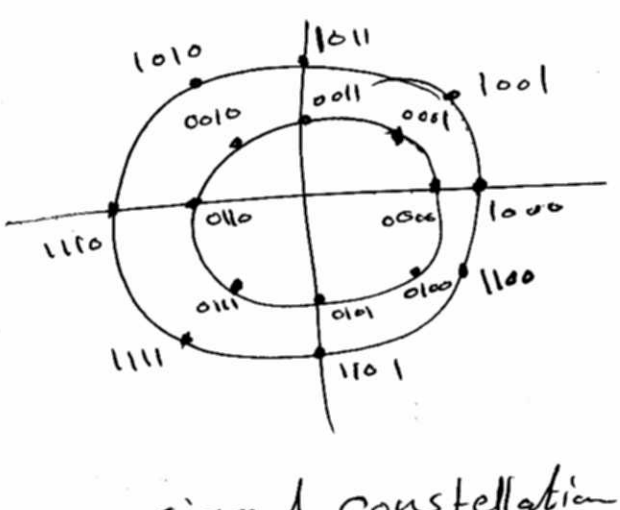
$\therefore$

$$S_B(f) = 2P_s * 4 * T_b \left[\frac{\sin(\pi f 4 T_b)}{4\pi f T_b} \right]^2$$

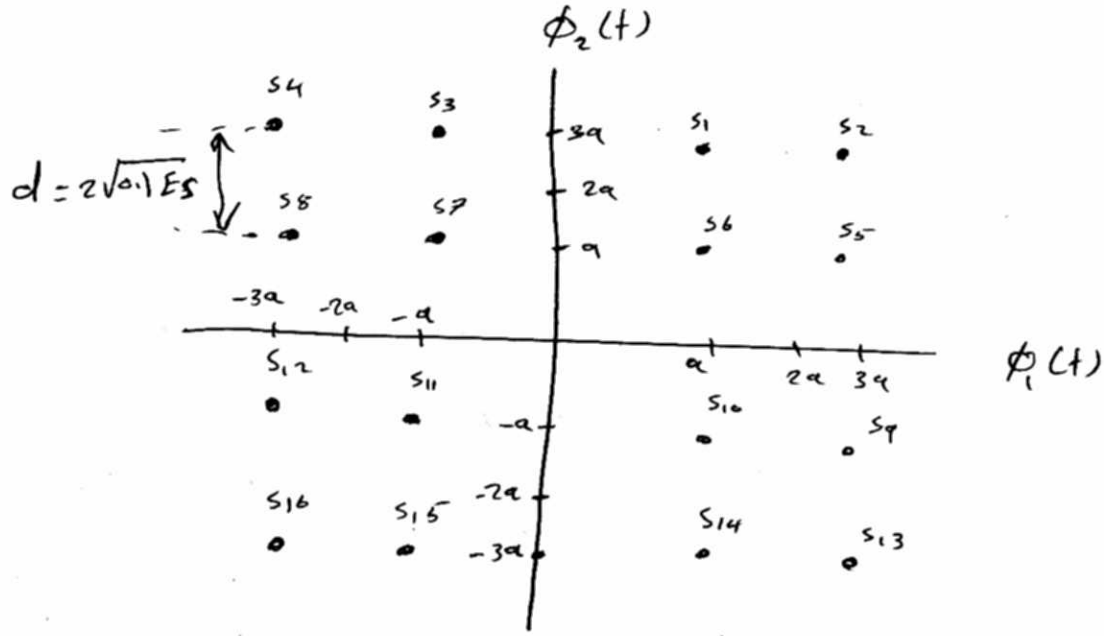
$$\boxed{S_B(f) = 8 P_s T_b \left[\frac{\sin(\pi 4 T_b)}{4\pi f T_b} \right]^2}$$

Quadrature amplitude shift keying (QASK) or QAM

If the amplitude of the signal is also varied, then the points will lie inside the circle also on the signal space diagram. This will increase the immunity of the system. This system ivolves phase as well as amplitude shift keying.



