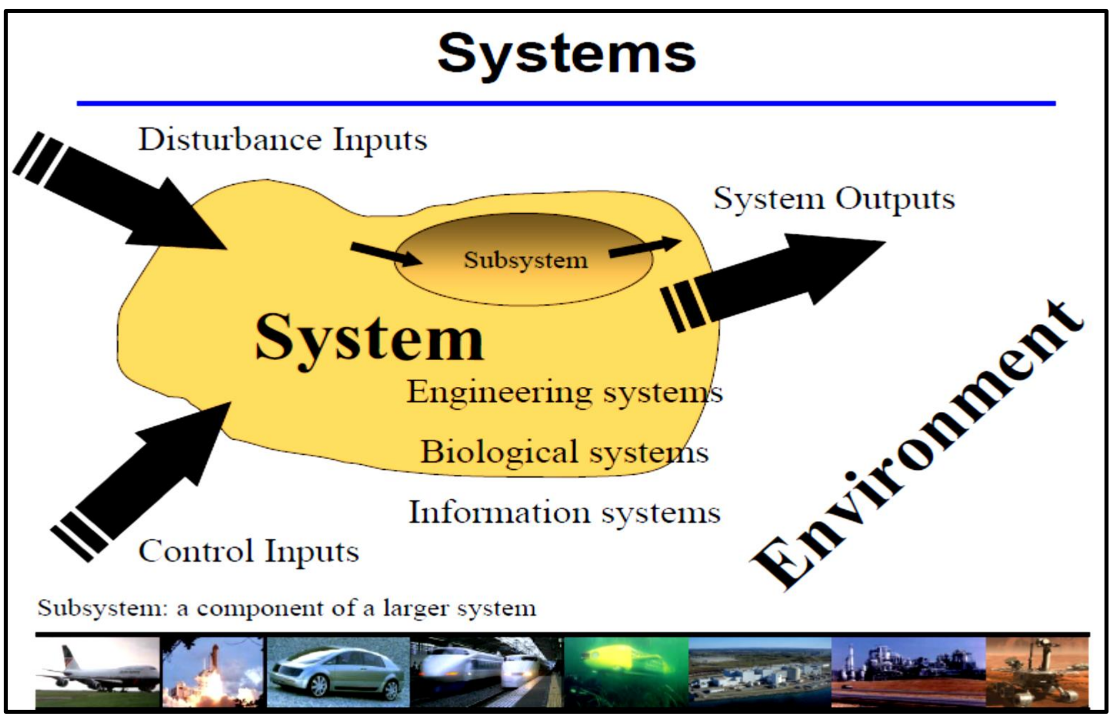
Modelling, simulation and analysis of physical systems

- ❖ **Introduction to modeling and simulation**
- ❖ **Objectives:**
  - To introduce methods for predicting the dynamic behavior of physical systems used in engineering.
  - For modeling, simulation and analysis of physical systems containing individual or mixed mechanical, electrical, thermal and fluid elements should be able to:
    - build mathematical models of physical systems from first principles.
    - analyze behavior of the system using built mathematical models.
    - use software tools (e.g. Matlab/Simulink) for modelling, simulation, and analysis.

❖ **Modeling and Analysis of Physical Systems** are the discipline of understanding and evaluating the interaction of parts of a real or theoretical system by;

- Designing its representation (model) and
- Executing (running) the model including the time and space dimension (simulation).

❖ **System** is a unit or process, which exists and operates in time and space through the interaction with the environment are described by inputs, outputs, and disturbances. System is a collection of components which are coordinated together to perform a function.



