

5-Reversible Polytropic Process

In this process, the working fluid (vapor or perfect gas) follows the following relation:

$$Pv^n = c$$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2} \right)^n$$

$$\frac{v_2}{v_1} = \left(\frac{P_1}{P_2} \right)^{1/n}$$

The work done during this process is defined as:

$$w = \frac{P_1 v_1 - P_2 v_2}{n-1}$$

$$W = \frac{m(P_1 v_1 - P_2 v_2)}{n-1} = \frac{P_1 V_1 - P_2 V_2}{n-1}$$

The heat flow is defined as:

$$Q = W + m(u_2 - u_1)$$

The change in entropy for a vapor is found by determining the end states

For **perfect gas** only:

$$Pv = RT$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{n-1} = \left(\frac{V_1}{V_2} \right)^{n-1}$$

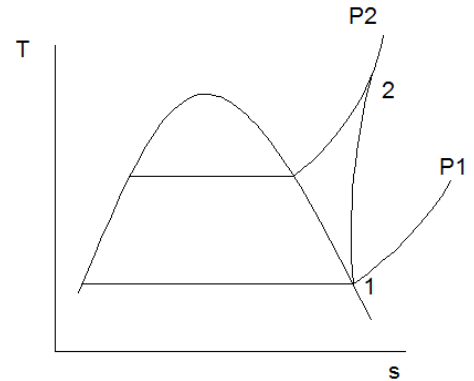
$$\frac{V_2}{V_1} = \frac{v_2}{v_1} = \left(\frac{T_1}{T_2} \right)^{1/(n-1)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}}$$

Furthermore, the work done can be calculated as:

$$w = \frac{P_1 v_1 - P_2 v_2}{n-1} = \frac{R(T_1 - T_2)}{n-1}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$$



$$q = \left(\frac{n-k}{1-k} \right) w$$

$$Q = \left(\frac{n-k}{1-k} \right) W \quad (\text{This equation is applied for ideal gas only})$$

To find the entropy change for a perfect gas in a general case (polytropic process), consider a non-flow energy equation (closed system)

$$\delta q - \delta w = du$$

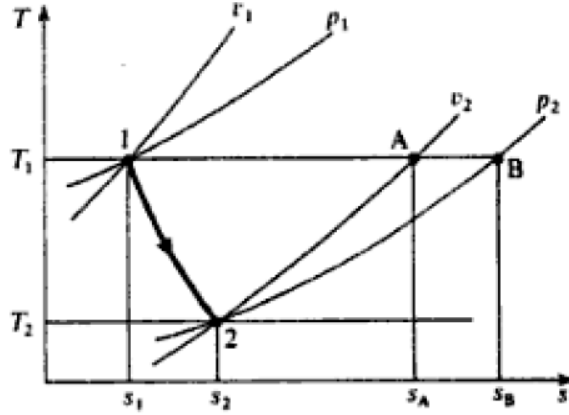
$$Tds - Pdv = C_v dT$$

$$Tds = Pdv + C_v dT$$

$$ds = \frac{Pdv}{T} + \frac{C_v dT}{T}$$

$$ds = \frac{Rdv}{v} + \frac{C_v dT}{T}$$

$$\Delta s = s_2 - s_1 = R \ln \left(\frac{v_2}{v_1} \right) + C_v \ln \left(\frac{T_2}{T_1} \right)$$



If we assume polytropic expansion, $T_1 > T_2$

$$s_2 - s_1 = R \ln \left(\frac{v_2}{v_1} \right) - C_v \ln \left(\frac{T_1}{T_2} \right)$$

The first term in RHS of the above equation represents the entropy change for isothermal process from v_1 to v_2 ,

$$s_A - s_1 = R \ln \left(\frac{v_2}{v_1} \right)$$

While, the second term stands for the entropy change for constant volume process from T_1 to T_2

$$s_A - s_2 = C_v \ln \left(\frac{T_1}{T_2} \right)$$

Thus,

$$s_2 - s_1 = (s_A - s_1) - (s_A - s_2)$$

The calculating the entropy change in a reversible polytropic process (state 1 to state 2) requires replacing this process by two simple reversible processes: from 1 to A and then from A to 2.

For example, going from 1 to B and then from B to 2 (when two pressure are known):

$$s_2 - s_1 = (s_B - s_1) - (s_B - s_2)$$

$$s_2 - s_1 = R \ln \left(\frac{P_2}{P_1} \right) - C_p \ln \left(\frac{T_1}{T_2} \right)$$

Ex: At the start of the compression process in the reciprocating compressor of a refrigeration plant, the dry saturated vapor of refrigerant R-134a is at 1.4 bar. The compression process is according to the law $Pv^{1.2}=c$, until the pressure is 6.0 bar. Calculate the entropy change, the work done, the heat transfer, and the degree of superheat of the refrigerant after compression?

Sol:

$P_1=1.4 \text{ bar}=140 \text{ kPa}$ (dry saturated)

Polytropic process of index $n=1.2$

$P_2=6.0 \text{ bar}=600 \text{ kPa}$

$\Delta s = s_2 - s_1 = ?$, $q=?$, $w=?$, Degree of superheat at state 2=?

