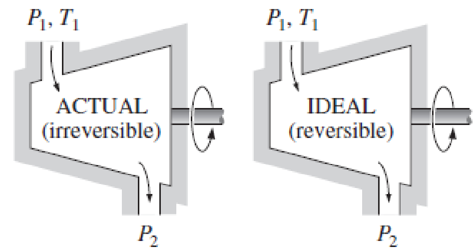
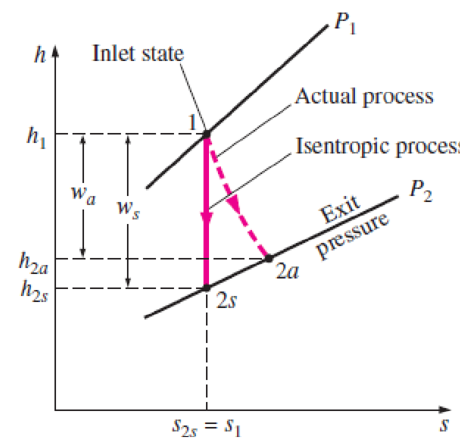
Isentropic Efficiencies of Steady-Flow Devices

Many steady-flow devices are intended to operate under adiabatic conditions. Furthermore, an ideal process should involve no irreversibilities since the effect of irreversibilities is always to downgrade the performance of engineering devices. Thus, the ideal process that can serve as a suitable model for adiabatic steady-flow devices is the isentropic process.
The **isentropic or adiabatic efficiency** is a measure of the deviation of actual processes from the corresponding idealized ones.



The isentropic efficiency of a turbine is defined as the ratio of the actual work output of the turbine to the isentropic work output:

$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s}$$



Usually the changes in kinetic and potential energies are small relative to the change in enthalpy and can be neglected. Then the work output of an adiabatic turbine simply becomes the change in enthalpy

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

where h_{2a} and h_{2s} are the enthalpy values at the exit state for actual and isentropic processes, respectively

$s_1 = s_{2s}$

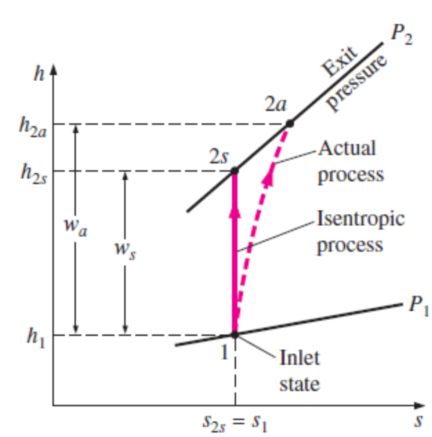
The increase in entropy due to irreversibilities is:

$$\Delta s = s_{2a} - s_1 = s_{2a} - s_2$$

For perfect gas:

$$s_{2a} - s_{2s} = C_p \ln\left(\frac{T_{2a}}{T_2}\right) = s_{2a} - s_1$$

$$\eta_T = \frac{T_1 - T_{2a}}{T_1 - T_{2s}}$$



The **isentropic efficiency of a compressor** is defined as the ratio of the work input required to raise the pressure of a gas to a specified value in an isentropic manner to the

actual work input:

$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

where h_{2a} and h_{2s} are the enthalpy values at the exit state for actual and isentropic compression processes, respectively

$s_1 = s_{2s}$

The increase in entropy due to irreversibilities is:

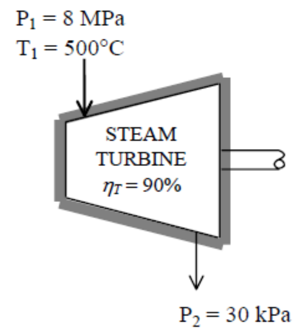
$$\Delta s = s_{2a} - s_1 = s_{2a} - s_2$$

For perfect gas:

$$s_{2a} - s_{2s} = C_p \ln\left(\frac{T_{2a}}{T_2}\right) = s_{2a} - s_1$$

$$\eta_c = \frac{T_{2s} - T_1}{T_{2a} - T_1}$$

Ex: Steam enters an adiabatic turbine at 8 MPa and 500°C with a mass flow rate of 3 kg/s and leaves at 30 kPa. The isentropic efficiency of the turbine is 0.90. Determine (a) the temperature at the turbine exit and (b) the power output of the turbine (c) the increase in entropy.



Sol:

P_1=8 MPa, T_1=500°C, $\dot{m} = 3\ kg/s$, P_2=30 kPa, $\eta_T = 0.9$

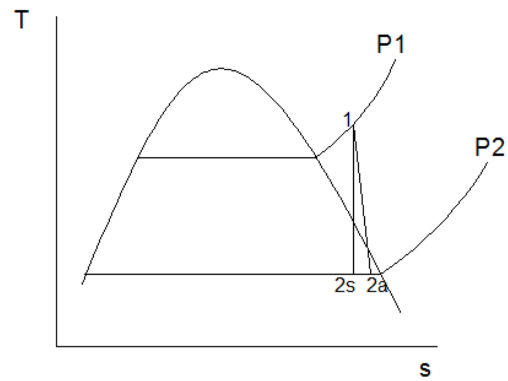
T_2=?, $\dot{W} = ?$, $\Delta s = s_{2a} - s_1 = ?$

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

$$\dot{Q} - \dot{W}_a = \dot{m}\left((h_{2a} - h_1) + \frac{1}{2}(Ve_2^2 - Ve_1^2) + g(z_2 - z_1)\right)$$

For adiabatic and neglect the change in potential and Kinetic energies, we get:

$$\dot{W} = \dot{m}(h_1 - h_{2a})$$



State 1:

P_1=8 MPa, T_1=500°C

At P_{sat}= P_1=8 MPa=8000 kPa, T_{sat}=295.01 °C

Since T_1> T_{sat}, the steam is superheated

h_1=3399.5 kJ/kg, s_1=6.7266 kJ/kg.K

State 2s:

P_2=30 kPa, s_{2s}=s_1=6.7266 kJ/kg.K

At P_{sat}= P_2=30 kPa, s_g=7.7675 kJ/kg.K, 0.9441 kJ/kg.K

Since s_f <s_{2s}< s_g , the steam is liquid-vapor mixture (wet)

s_{2s} =s_f+x_{2s} (s_{fg})

6.7266=0.9441+ x_{2s} (6.8234), x_{2s}=0.847

h_{2s} =h_f+x_{2s} (h_{fg})=289.27+0.847(2335.3)=2267.27 kJ/kg

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

$$0.9 = \frac{3399.5 - h_{2a}}{3399.5 - 2267.27}$$, h_{2a}=2384.093 kJ/kg

$$\boldsymbol{\dot{W}_a = \dot{m}(h_1 - h_{2a}) = 3(3399.5 - 2384.093) = 3046.22\, kW}$$

State 2a:

P_2=30 kPa, h_{2a}=2384.093 kJ/kg

At P_{sat}= P_2=30 kPa, h_g=2624.6 kJ/kg, h_f=289.27 kJ/kg

Since h_f < h_{2a}< h_g , the steam is liquid-vapor mixture (wet)

h_{2a} =h_f+x_{2a} (h_{fg})

2384.093 =289.27+ x_{2a} (2335.3), x_{2a} =0.897

s_{2a} =s_f+x_{2a} (s_{fg}) = 0.9441+ 0.897 (6.8234) =7.064 kJ/kg.K

$$\Delta s = s_{2a} - s_1 = 7.064 - 6.7266 = 0.3374\ kJ/kg.K$$

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$