Definitions of Transient-Response Specifications:

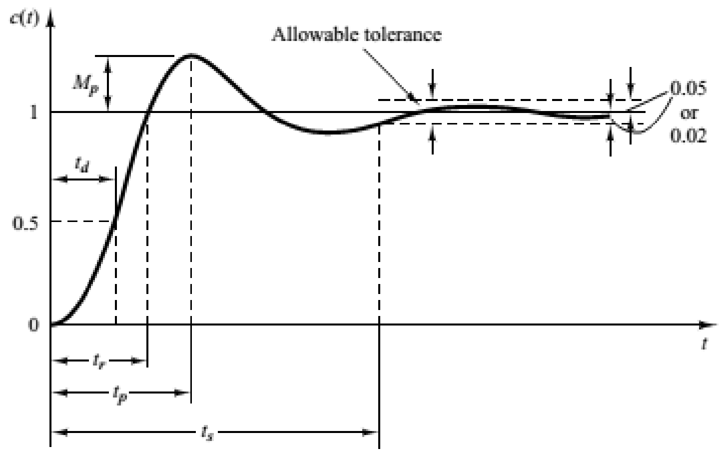
Frequently, the performance characteristics of a control system are specified in terms of the transient response to a unit-step input. If the response to a step input is known, it is mathematically possible to compute the response to any input. The transient response of a system to a unit-step input with standard initial condition that the system is at rest initially. Then the response characteristics of many systems can be easily compared. The transient response of a practical control system often exhibits damped oscillations before reaching steady state. In specifying the transient-response characteristics of a control system to a unit-step input, it is common to specify the following .

1. **Delay time, t_d :** The delay time is the time required for the response to reach half the final value the very first time.
2. **Rise time, t_r :** The rise time is the time required for the response to rise from 10 to 90%, 5% to 95%, or 0% to 100% of its final value. For underdamped second order systems, the 0%to 100%rise time is normally used. For overdamped systems, the 10% to 90%rise time is commonly used.
3. **Peak time, t_p :**The peak time is the time required for the response to reach the first peak of the overshoot.
4. **Maximum (percent) overshoot, M_p:** The maximum overshoot is the maximum peak value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. It is defined by The amount of the maximum (percent) overshoot directly indicates the relative stability of the system.

$$Maximum\ percent\ overshoot = \frac{c(t_p)-c(\infty)}{c(\infty)} \times 100\%$$

5. **Settling time, t_s:** The settling time is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2%or 5%). The settling time is related to the largest time constant of the control system .

The time-domain specifications just given are quite important, since most control systems are time-domain systems; that is, they must exhibit acceptable time responses. (This means that, the control system must be modified until the transient response is satisfactory). Note that not all these specifications necessarily apply to any given case. For example, for an overdamped system, the terms peak time and maximum overshoot do not apply



Unit-step response curve

Second-order systems and transient-response specifications:

The rise time, peak time, maximum overshoot, and settling time of the second-order system given by equation :

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

These values will be obtained in terms of ζ and ω_n. The system is assumed to be **underdamped**

$$C(s) = \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)s}$$

Rewrite the equation in the following form:

$$C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$
$$= \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

Where $\omega_d = \omega_n\sqrt{1-\zeta^2}$ Damped natural frequency.

The inverse laplace transform for each term is

$$\mathcal{L}^{-1}\left[\frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}\right] = e^{-\zeta\omega_n t}\cos\omega_d t$$

$$\mathcal{L}^{-1}\left[\frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2}\right] = e^{-\zeta\omega_n t}\sin\omega_d t$$

$$\mathcal{L}^{-1}[C(s)] = c(t)$$
$$= 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right)$$
$$= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right), \qquad \text{for } t \geq 0$$

Rise time, t_r :. For underdamped second order systems, the t_r from 0% to 100% . by letting c(t_r)=1

$$c(t_r) = 1 = 1 - e^{-\zeta\omega_n t_r}\left(\cos\omega_d t_r + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r\right)$$

$$\cos\omega_d t_r + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r = 0$$ Then $$t_r = \frac{1}{\omega_d}tan^{-1}\left(\frac{\omega_d}{-\zeta\omega_n}\right)$$

Peak time t_p : To obtain the peak time, c(t) will be differentiated with respect to time and letting this derivative equal zero.

$$\frac{dc}{dt} = \zeta\omega_n e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right)$$
$$+ e^{-\zeta\omega_n t}\left(\omega_d\sin\omega_d t - \frac{\zeta\omega_d}{\sqrt{1-\zeta^2}}\cos\omega_d t\right)$$

Then at t equal to t_p

$$\left.\frac{dc}{dt}\right|_{t=t_p} = (\sin\omega_d t_p)\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t_p} = 0$$

$$\sin\omega_d t_p = 0$$

$$\omega_d t_p = 0, \pi, 2\pi, 3\pi, \ldots$$

$$t_p = \frac{\pi}{\omega_d}$$

Maximum overshoot Mp: The maximum overshoot occurs at the peak time or at t_p. Assuming that the final value of the output is unity.

$$M_p = c(t_p) - 1$$

$$= -e^{-\zeta\omega_n(\pi/\omega_d)}\left(\cos\pi + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\pi\right)$$

$$= e^{-(\zeta/\sqrt{1-\zeta^2})\pi}$$

Settling time T_s: For a given ω_n, the settling time is a function of the damping ratio ζ. For

2% criterion.

$$\boxed{T_s = \frac{4}{\zeta\omega_n}}$$