signal constellation



Geometrical representation of 16 signals in QASK system

Considering first quadrant:-

$$E_s = \frac{1}{4}\left[(a^2+a^2)+(9a^2+a^2)+(a^2+9a^2)+(9a^2+9a^2)\right]$$

$$E_s = 10a^2$$

$$a = \sqrt{0.1 E_s}$$

$$d = 2a \quad \text{sub } (a)$$

$$d = 2\sqrt{0.1 E_s} \Rightarrow d = \sqrt{0.4 E_s}$$

$$E_s = 4E_b$$

$$d_{QASK} = 2\sqrt{0.4 E_b} \Rightarrow d = \sqrt{1.6 E_b}$$

$$d_{16\text{-}PSK} = 2\sqrt{0.15 E_b}$$

(16 [redacted]PSK)

$$d_{QASK} > d_{16\text{-}PSK}$$

(QASK [redacted])

Transmitter and Receiver of QASK :-

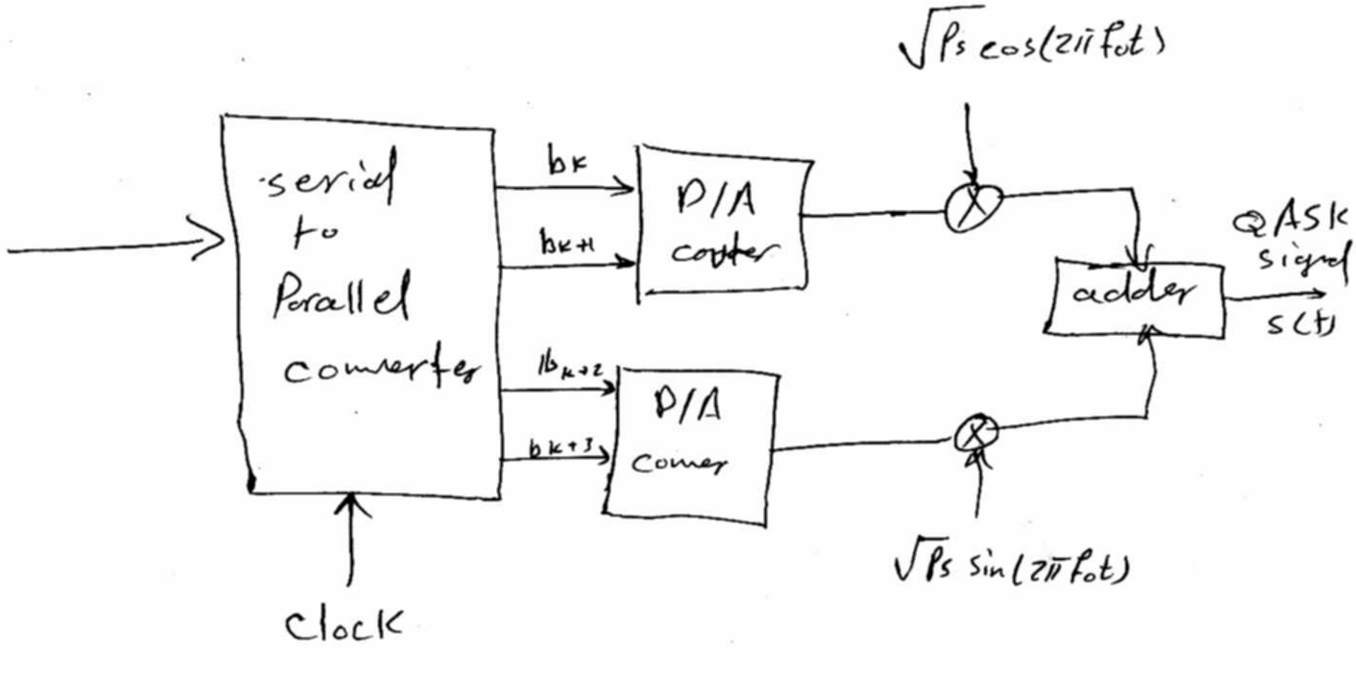
$$s(t) = K_1 \sqrt{0.2 P_s} \cos(2\pi f_0 t) + K_2 \sqrt{0.2 P_s} \sin(2\pi f_0 t)$$