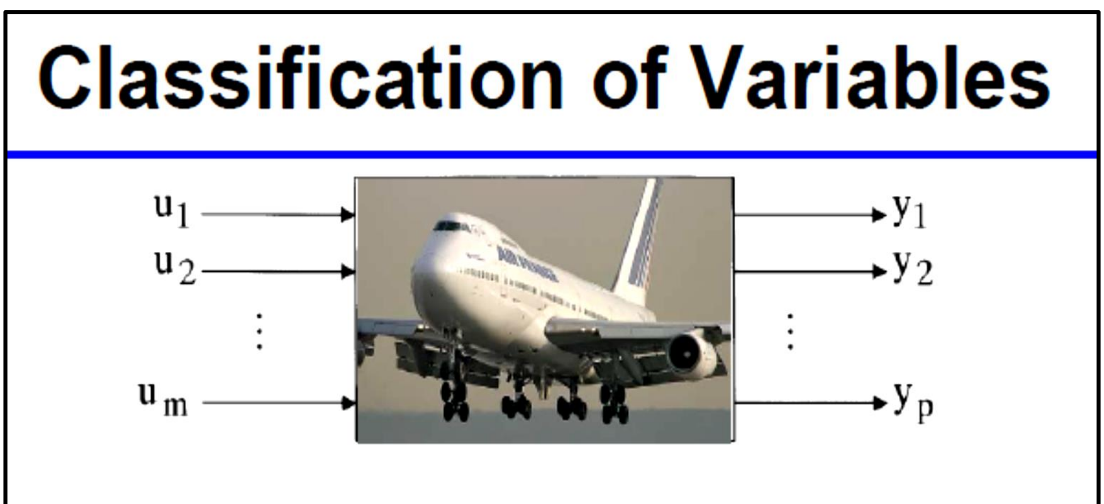
❖ **Dynamic system:** A system with a memory, i.e., the input value at time t will influence the output at future instants (or a system that changes over time).

Examples of dynamic system:

- An aircraft
- An automobile/car
- A robot …

Classification of Dynamic Systems

Temporal	**Dynamic** / Static
Spatial	**Lumped** / Distributed
Linearity	**Linear / Nonlinear**
Continuity of time	**Continuous** / Discrete-time / Hybrid
Parameter variation	**Fixed / Time-varying**
Quantization of dependent variables	**Nonquantized (Analog) /** Quantized (Digital)
Determinism	**Deterministic/** Nondeterministic



Input / output system model

Inputs, u: External influences on the system (force, current, ...)

Outputs, y: Variables of interest to be calculated or measured (position, velocity, ...)
State variables, x: Represent the status or memory of the system
Initial states $x(t0)$ and inputs $u(t)$ completely determine future output $y(t)$ and states $x(t)$, $t \sim t0$ (cause and effect)

❖ Classification of Systems

❖ Temporal characteristics:

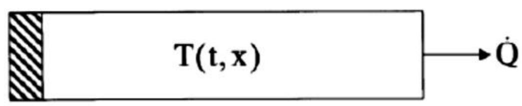
Static: Steady state system with no states, e.g. $y = c(u)$

Dynamic: Transient system that varies with time, e.g. $\frac{dx}{dt} = f(t,x,u)$, $y = c(t,x,u)$

❖Spatial characteristics:

Lumped: Can be described by a finite number of state variables
Distributed: Cannot be described by a finite number of states, e.g

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$



❖Linearity property:

Linear: Systems that have a special linearity property at rest,

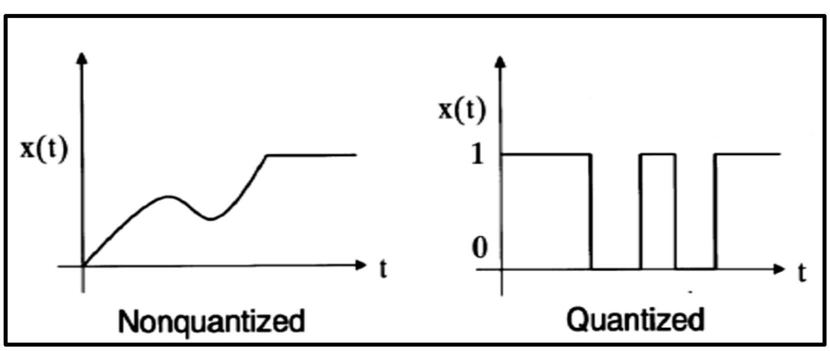
❖Continuity of time variable:

Hybrid system: Contains both continuous and discrete subsystems

❖Parameter Variation:

Fixed: System parameters do not change over time, e.g. $x = ax$
Time Varying: Parameters such as resistance vary with time, e.g. $x=a(t)\,x$

❖Quantization of the dependent variables:



Determinism:

Deterministic: The system changes in a predetermined manner

Nondeterministic:

The system changes in a random manner as a result of noise and other unpredictable factors

These systems often called stochastic problems are solved in terms of probability distributions, e.g.

$$\dot{x} = a(t)x + d \quad , \quad \bar{d} = 0 \quad , \quad \sigma_d = 1$$

❖ **Model** is a simplified representation of a real or theoretical system at some particular point in time or space intended to provide understanding of the system. The model should capture the essential information about the system.

❖ Level of model detail

- Whether a model is good or not depends on the extent to which it provides understanding.
- All the models are simplification of reality.
- Exact copy of a reality can only be the reality itself.
- There is always a trade off as to what level of detail is included in the model:
  - Too little detail: risk of missing relevant interactions.
  - Too much detail: Overly complicated to understand.

❖ Model Types

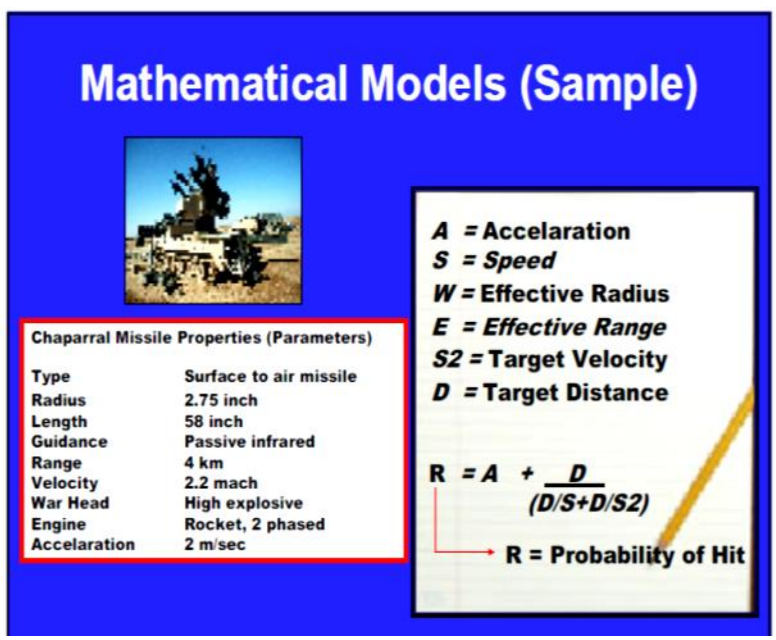
- Mathematical Models
- Physical Models
- Process Models

