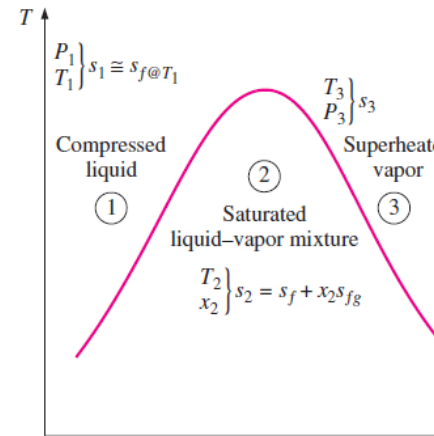


## Entropy Change of Pure Substances

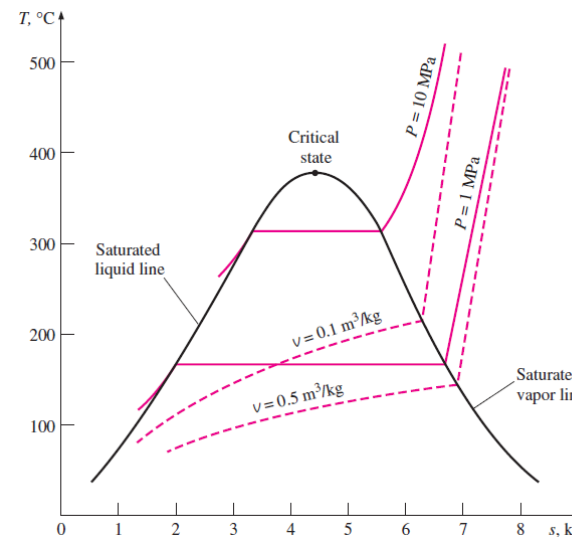
The entropy change of a substance can be expressed in terms of other properties. But in general, these relations are too complicated and are not practical to use for hand calculations. Therefore, using a suitable reference state, the entropies of substances are evaluated from measurable property data and are tabulated in the same manner as the other properties such as  $v$ ,  $u$ , and  $h$ .

The entropy values in the property tables are given relative to an arbitrary reference state. In steam tables the entropy of saturated liquid  $s_f$  at  $0.01^\circ\text{C}$  is assigned the value of zero.

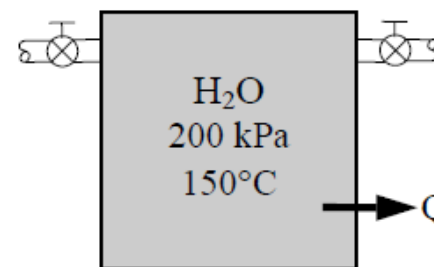


The (T-s) diagram shows that:

- the constant volume lines are steeper than the constant-pressure lines.
- the constant pressure lines are parallel to the constant-temperature lines in the saturated liquid-vapor mixture region.



**Ex:** The radiator of a steam heating system has a volume of 20 L and is filled with superheated water vapor at 200 kPa and  $150^\circ\text{C}$ . At this moment both the inlet and the exit valves to the radiator are closed. After a while the temperature of the steam drops to  $40^\circ\text{C}$  as a result of heat transfer to the room air. Determine the entropy change of the steam during this process and the amount of heat.



**Sol:**

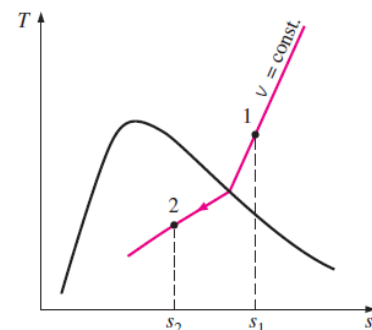
$V=20 \text{ L}$ ,  $P_1=200 \text{ kPa}$ ,  $T_1=150^\circ\text{C}$  (superheated)

$T_2=40^\circ\text{C}$

$\Delta S = ?$   $Q = ?$

$Q-W=U_2-U_1$ ,  $W=0$  (rigid vessel and constant volume)

$Q=m(u_2-u_1)$



$$V=20 \text{ L}=20*10^{-3}=0.02 \text{ m}^3$$

**State 1:** superheated vapor  $P_1=200 \text{ kPa}=0.2 \text{ Mpa}$ ,  $T_1=150^\circ\text{C}$

From superheated vapor table:

$$s_1=7.281 \text{ kJ/kg}, v_1=0.95986 \text{ m}^3/\text{kg}, u_1=2577.1 \text{ kJ/kg}$$