K_1, K_2 = amplitude of the modulated signal

The input bit stream is applied to a serial to Parallel converter. Four succesive bits are applied to D/A converters. These bits are applied after every T_s second. T_s is the symbol Period and $T_s = 4T_b$

b_k, b_{k+1} → upper D/A

b_{k+2}, b_{k+3} → lower D/A

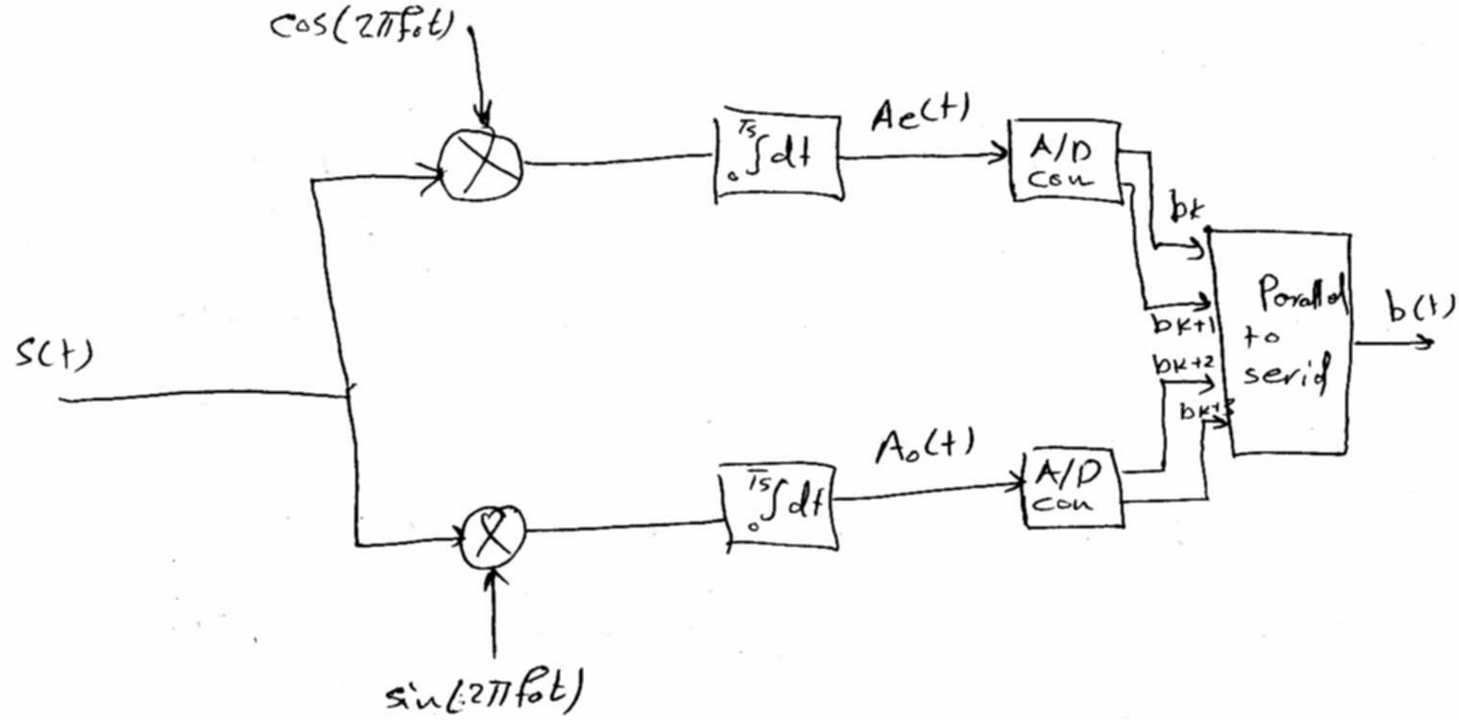


$$s(t) = A_e(t) \sqrt{P_s} \cos(2\pi f_0 t) + A_o(t) \sqrt{P_s} \sin(2\pi f_0 t)$$

$A_e(t)$ and $A_o(t) = \mp\sqrt{0.2}$ or $\mp 3\sqrt{0.2}$

(depending on D/A Converter)