▪ Mathematical Models

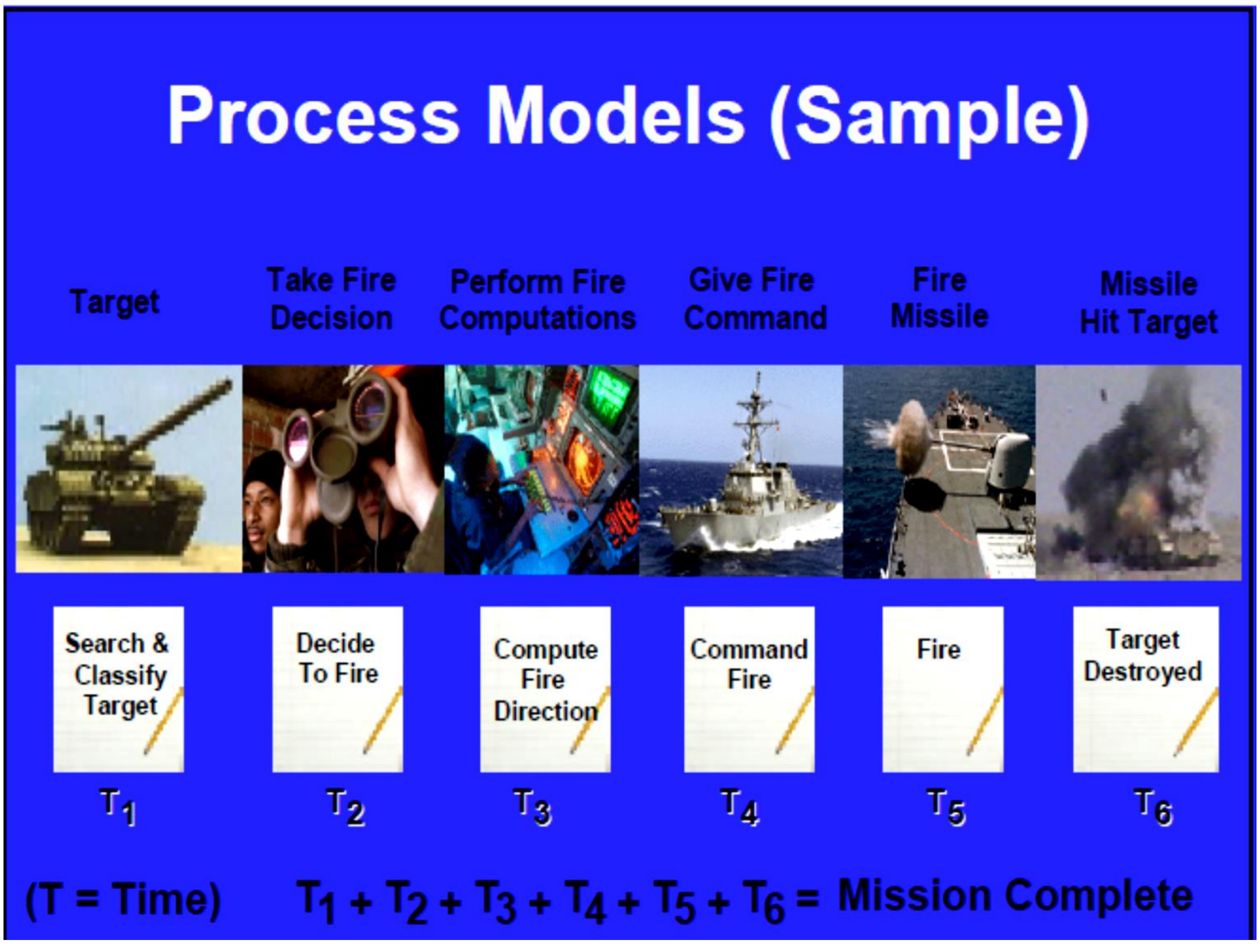
- Models, properties of which are described by mathematical symbols and relations.
- Constructed using:

– Procedures (algorithms)

– Mathematical equations.



- **Physical Models**
  - Models, properties of which are described by physical structures and relations.
  - Usually applied to high fidelity (detailed) system simulations such as simulators.
- **Process Models**
  - Models the process a system performs.
  - Represents dynamic relations by mathematical and logical functions.



❖ **Modelling**: Development of a mathematical representation for a physical system.

- **Mental, intuitive or verbal models**
  - e.g., driving a car
- **Graphs and tables**
  - e.g., Bode plots and step responses
- **Mathematical models**

- A class of model that the relationships between quantities (distances, currents, temperatures etc.) that can be observed in the system are described as mathematical relations
- e.g., differential and difference equations, which are well-suited for modeling dynamic systems

Why Mathematical Models are Needed?

