❖ Type of Models and System Modeling Approaches

- **Models**:
  mathematical –other
  parametric–nonparametric
  continuous-time –discrete-time
  input/output –state-space
  linear –nonlinear
  dynamic – static
  time-invariant – time-varying
  SISO – MIMO

- **Modelling / System Identification**:
- **physical (theoretical)** – experimental
  white-box – **grey-box** – black-box
- **structure determination** – parameter estimation
- **time-domain** – frequency-domain

❖ Types of Mathematical Models

• Parametric and Non-parametric Models

Many approaches to system modelling, depending on model class
- linear/nonlinear
- parametric/nonparametric

Non-parametric methods try to estimate a generic model of a system
– step responses, impulse responses, frequency responses, etc.
Parametric methods estimate parameters in a user- specified model
– parameters in transfer functions, state-space matrices of a given order, etc.

• Linear and Nonlinear Models

The system modelling methods are characterized by model type:

A. Linear model: Classical system identification

B. Neural network: Strongly non-linear systems with complicated structures – no relation to the actual physical structures/parameters (will not be covered)

C. General simulation model: Any mathematical model, that can be simulated e.g. with Matlab/Simulink. It requires a realistic physical model structure, typically developed by theoretical modelling

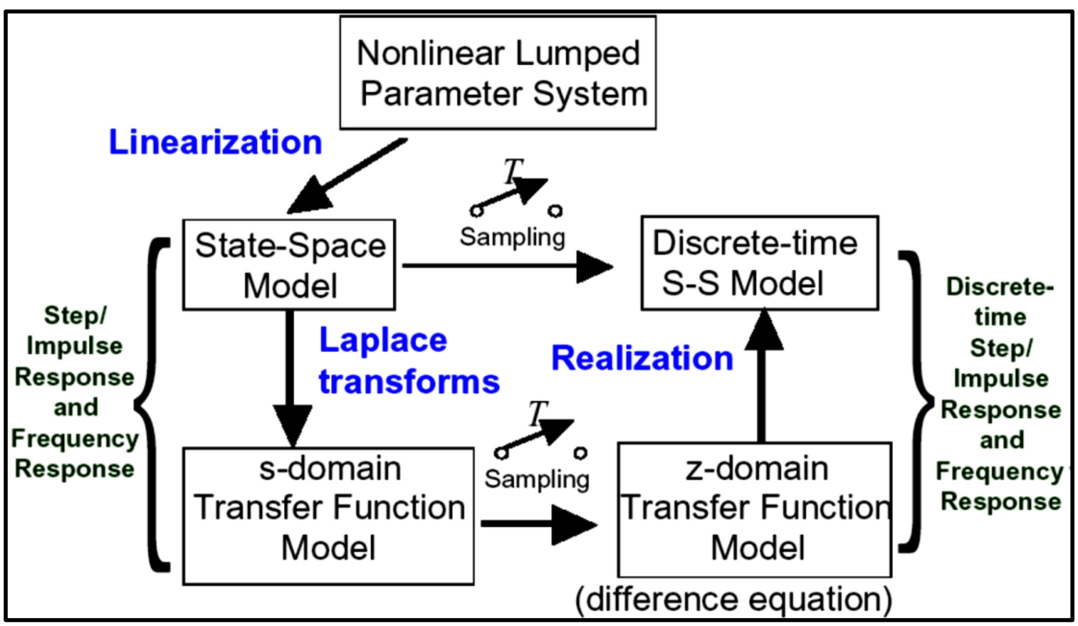
- **Purpose of Models**

 Models can also be classified according to purpose:

- **Models to assist plant design and operation**
  - Detailed, physically based, often non-dynamic models to assist in fixing plant dimensions and other basic parameters
  - Economic models allowing the size and product mix of a projected plant to be selected
  - Economic models to assist decisions on plant renovation

- **Models to assist control system design and operation**
  - Fairly complete dynamic model, valid over a wide range of process operation to assist detailed quantitative design of a control system
  - Simple models based on crude approximation to the plant, but including some economically quantifiable variables, to allow the scope and type of a proposed control system to be decided
  - Reduced dynamic models for use on-line as part of a control system

Systems/Models Representations



The Modeling Process

1. **Define the purpose or objective of the model**

Identify system boundaries, functional blocks, interconnecting variables, inputs and outputs. Construct a functional block diagram.

2. Determine the model for each component or subsystem

Apply known physical laws when possible, otherwise use experimental data to identify input-output relationships - system identification.

3. Integrate the subsystem models into an overall system model

Combine equations, eliminate variables, check for sufficient equations to solve the system.

