

Entropy

The Inequality of Clausius

“the cyclic integral of $\delta Q/T$ is always less than or equal to zero”

$$\oint \frac{\delta Q}{T} \leq 0$$

This inequality is valid for all cycles, reversible or irreversible heat engine or refrigerators. The symbol \oint is used to indicate that the integration is to be performed over the entire cycle.

Consider a reversible (Carnot) Heat engine cycle.

The cyclic integral of the heat transfer is greater than zero

$$\oint \delta Q = (Q_H - Q_L) > 0$$

Since T_H and T_L are constant, and from the definition of the absolute (Kelvin) temperature scale, we get:

$$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0$$

Now consider an irreversible cyclic heat engine operating Between the same T_H and T_L , and receiving the same Quantity of heat Q_H .

$$W_{irr} < W_{rev}$$

$$(Q_H - Q_{Lirr}) < (Q_H - Q_{Lrev})$$

And therefore

$$Q_{Lirr} > Q_{Lrev}$$

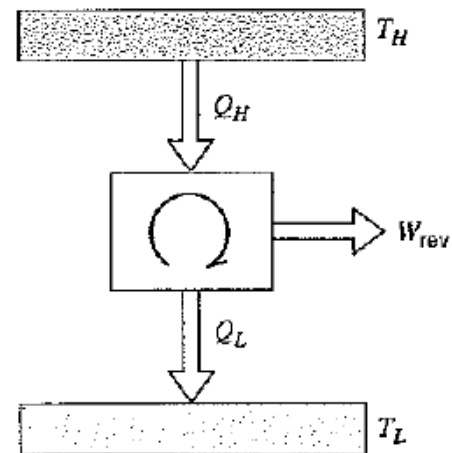
Consequently, for the irreversible cyclic engine,

$$\oint \delta Q = (Q_H - Q_{Lirr}) > 0$$

$$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_{Lirr}}{T_L} < 0$$

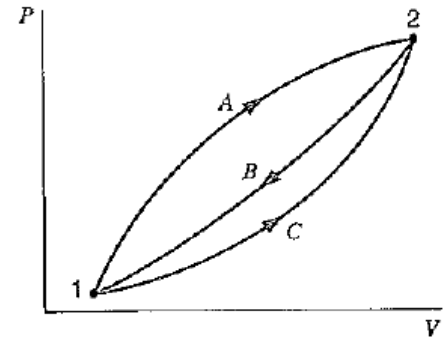
Therefore, for reversible and irreversible cycle,

$$\oint \frac{\delta Q}{T} \leq 0 \begin{cases} = 0, & \text{for reversible or internally reversible cycles} \\ < 0, & \text{for irreversible cycles} \end{cases}$$



Entropy as a System Property

Let a system undergoes a reversible process from State 1 to state 2 along a path A, and let the cycle be Completed along path B, which is also reversible



For reversible cycle, we can write

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T} \right)_A + \int_2^1 \left(\frac{\delta Q}{T} \right)_B = 0$$

Now consider another reversible cycle which proceeds first along path C and is then completed along path B

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T} \right)_C + \int_2^1 \left(\frac{\delta Q}{T} \right)_B = 0$$

We get

$$\int_1^2 \left(\frac{\delta Q}{T} \right)_A = \int_1^2 \left(\frac{\delta Q}{T} \right)_C$$

Since quantity $\frac{\delta Q}{T}$ is the same for all reversible paths between states 1 and

2, we conclude that this quantity is independent on the path and is a function of end states only; it is therefore a property called entropy (S, s).

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \quad (kJ/K) \quad \text{total entropy}$$

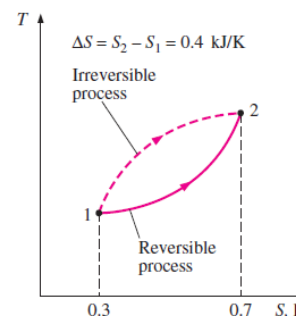
Entropy is viewed as a measure of the disorder (or “mixed-up-ness”).

The entropy change of a system during a process can be determined by integrating above equation between the initial and the final states:

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \quad (kJ/K) \quad (1)$$

$$\Delta s = \frac{\Delta S}{m} = \int_1^2 \left(\frac{\delta q}{T} \right)_{\text{int rev}} \quad (kJ/kg.K) \quad \text{change in specific entropy}$$

$$\Delta \dot{S} = \dot{m} \Delta S = \int_1^2 \left(\frac{\delta \dot{Q}}{T} \right)_{\text{int rev}} \quad (kW/K) \quad \text{rate of change in entropy}$$



To perform the integration in Eq. (1), one needs to know the relation between Q and T during a process.

Note that entropy is a property, and like all other properties, it has fixed values at fixed states. Therefore, the entropy change ΔS between two specified states is the same no matter what path, reversible or irreversible, is followed during a process.

e.g: Internally Reversible Isothermal Heat Transfer Processes at constant temperature T_o

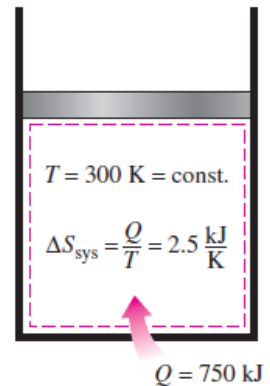
$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}}$$

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T_o} \right)_{\text{int rev}} = \frac{1}{T_o} \int_1^2 (\delta Q)_{\text{int rev}} = \frac{Q}{T_o}$$

Ex: A piston–cylinder device contains a liquid–vapor mixture of water at 300 K. During a constant-pressure process, 750 kJ of heat is transferred to the water. As a result, part of the liquid in the cylinder vaporizes. Determine the entropy change of the water during this process.

Sol:

We note that the temperature of the system remains constant at 300 K during this process since the temperature of a pure substance remains constant at the saturation value during a phase change process at constant pressure.



$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = \frac{Q}{T} = \frac{750}{300} = +2.5 \text{ kJ/kg}$$

Ex: Air is compressed by a 12-kW compressor from P_1 to P_2 . The air temperature is maintained constant at 25°C during this process as a result of heat transfer to the surrounding medium at 10°C . Determine the rate of entropy change of the air. State the assumptions made in solving this problem.

Sol:

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m} \left(h + Ve^2/2 + gz \right) - \sum_{in} \dot{m} \left(h + Ve^2/2 + gz \right)$$

Neglect the change in PE and KE and since the temperature of the system remains constant, i.e. $\Delta h = 0$, thus:

$$\dot{Q} = \dot{W} = -12 \text{ kW}$$

$$\Delta \dot{S} = \int_1^2 \left(\frac{\delta \dot{Q}}{T} \right)_{int \ rev} = \frac{\dot{Q}}{T} = \frac{-12}{283} = -0.0424 \text{ (kW/K)}$$