Receiver of QASK signal :-



4-bit QASK receiver BD

Bw

$$Bw = \frac{2f_b}{N}$$ similar to M-ary PSK

M-ary FSK :-

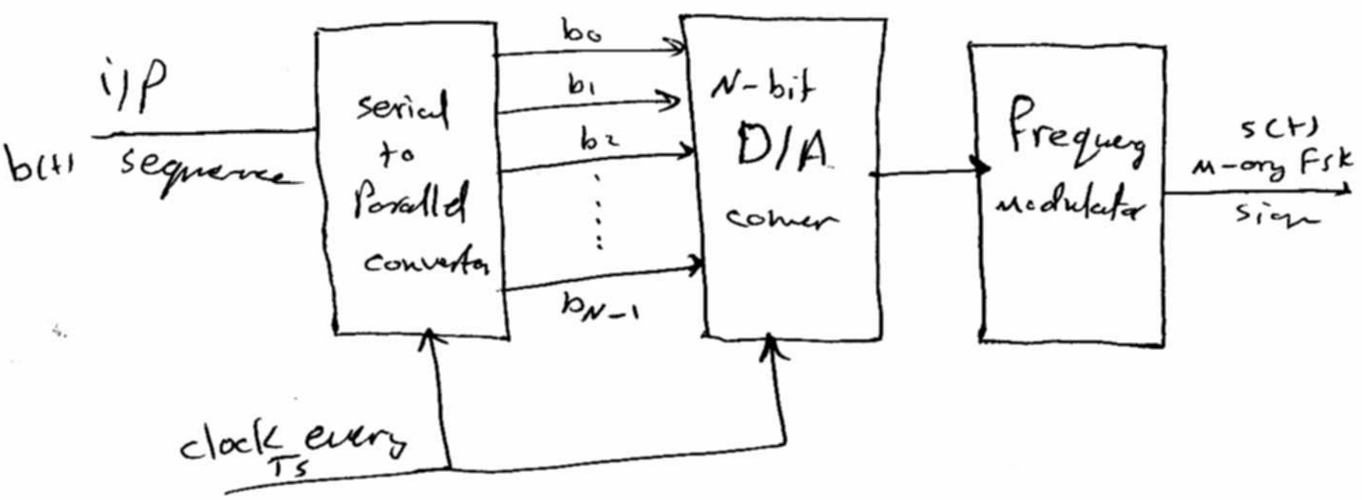
The Principle of BFSK is extended further to "N" succesive bits. These "N" bits from $2^N = M$ different symbols Every symbol uses separate frequency transmission and reception of M-ary FSK.

Transmitter and Receiver of M-ary FSK :-

Transmitter:-

There are a total of $2^N = M$ Possible Symbols. The Symbol is Presented every $T_s = NT_b$. The output of digital to analog converter is given to a frequency modulator. depending on the Symbol, the frequency modulator generats the o/p frequency. For every symbol, the frequency modulator Producess differet frequency output.

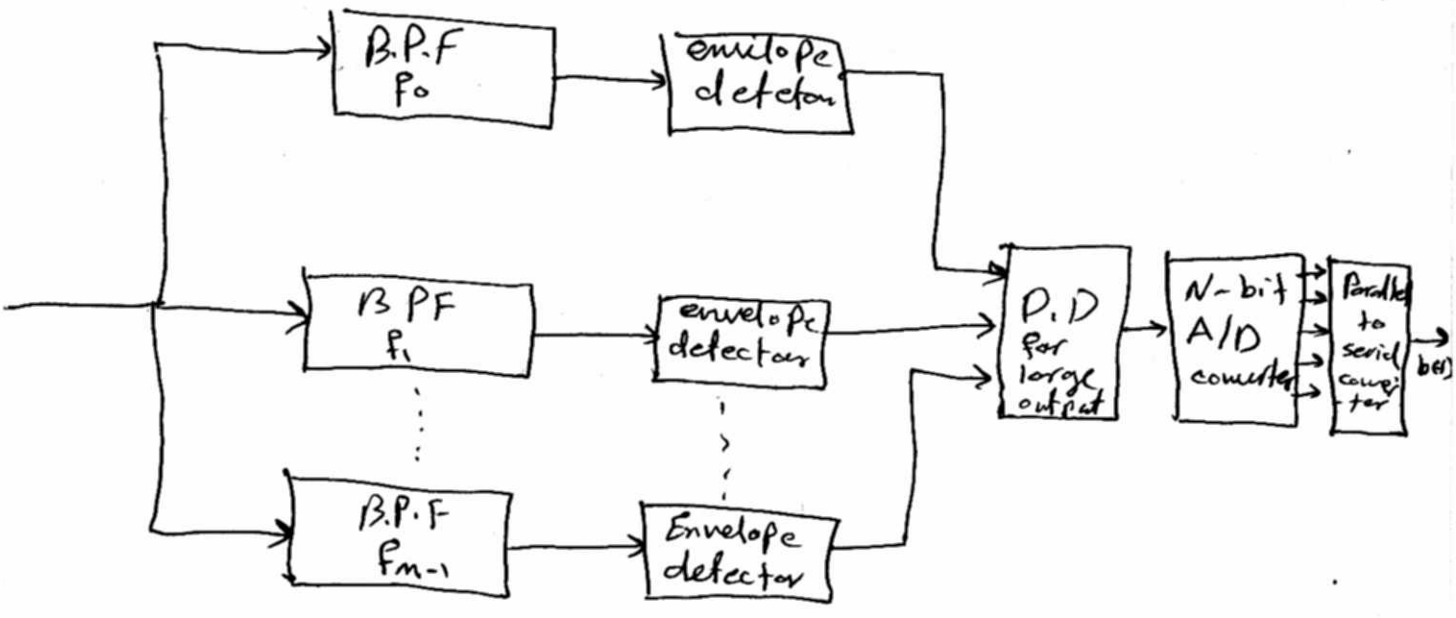
The transmitted frequencies are, f_0, f_1, f_2, f_{m-1}, depending upon the input Symbol.