4. Verify the model validity and accuracy

Implement a *simulation* of the model equations and compare with experimental data for the same conditions

5. Make simplifications to create an approximate model suitable for design

–Linearization of model equations

–Reduce the order of the model by eliminating unimportant dynamics dynamics

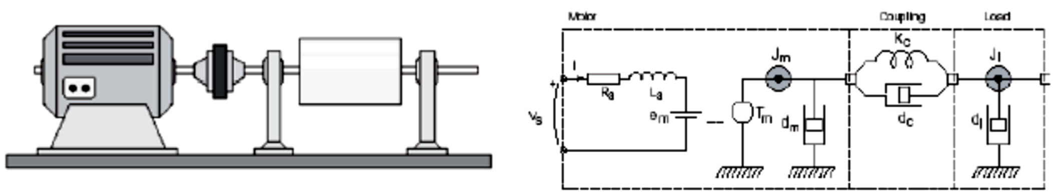
trade-off

Model Complexity ⟷ Model Accuracy

❖ Specified Procedure of System Modeling

- Divide the system into idealized components
- Apply physical laws to the elements
- Apply interconnection laws between elements
- Combine the equations to obtain the mode

More detailed schematic



State equations

$$L_a \frac{di}{dt} = V_s - R_a i - k_m \omega_m$$

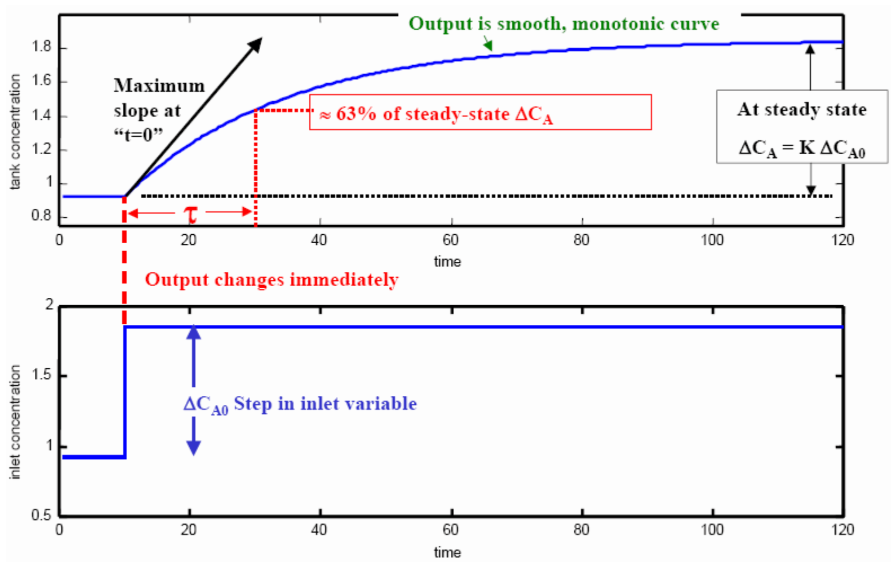
$$\frac{d\theta_m}{dt} = \omega_m \qquad J_m \frac{d\omega_m}{dt} = k_m i - d_m \omega_m - k_c(\theta_m - \theta_l) - d_c(\omega_m - \omega_c)$$

$$\frac{d\theta_l}{dt} = \omega_l \qquad J_l \frac{d\omega_l}{dt} = -d_l \omega_l - k_c(\theta_l - \theta_m) - d_c(\omega_l - \omega_m)$$

Analysis of Systems

Dynamic models obtained from modelling step will involve differential/algebraic equations

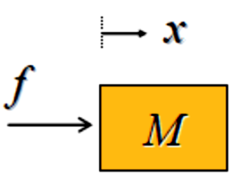
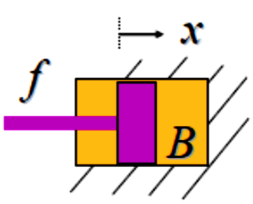
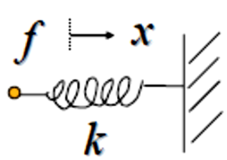
- We can solve simple models analytically to provide information on relationship between process and dynamic response
- We can solve complex models numerically, e.g. using Euler or Runge- Kutta method with computer simulation – relevant to ENGR 391- numerical methods in engineering.



