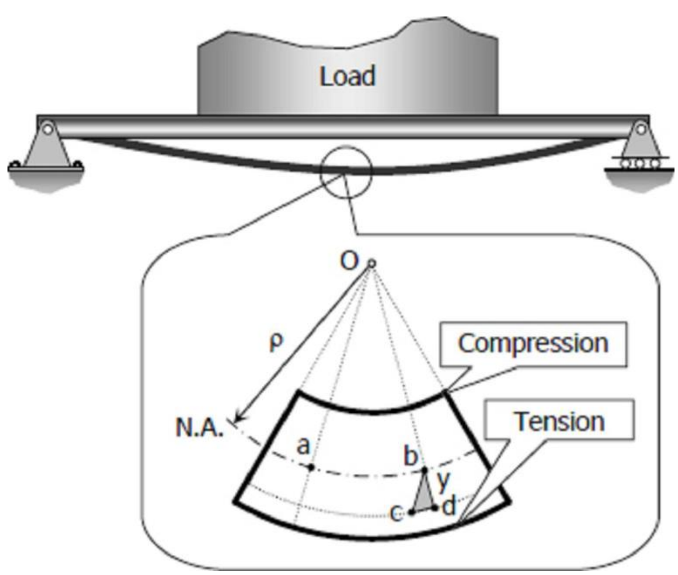
Stresses in Beams

Forces and couples acting on the beam cause bending (flexural stresses) and shearing stresses on any cross section of the beam and deflection perpendicular to the longitudinal axis of the beam. If couples are applied to the ends of the beam and no forces act on it, the bending is said to be pure bending. If forces produce the bending, the bending is called ordinary bending.

In using the following formulas for flexural and shearing stresses, it is assumed that a plane section of the beam normal to its longitudinal axis prior to loading remains plane after the forces and couples have been applied, and that the beam is initially straight and of uniform cross section and that the moduli of elasticity in tension and compression are equal.

Flexure Formula

Stresses caused by the bending moment are known as flexural or bending stresses. Consider a beam to be loaded as shown:



Consider a fiber at a distance *y* from the neutral axis, because of the beam's curvature, as the effect of bending moment; the fiber is stretched by an amount of *cd*. Since the curvature of the beam is very small, *bcd* and *Oba* are considered as similar triangles. The strain on this fiber is:

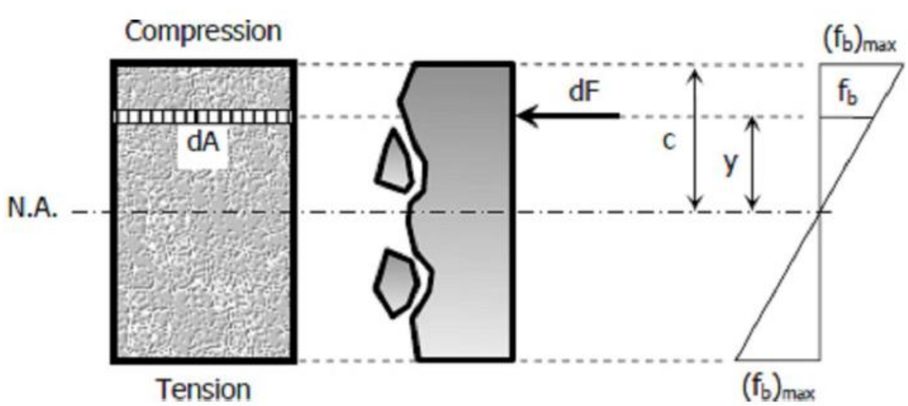
$$\varepsilon = \frac{cd}{ab} = \frac{y}{\rho}$$

By Hooke's law, $\varepsilon = \sigma / E$, then

$$\frac{\sigma}{E} = \frac{y}{\rho}\,;\ \ \sigma = \frac{y}{\rho}E$$

Which means that the stress is proportional to the distance *y* from the neutral axis.

For this section, the notation f_b will be used instead of σ.



Considering a differential area *dA* at a distance *y* from N.A., the force acting over the area is:

$$dF = f_b\, dA = \frac{y}{\rho} E\, dA = \frac{E}{\rho} y\, dA$$

The resultant of all the elemental moment about N.A. must be equal to the bending moment on the section.

$$M = \int M\, dF = \int y \left(\frac{E}{\rho} y\, dA\right)$$
$$M = \frac{E}{\rho} \int y^2\, dA$$

but $\int y^2\, dA = I$ then

$$M = \frac{EI}{\rho} \text{ or } \rho = \frac{EI}{M}$$

substituting $\rho = Ey/f_b$

$$\frac{Ey}{f_b} = \frac{EI}{M}$$

Then:

$$f_b = \frac{My}{I}$$

And:

$$(f_b)_{max} = \frac{Mc}{I}$$

The bending stress due to beams curvature is:

$$f_b = \frac{Mc}{I} = \frac{\frac{EI}{\rho} c}{I}$$

$$f_b = \frac{Ec}{\rho}$$

The **beam curvature** is:

$$k = \frac{1}{\rho}$$

Where ρ is the radius of curvature of the beam in mm (in), M is the bending moment in N·mm (lb·in), f_b is the flexural stress in MPa (psi), I is the centroidal moment of inertia in mm^4 (in^4), and c is the distance from the neutral axis to the outermost fiber in mm (in).

<u>Section Modulus:</u>

In the formula

$$(f_b)_{max} = \frac{Mc}{I} = \frac{M}{I/c}$$

The ratio I/c is called the section modulus and is usually denoted by S with units of mm^3 (in^3). The maximum bending stress may then be written as:

$$(f_b)_{max} = \frac{M}{S}$$

This form is convenient because the values of S are available in handbooks for a wide range of standard structural shapes.

Example 1:

A cantilever beam, 50 mm wide by 150 mm high and 6 m long, carries a load that varies uniformly from zero at the free end to 1000 N/m at the wall. (a) Compute the magnitude and location of the maximum flexural stress. (b) Determine the type and magnitude of the stress in a fiber 20 mm from the top of the beam at a section 2 m from the free end.

Example 2:

A simply supported beam, 2 in wide by 4 in high and 12 ft long is subjected to a concentrated load of 2000lb at a point 3ft from one of the supports. Determine the maximum fiber stress and the stress in a fiber located 0.5 in from the top of the beam at midspan.

Example 3:

Determine the minimum height h of the beam shown in Fig. P-508 if the flexural stress is not to exceed 20MPa.