$$m=V_1/v_1=0.02/0.95986=0.0208 \text{ kg}$$

**State 2:**  $T_2=40^\circ\text{C}$ , since the vessel is rigid  $v_2=v_1=0.95986 \text{ m}^3/\text{kg}$

From saturation temperature table, at  $T_2=T_{\text{sat}}=40^\circ\text{C}$ ,

$$v_f=0.001008 \text{ m}^3/\text{kg}, v_g=19.515 \text{ m}^3/\text{kg}$$

since  $v_f < v_2 < v_g$ , the vapor is wet

$$v_2=v_f+x_2(v_g-v_f)$$

$$0.95986=0.001008+x_2(19.515-0.001008), x_2=0.0491$$

$$s_2=s_f+x_2 s_{fg}=0.5724+0.0491(7.6832)=0.9496 \text{ kJ/kg}$$

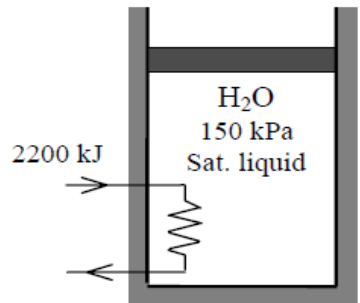
$$u_2=u_f+x_2 u_{fg}=167.53+0.0491(2261.9)=278.59 \text{ kJ/kg}$$

$$\Delta s=s_2-s_1=0.950-7.281=-6.331 \text{ kJ/kg.K}$$

$$\Delta S=m \Delta s=0.0208(-6.331)=-0.132 \text{ kJ/K}$$

$$Q=m(u_2-u_1)=0.0208(278.59-2577.1)=-47.8 \text{ kJ (heat rejected)}$$

**Ex:** An insulated piston–cylinder device contains 5 L of saturated liquid water at a pressure of 150 kPa. An electric resistance heater inside the cylinder is now turned on, and 2200 kJ of energy is transferred to the system while the pressure remains constant. Determine the entropy change of the water during this process.



Sol:

$$Q=0 \text{ (insulated)}, V_1=5 \text{ L}=0.005 \text{ m}^3, P_1=150 \text{ kPa (sat. liq)}, W_s=-2200 \text{ kJ}$$

$$P_2=P_1$$

$$\Delta S=?$$

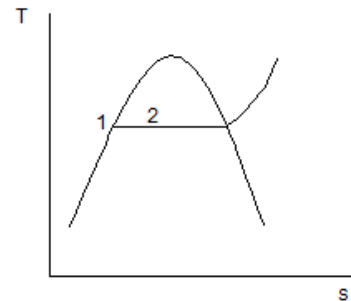
$$Q-W=U_2-U_1$$

$$W=W_b+W_e,$$

$$W_b=mP(v_2-v_1)$$

$$Q-W_e=m(h_2-h_1)$$

$$-W_e=m(h_2-h_1), h_2=h_1-W_e/m$$



**Sate 1:** Saturated liquid at  $P_1=150 \text{ kPa}$  and from saturated pressure table

$$h_1=h_f=467.13 \text{ kJ/kg}$$

$$s_1=s_f=1.4337 \text{ kJ/kg.K}$$

$$v_1=v_f=0.001053 \text{ m}^3/\text{kg}$$

$$m = V_1/v_1 = 0.005/0.001053 = 4.748 \text{ kg}$$

$$h_2 = h_1 - W_e/m = 467.13 - (-2200/4.748) = 930.483 \text{ kJ/kg}$$

**Sate 2:**  $P_2 = 150 \text{ kPa}$ ,  $h_2 = 930.483 \text{ kJ/kg}$   
 From saturation pressure table at  $P_2 = P_{\text{sat}} = 150 \text{ kPa}$ ,  
 $h_g = 2693.1 \text{ kJ/kg}$ ,  $h_f = 467.13 \text{ kJ/kg}$   
 since  $h_f < h_2 < h_g$ , the vapor is wet or (liquid-vapor mixture)  
 $h_2 = h_f + x_2 h_{fg}$   
 $930.483 = 467.13 + x_2 (2226)$ ,  $x_2 = 0.208$   
 $s_2 = s_f + x_2 s_{fg} = 1.4337 + 0.208 (5.7894) = 2.638 \text{ kJ/kg.K}$   
 $\Delta s = s_2 - s_1 = 2.638 - 1.4337 = 1.2043 \text{ kJ/kg.K}$   
 $\Delta S = m \Delta s = 4.748(1.2043) = 5.718 \text{ kJ/K}$

**Ex:** A  $0.5\text{-m}^3$  rigid tank contains refrigerant-134a initially at  $200 \text{ kPa}$  and  $40$  percent quality. Heat is transferred now to the refrigerant from a source at  $35^\circ\text{C}$  until the pressure rises to  $400 \text{ kPa}$ . Determine (a) the entropy change of the refrigerant, (b) the entropy change of the heat source, and (c) the total entropy change for this process.