- Do not require a physical system
  - Can treat new designs/technologies without prototype
  - Do not disturb operation of existing system
- Easier to work with than real world
  - Easy to check many approaches, parameter values, ...
  - Flexible to time-scales
  - Can access un-measurable quantities
- Support safety
  - Experiments may be dangerous
  - Operators need to be trained for extreme situations
- Help to gain insight and better understanding
- Analogous Systems
  - Can have the same mathematical model though different types of physical systems
  - Common analysis methods and tools can be used

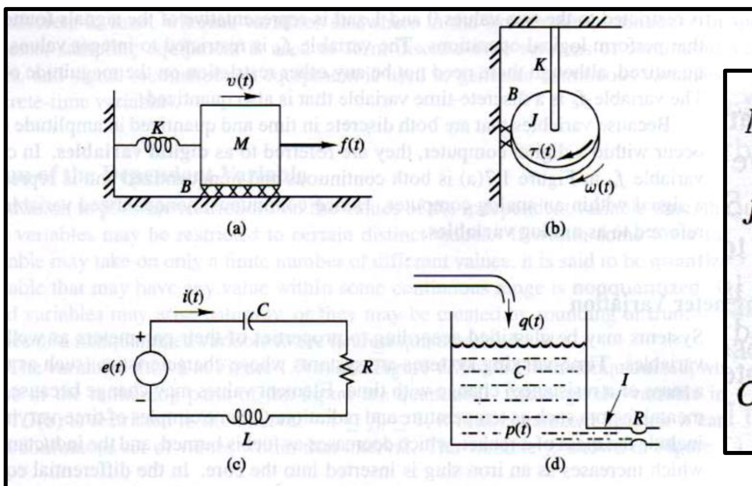


Figure 1.8 Analogous systems. (a) Translational mechanical. (b) Rotational mechanical. (c) Electrical. (d) Hydraulic.

$$M\frac{dv}{dt}+Bv(t)+K\int_0^t v(\lambda)d\lambda = f(t)$$

$$J\frac{d\omega}{dt}+B\omega(t)+K\int_0^t \omega(\lambda)d\lambda = \tau(t)$$

$$L\frac{di}{dt}+Ri(t)+\frac{1}{C}\int_0^t i(\lambda)d\lambda = e(t)$$

$$C\frac{dp}{dt}+\frac{1}{R}p(t)+\frac{1}{I}\int_0^t p(\lambda)d\lambda = q(t)$$

How to Build Mathematical Models?

Two basic approaches:

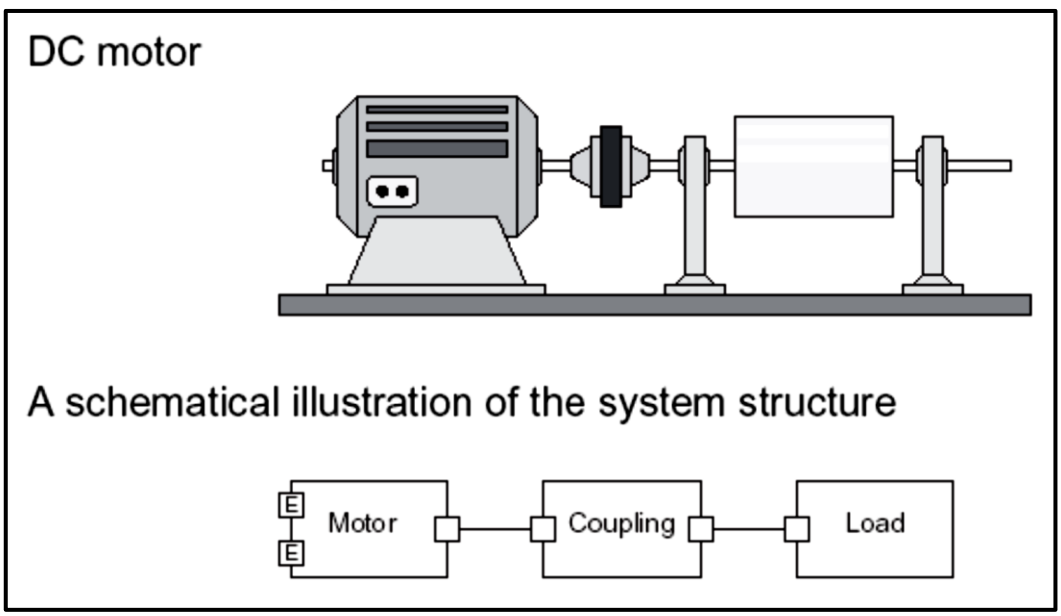
- **Physical/Theoretical modeling** – main topic in this course
  - Use first principles, laws of nature, etc. to model components
  - Need to understand system and master relevant facts!
- **Experimental modeling – System identification** – not covered in this course
  - Use experiments and observations to deduce model
  - Need prototype or real system!