Sample time response analysis of a system

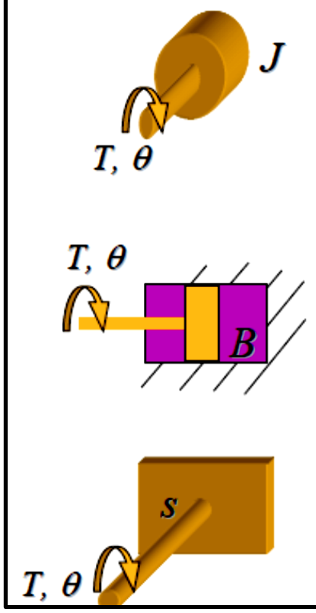
❖ Overview of Element Models in Physical Systems

Mechanical Translational Models

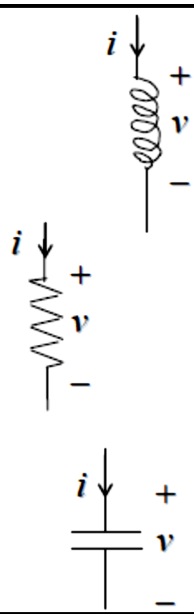
	force/velocity	force/position
Mass	$f = M\,dv/dt$	$f = M\,dx^2/dt^2$
Viscous friction	$f = B\,v$	$f = B\,dx/dt$
Spring	$f = k\int v\,dt$	$f = k\,x$

Mechanical Rotational Models



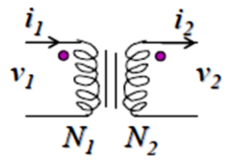
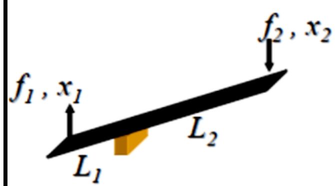
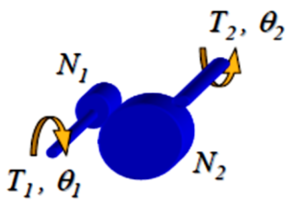
	torque/velocity	torque/position
Inertia	$T = J\,d\omega/dt$	$T = J\,d\theta^2/dt^2$
Viscous friction	$T = B\,\omega$	$T = B\,d\theta/dt$
Stiffness	$T = s\int \omega\,dt$	$T = s\;\theta$

Electrical Component Models



	voltage/current	voltage/charge
Inductance	$v = L\,di/dt$	$v = L\,dq^2/dt^2$
Resistance	$v = R\,i$	$v = R\,dq/dt$
Capacitance	$v = 1/C\int i\,dt$	$v = 1/C\,q$

Transformation Models

	Transformer	$\frac{v_1}{v_2}=\frac{N_1}{N_2}$	$\frac{i_1}{i_2}=\frac{N_2}{N_1}$
	Lever	$\frac{f_1}{f_2}=\frac{L_2}{L_1}$	$\frac{x_1}{x_2}=\frac{L_1}{L_2}$
	Gears	$\frac{T_1}{T_2}=\frac{N_1}{N_2}$	$\frac{\theta_1}{\theta_2}=\frac{N_2}{N_1}$

❖ Mathematical Modelling of Mechanical Systems

Elementary parts

- •A means for storing kinetic energy (mass or inertia)
- •A means for storing potential energy (spring or elasticity)
- •A means by which energy is gradually dissipated (damper)

Motion in mechanical systems can be

- Translational
- Rotational, or
- Combination of above

Mechanical systems can be of two types

- Translational systems
- Rotational systems

Variables that describe motion

- Displacement, x
- Velocity, v
- Acceleration, a

❖ Modeling of translational mechanical systems

Key concepts to remember

- **Three primary elements of interest**
  - – Mass (inertia) *m*
  - – Stiffness (spring) *K*
  - – Friction - Dissipation (damper) *B*
  - – Usually we deal with "equivalent" *m, B, K*
    - Distributed mass -> lumped mass
- **Lumped parameters**
  - – Mass maintains motion
  - – Stiffness restores motion
  - – Damping eliminates motion

Modeling of translational mechanical systems

Variables

- x: displacement (m)
- v: velocity (m/sec)
- a: acceleration (m/sec^2)
- f: force (N)
- p: power (Nm/sec)
- w: work (energy) (Nm)

All these variables are functions of time, *t*

Element Laws

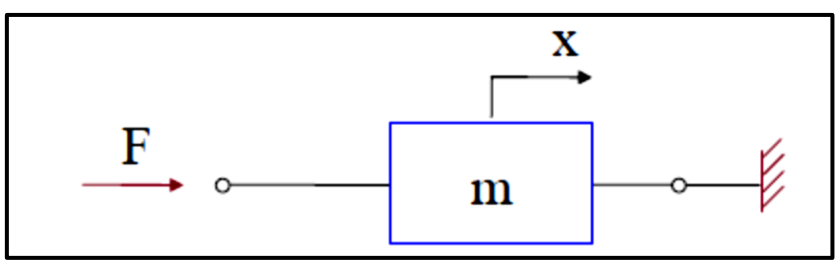
Mass: Property or means of kinetic energy is stored

