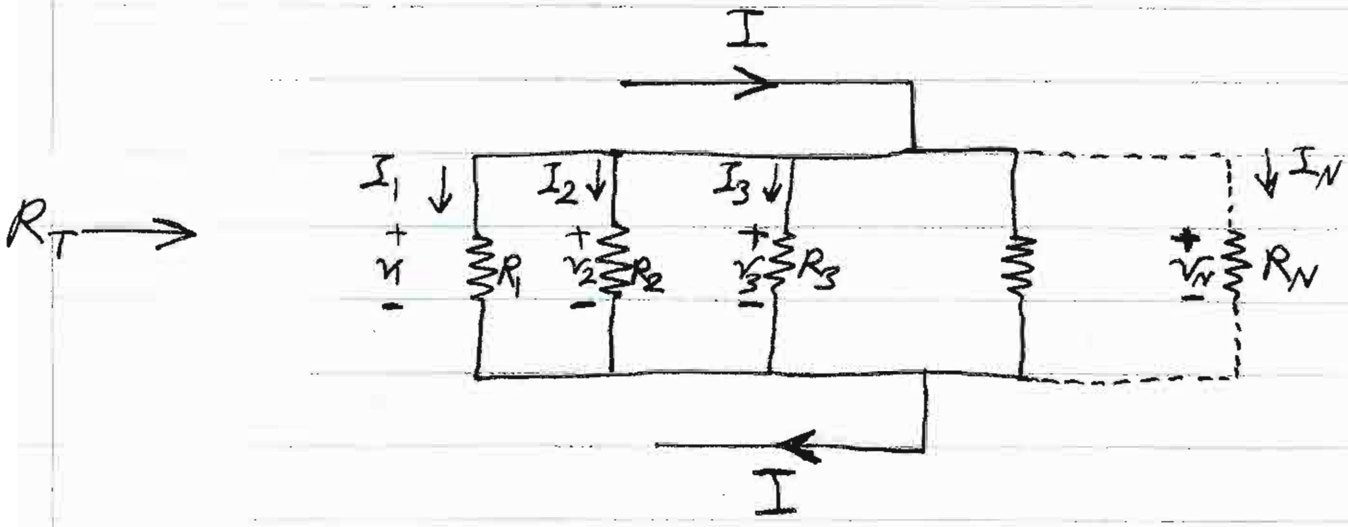
Current Divider Rule (CDR):



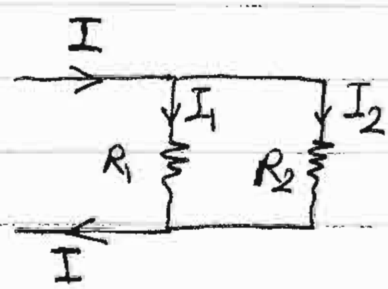
$V_1 = V_2 = V_3 = \cdots = V_N$

$I = I_1 + I_2 + I_3 + \cdots + I_N$

$\because I = \frac{V}{R_T} = \frac{I_X R_X}{R_T} \Longrightarrow \boxed{I_X = \frac{R_T}{R_X} I}$

→ General form of CDR

The CDR for 2 parallel resistors:



$\because R_T = \frac{R_1 R_2}{R_1 + R_2}$

& $I_1 = \frac{R_T}{R_1} I = \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_1} * I$

$\therefore I_1 = I * \frac{R_2}{R_1 + R_2}$

Note the difference in subscript.

& $I_2 = I * \frac{R_1}{R_1 + R_2}$

Example: A 100V dc source supplies a current of 5mA to two resistors in parallel. If $R_1 = 100k\Omega$, calculate the:

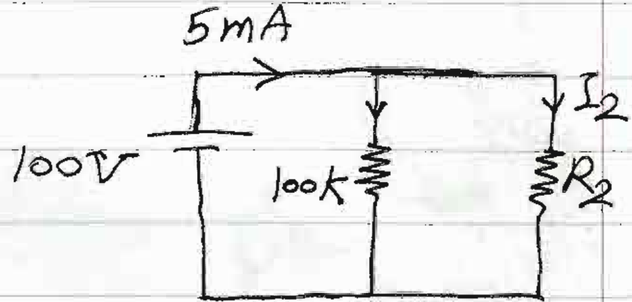
a) current in R_2

b) Value of R_2

c) Equivalent circuit resistance

d) current in R_2 using the CDR

Sol:



$$I_{100k} = \frac{100V}{100k\Omega} = 1mA$$

from KCL $\Rightarrow I_T = I_1 + I_2 \Rightarrow I_2 = I_T - I_1 = 5mA - 1mA = 4mA$