"M-ary transmitter"

Receiver

The block diagram is shown in fig below. The M-ary FSK signal is given to the set M - B.P.F The center frequencies of those are f_0, f_1, f_2, f_{M-1}. The decision device Produces output depending on the highest input. Depending upon the Particular Symbol, only one envelope detector will have higher o/p, the rest of the detector are very low.



"Block diagram of M-ary FSK system"

Power SPectral density and Bandwidth of M-ary FSK

There are M-symbol $(f_0, f_1, f_2 --- f_{M-1})$ frequencies are used for transmission. The Pe is minimized by selecting those frequencies such that transmitted signals are mutually orthogonal.

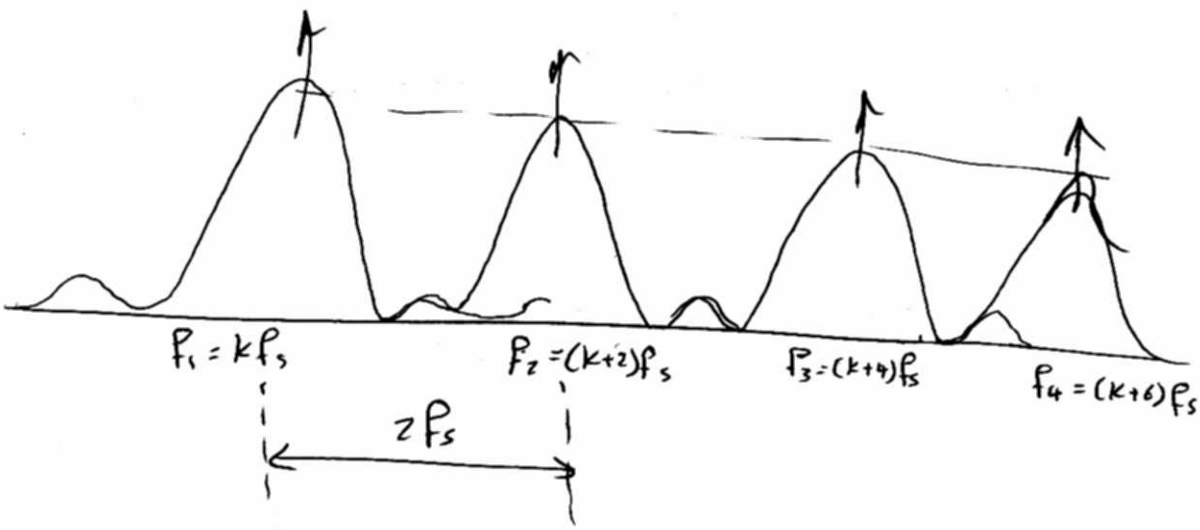
(77) $f_0 = K f_s$

$f_1 = (K+2) f_s$

$f_2 = (K+4) f_s$ ..... etc

} orthogonal (harmonics)