F = Mass * Acceleration

$= m\ddot{x}$

$= m\dot{v}$

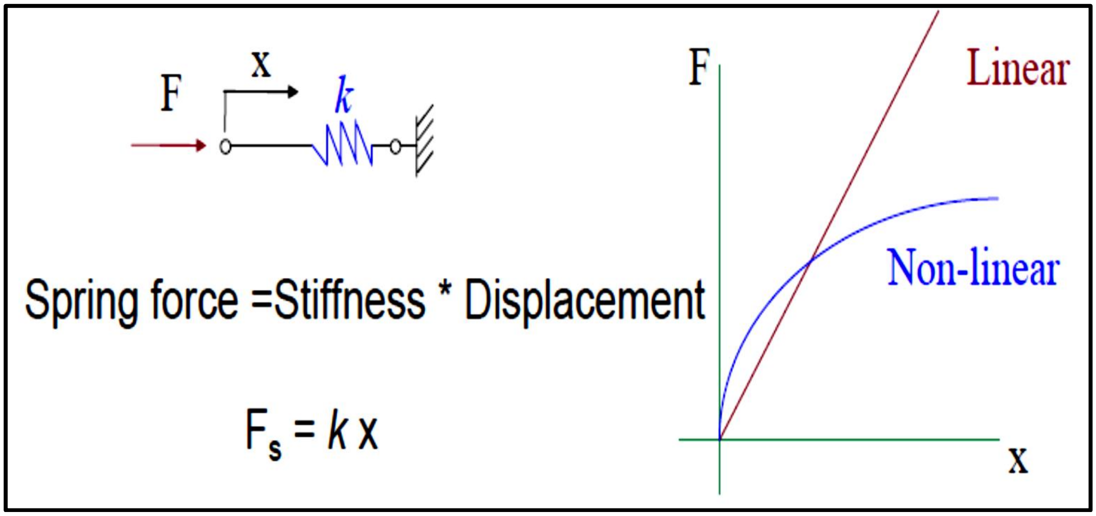
$= m\,a$



Stiffness is the resistance of an elastic body to deflection or deformation by an applied force

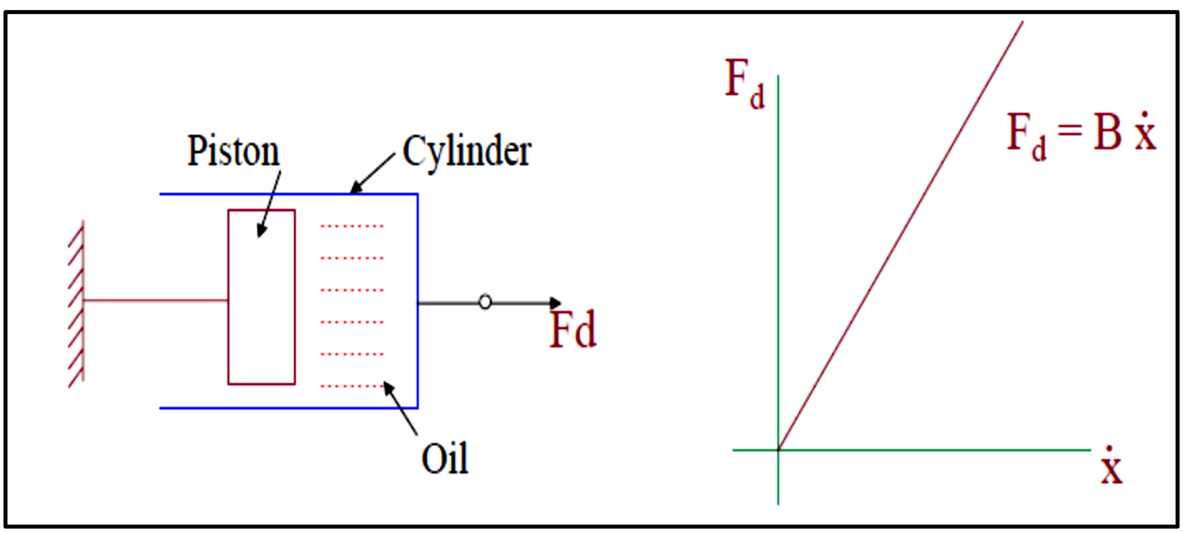
The most common stiffness element is the spring

Spring force is proportional to *displacement*



Friction: is the forcethat opposes the relative motion or tendency of such motion of two surfaces in contact
Exists in all systems and opposes the motion of the mass

- Static friction: occurs when the two objects are moving relative to each other (like a book on a desk)
- Coulomb friction: the classical approximation of the force of friction is known as Coulomb friction (dry friction)
- Viscous friction: a mass sliding on an oil film is subject to viscous friction
- Viscous Friction (Damping)
  - Viscous Damping: Means by which energy is absorbed
  - Damping Force is proportional to *velocity*



Damping Force = Damping Coefficient * Velocity

$$F_d = B\,\dot{x}$$

A Translational System Example