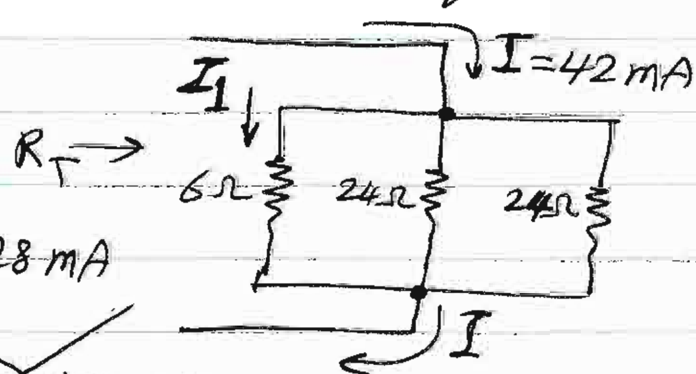
b) $R_2 = \frac{E}{I_2} = \frac{100V}{4mA} = 25k\Omega$

c) $R_T = \frac{100k * 25k}{(100+25)k} = 20k\Omega$

d) $I_2 = I * \frac{R_1}{R_1+R_2} = 5mA * \frac{100k\Omega}{(100+25)k\Omega} = 4mA$

Example: Find the current I_1 for the following network:

Sol:)



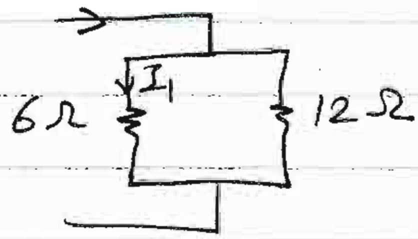
$$I_1 = \frac{R_T}{R_1} I$$

$$\therefore I_1 = \frac{4\Omega}{6\Omega} * 42 * 10^{-3} mA = 28mA$$

We cannot use ~~$\frac{24}{6+24} * 42mA$~~

But we can do the following

$24\Omega \,//\, 24\Omega = 12\Omega$

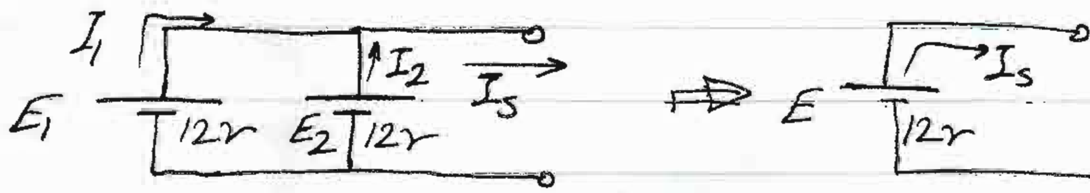


$$I_1 = \frac{12\Omega}{(6+12)\Omega} * 42\,mA$$

$\therefore I_1 = 28\,mA$

Voltage Sources in parallel:

The primary reason for placing two or more batteries in parallel of the same terminal voltage is to increase the current rating of the source, as shown bellow:

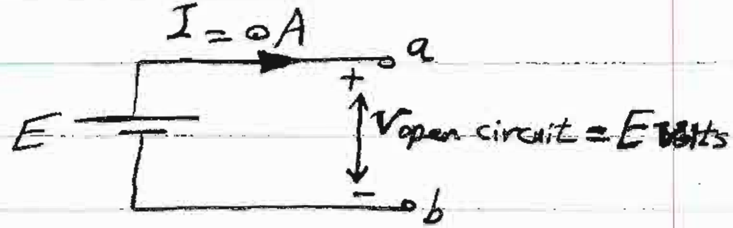


$I_S = I_1 + I_2$

The resulting power rating is twice that available with one supply.

Open and Short Circuits:

The following figure shows an open circuit exists between terminals a and b.

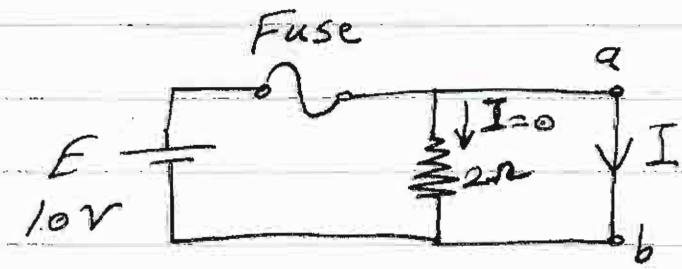


$V_{ab} = E$

An open circuit is defined as infinite resistance (Zero Conductance).

$\because I = \frac{V}{R}$ and $R = \infty$, then the current is Zero.