Sol:

$V = 0.5 \text{ m}^3$ , rigid tank,  $V_1 = V_2$ ,  $v_1 = v_2$   
 Working fluid R-134a  
 $P_1 = 200 \text{ kPa}$ ,  $x_1 = 40\% = 0.4$  (wet vapor)  
 $T_h = 35^\circ\text{C}$ ,  $P_2 = 400 \text{ kPa}$

$$\Delta S_{\text{system}} = ?, \Delta S_{\text{source}} = ?, \Delta S_{\text{total}} = ?$$

$$\Delta S_{\text{system}} = m (s_2 - s_1)$$

$$\Delta S_{\text{source}} = \frac{Q_H}{T_H}$$

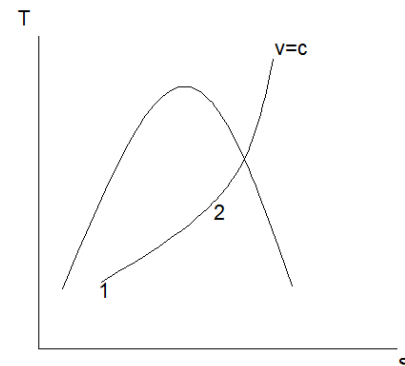
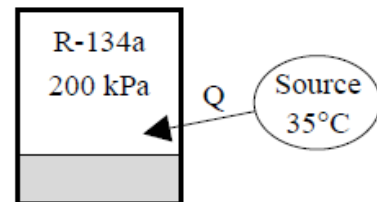
the heat out from the source equal in magnitude but opposite in direction with the heat received from the system

$$Q_{\text{in}} = m (u_2 - u_1)$$

$$Q_H = -Q_{\text{in}}$$

$$\Delta S_{\text{total}} = \Delta S_{\text{system}} + \Delta S_{\text{source}}$$

**State 1:** wet vapor  $P_1 = 200 \text{ kPa}$ ,  $x_1 = 0.4$   
 $s_1 = s_f + x_1 s_{fg} = 0.15457 + 0.4(0.78316) = 0.468 \text{ kJ/kg.K}$   
 $u_1 = u_f + x_1 u_{fg} = 38.28 + 0.4(186.21) = 112.764 \text{ kJ/kg}$   
 $v_1 = v_f + x_1 (v_g - v_f) = 0.0007533 + 0.4(0.099867 - 0.0007533)$



$$=0.0404 \text{ m}^3/\text{kg}$$

$$m=V/v_1=0.5/0.0404=12.376 \text{ kg}$$

**State 2:**  $P_2=400 \text{ kPa}$ ,  $v_2= v_1=0.0404 \text{ m}^3/\text{kg}$

At  $P_{\text{sat}}=P_2=400 \text{ kPa}$ ,  $v_f=0.0007907 \text{ m}^3/\text{kg}$ ,  $v_g=0.051201 \text{ m}^3/\text{kg}$

Since  $v_f < v_2 < v_g$ , the vapor is also wet vapor

$$v_2= v_f + x_2 (v_g-v_f)$$

$$0.0404=0.0007907+ x_2(0.051201-0.0007907), x_2=0.786$$

$$s_2=s_f + x_2 s_{fg}=0.24761+0.786 (0.67929) =0.782 \text{ kJ/kg.K}$$

$$u_2=u_f + x_2 u_{fg}=63.62+0.786 (171.45) =198.38 \text{ kJ/kg}$$

$$\Delta S_{\text{system}} = m(s_2 - s_1) = 12.376(0.782 - 0.468) = 3.886 \text{ kJ / K}$$

$$Q_{\text{in}}=m (u_2-u_1) =12.376 (198.38-112.764) =1059.584 \text{ kJ}$$

$$Q_H=-Q_{\text{in}}=-1059.584 \text{ kJ}$$

$$\Delta S_{\text{source}} = \frac{Q_H}{T_H} = \frac{-1059.584}{(35 + 273)} = -3.44 \text{ kJ / K}$$

$$\Delta S_{\text{total}} = \Delta S_{\text{system}} + \Delta S_{\text{source}} = 3.886 + (-3.44) = 0.446 \text{ kJ / K}$$

## Property Diagrams Involving Entropy

The two diagrams commonly used in the second-law analysis are the temperature-entropy and the enthalpy-entropy diagrams.

Recall the defining equation of entropy:

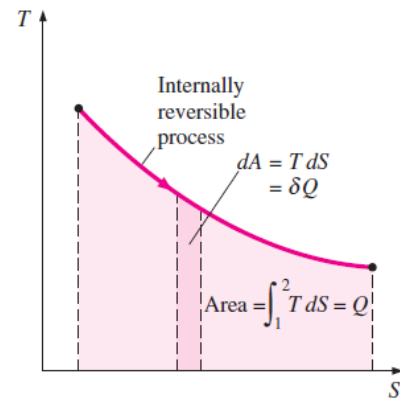
$$dS = \left( \frac{\delta Q}{T} \right)_{\text{int rev}} \quad (\text{kJ / K})$$

$$\delta Q_{\text{int,rev}} = T dS \quad (\text{kJ})$$

The total heat transfer during an internally reversible process is determined by integration to be:

$$Q_{\text{int,rev}} = \int_1^2 T dS \quad (\text{kJ})$$

This corresponds to the area under the process curve on a T-S diagram. Therefore, we conclude that *the area under the process curve on a T-S diagram represents heat transfer during an internally reversible process.* Per unit mass, we get:



$$\delta q_{\text{int},rev} = T ds \quad (kJ/kg)$$

$$q_{\text{int},rev} = \int_1^2 T ds \quad (kJ/kg)$$

To perform the integrations in above equations, one needs to know the relationship between  $T$  and  $s$  during a process.