f_L and f_H are separated by $2f_s$



f_s : is the symbol frequency not f_b

The separation between the two nearest main lobes is $2f_s$

Bw of Mary FSK :- There will be M- lobes

$$Bw = M(2f_s)$$

$$2^N = M$$

$$f_s = \frac{f_b}{N}$$

$$Bw = 2 \cdot 2^N \frac{f_b}{N}$$

$$Bw = \frac{2^{N+1} \cdot f_b}{N}$$

$$Bw_{M\text{-ary FSK}} > Bw_{M\text{-ary PSK}}$$

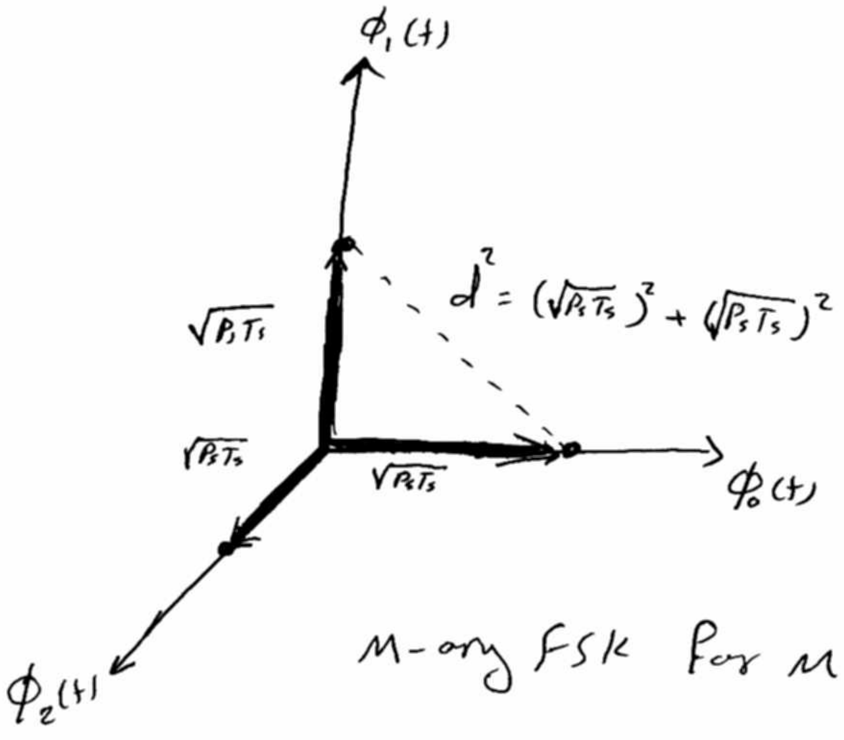
Geometrical Representation of M-ary FSK signal space

$$S_0(t) = \sqrt{P_s T_s}\ \phi_0(t)$$
$$S_1(t) = \sqrt{P_s T_s}\ \phi_1(t)$$
$$S_2(t) = \sqrt{P_s T_s}\ \phi_2(t)$$
$$\vdots$$
$$S_m(t) = \sqrt{P_s T_s}\ \phi_{M-1}(t)$$

where:

$\phi_0(t), \phi_1(t), \phi_2(t) \text{ ---- } \phi_{M-1}(t)$ are orthogonal.

$$\phi_0(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_0 t)$$
$$\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_1 t)$$
$$\vdots$$
$$\phi_{M-1}(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_{M-1} t)$$