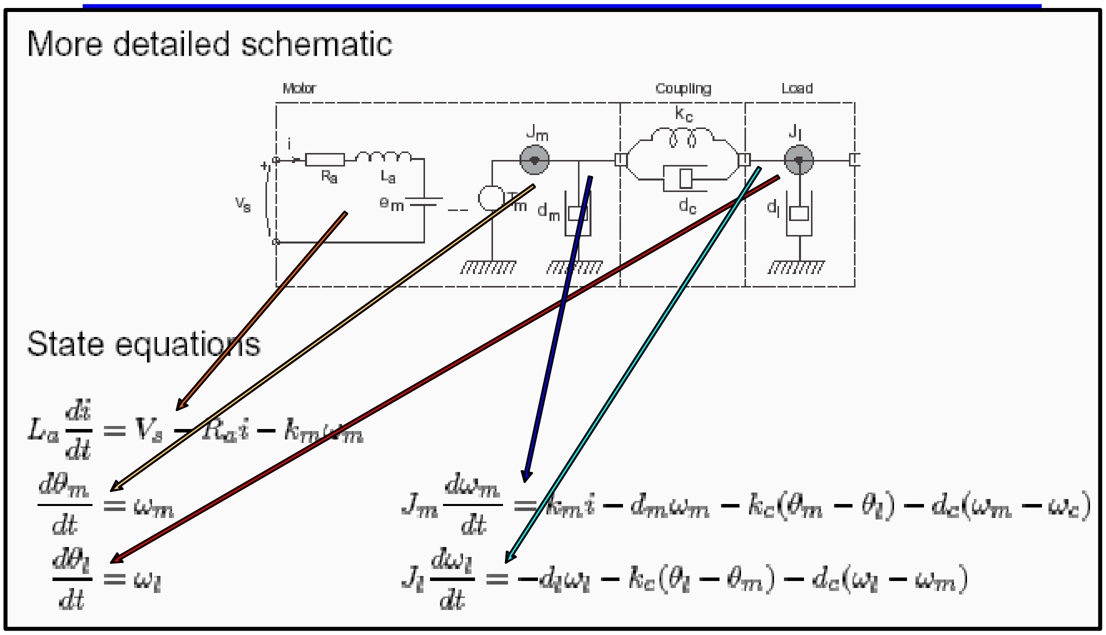
❖ Principle of Physical Modeling

- **Basic idea**: use physics to model system dynamics
  - balance equations and constitutive relations
  - e.g. Newton's laws, Kirchhoff's laws etc.
  - requires detailed knowledge about physics, brings much insight
- Naturally done in continuous-time, leads to
  ODEs (Ordinary Differential Equations) or
  DAEs (Differential Algebraic Equations)

$$\text{ODEs}: \ \dot{x}(t) = f(t,x) \ \text{ or } \ \text{DAEs}: \ F(\dot{z},z,t) = 0$$

Example – Physical Modeling





These are ODEs. How about other forms of mathematical models?

❖ **Mathematical model descriptions**

- Transfer functions
- State space
- Block diagrams

Notation for continuous-time and discrete-time models

Complex Laplace transform variable *s* and differential operator *p*:

$$\dot{x}(t) = dx(t)/dt = px(t)$$

Complex z-transform variable *z* and shift operator *q*:

$$x(k+1) = qx(k)$$

Block diagram of a nonlinear system (DC-motor):