State 1:

$P_1=1.4 \text{ bar}=140 \text{ kPa}$ (dry saturated)

$s_1=s_g=0.94456 \text{ kJ/kg.K}$

$u_1=u_g=219.54 \text{ kJ/kg}$

$v_1=v_g=0.14014 \text{ m}^3/\text{kg}$

State 2:

$P_2=6.0 \text{ bar}=600 \text{ kPa}=0.6 \text{ MPa}$

$$\frac{v_2}{v_1} = \left(\frac{P_1}{P_2} \right)^{1/n}$$

$$\frac{v_2}{0.14014} = \left(\frac{1.4}{6.0} \right)^{1/1.2}, \quad v_2 = 0.0417 \text{ m}^3/\text{kg}$$

At $P_{\text{sat}}=P_2=6.0 \text{ bar}=600 \text{ kPa}$

$T_{\text{sat}}=21.55 \text{ }^\circ\text{C}$

$$v_g = 0.034295 \text{ m}^3/\text{kg}$$

Since $v_2 > v_g$, the vapor is superheat

By using linear

v	u	T	s
0.041389	275.15	60	1.0417
0.0417	u_2	T_2	s_2
0.043069	283.89	70	1.0705

interpolation:

$$\frac{u_2 - 275.15}{283.89 - 275.15} = \frac{0.0417 - 0.041389}{0.043069 - 0.041389}$$

$$\frac{T_2 - 60}{70 - 60} = \frac{0.0417 - 0.041389}{0.043069 - 0.041389} \quad \frac{s_2 - 1.0417}{1.0705 - 1.0417} = \frac{0.0417 - 0.041389}{0.043069 - 0.041389}$$

$$u_2 = 276.7684 \text{ kJ/kg}, T_2 = 61.85^\circ\text{C}, s_2 = 1.047 \text{ kJ/kg.K}$$

$$w = \frac{P_1 v_1 - P_2 v_2}{n - 1} = \frac{140 * 0.14014 - 600 * 0.0417}{1.2 - 1} = -27.0 \text{ kJ/kg}$$

$$q = w + (u_2 - u_1) = -27 + (276.768 - 219.54) = 16.063$$

$$\Delta s = s_2 - s_1 = 1.047 - 0.94456 = 0.1024 \text{ kJ/kg.K}$$

$$\text{Degree of superheat} = T_2 - T_{\text{sat}} = 61.85 - 21.55 = 40.3^\circ\text{C}$$

Ex: A piston–cylinder device contains 1.2 kg of nitrogen gas at 120 kPa and 27°C . The gas is now compressed slowly in a polytropic process during which $PV^{1.2} = \text{constant}$. The process ends when the volume is reduced by one-half. Determine the entropy change of nitrogen during this process, the work done and heat transfer.

Sol:

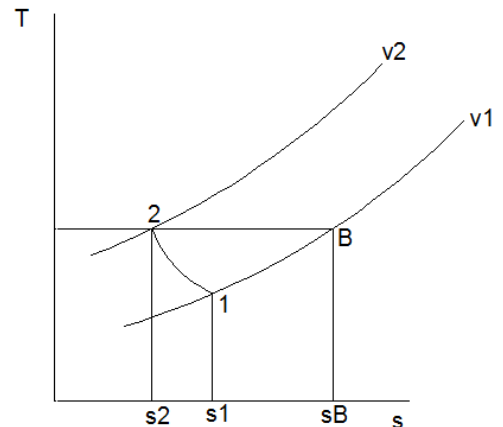
$$m = 1.2 \text{ kg}, P_1 = 120 \text{ kPa}, T_1 = 27^\circ\text{C} = 300 \text{ K}$$

Polytropic process of $n = 1.2$,

$$V_2 = 1/2 V_1$$

$$\Delta s = s_2 - s_1 = ?, Q = ?, W = ?$$

$$W = \frac{mR(T_1 - T_2)}{n - 1}$$



$$Q = \left(\frac{n-k}{1-k} \right) W$$

$$s_2 - s_1 = (s_B - s_1) - (s_B - s_2)$$

$$s_2 - s_1 = C_v \ln \left(\frac{T_B}{T_1} \right) - R \ln \left(\frac{V_B}{V_2} \right)$$

For nitrogen:

$$R = 0.2968 \text{ kJ/kg.K}, C_v = 0.743 \text{ kJ/kg.K}, k = 1.4$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{n-1}$$

$$\frac{T_2}{300} = \left(\frac{V_1}{0.5V_1} \right)^{1.2-1}, T_2 = T_B = 344.61 \text{ K}$$

$$\begin{aligned} s_2 - s_1 &= C_v \ln \left(\frac{T_B}{T_1} \right) - R \ln \left(\frac{V_1}{V_2} \right) \\ &= 0.743 \ln \left(\frac{344.61}{300} \right) - 0.2968 \ln \left(\frac{V_1}{0.5V_1} \right) = -0.1027 \text{ kJ/kg.K} \end{aligned}$$

$$S_2 - S_1 = m (s_2 - s_1) = 1.2 (-0.1027) = -0.123 \text{ kJ/K}$$

$$W = \frac{mR(T_1 - T_2)}{n-1} = \frac{1.2 * 0.2968(300 - 344.61)}{1.2-1} = -79.44 \text{ kJ}$$

$$Q = \left(\frac{n-k}{1-k} \right) W = \left(\frac{1.2-1.4}{1-1.4} \right) (-79.44) = -39.72 \text{ kJ}$$