M-ary FSK for M=3

(79) $d^2 = 2 P_s T_s$

$d = \sqrt{2 E_s}$ $E_s = N E_b$

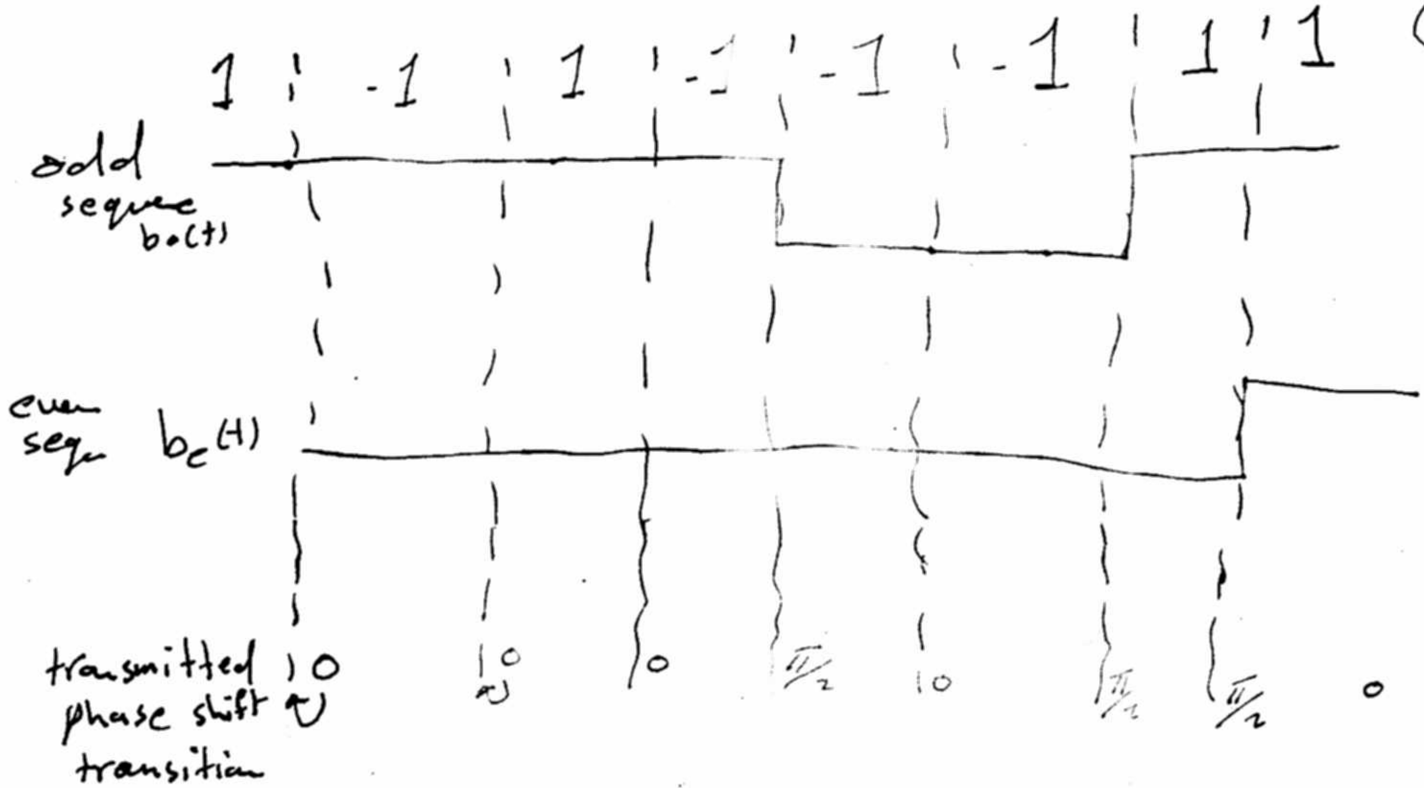
$$\boxed{d = \sqrt{2 N E_b}}$$

distance between any two signal points.

The distance for M-ary FSK is greater compared to M-ary PSK and QPSK

Q/ For input binary sequence {1, -1, 1, -1, -1, -1, 1, 1} find the transmitted phase shift using QPSK

(80)



M_ary PSK :- BPSK transmits one bit at a time and has only two symbols

$$\text{phase shift} = \frac{2\pi}{\text{Number of symbols}} = \frac{2\pi}{2} = 180$$

$$\text{phase shift}_{QPSK} \quad \frac{2\pi}{4} = \pi/2 \text{ or } 90^\circ$$

if "N" Number of symbols then the phase shift is:-

$$\text{phase shift M-ary PSK} = \frac{2\pi}{\text{number of symbols}} = \frac{2\pi}{M}$$

The G.E for M-ary PSK

$$S(t) = \sqrt{2P_s} \cos(2\pi f_0 t + \phi_m)$$

$m = 0, 1, 2 \ldots M-1$

The symbol phase $\phi_m = (2m+1)\frac{\pi}{M}$