* A short circuit is defined as a resistance of zero ohms (infinite conductance)



$R_T = \frac{2 * 0}{2 + 0} = 0\,\Omega$

$\therefore I = \frac{E}{0} = \infty\,A$

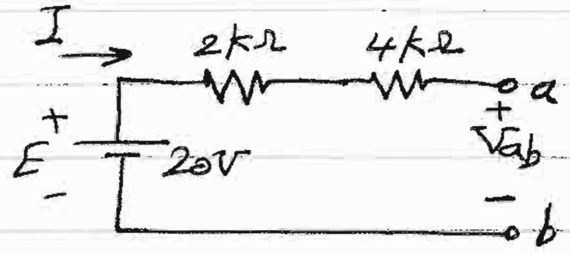
$V_{ab} = I * R_{ab} = 0$

$I_{2\Omega} = 0\,\Omega$

$\therefore R_{2\Omega}$ is shorted by the connection (a b).

Example: Determine the voltage V_{ab} for the following network.

Sol:)



The open circuit requires that I be zero amperes.

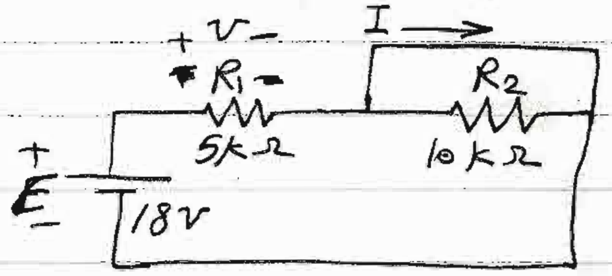
$\therefore V_{2k\Omega} = V_{4k\Omega} = IR = (0)R = 0V$

By applying KVL around the closed loop,

$$V_{ab} = E = 20V$$

Example: Calculate the current I and the voltage V for the following network.

Sol:)



The 10kΩ has been shorted out.

$\therefore I = \frac{E}{R_1} = \frac{18V}{5k\Omega} = 3.6\,mA$ & $V = RI = 5k\Omega * 3.6mA$

$\therefore V = 18V$

OR $V = E = 18V$

Home work:

Q1) Two resistors must be selected so that the current in one is four times the current in the other. If their equivalent parallel resistance is 5kΩ, calculate R_1 and R_2.

[Ans: 6.25K, 25K]

Q2) A resistance R_1 of $5 k\Omega$ is shunted by a conductance G of $50 \mu S$. The combination draws a current of 10 mA from a d.c. supply. Calculate:

a) G_T and R_T

b) I_1 and I_2

c) The voltage across each resistor and the supply voltage.

[Ans: a- $250 \mu S$, $4 k\Omega$
b- 8mA, 2 mA
c- 40V]

Q3) Two conductances G_1 and G_2 having values of 600 and 900 μS respectively, are connected in parallel. The combination is then connected in parallel with 800Ω resistor. Calculate the:

a) Equivalent resistance of the parallel circuit. [Ans: 363.63Ω]

b) Current in each branch if G_1 carries 2mA. [Ans: 3mA, 4.16mA]

c) power dissipated in each branch. [Ans: 6.6mw, 10mw, 13.8mw]

d) Voltage across each component. [Ans: 3.33 V].

e) Total power dissipated. [Ans: 30.5 mW].

Q4) A 6V battery, delivers 10mA to three resistors in parallel, $R_1 = 1 k\Omega$, $R_2 = 2 k\Omega$, $R_3 = R_x$.

Calculate: a) Current in R_x, that is I_x. [Ans: 1 mA].

b) Power dissipated by R_x and R_1. [Ans: 6mW, 36mW].

c) Resistance ratio R_x/R_1. [Ans: 6/1].

d) Current ratio I_1/I_x. [Ans: 6/1].

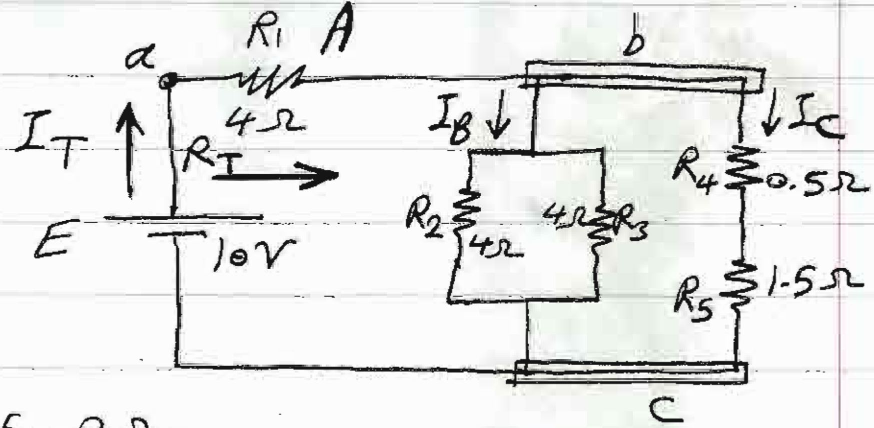
Series-Parallel Circuits:

A series-parallel circuit consists of a combination of any number of series and parallel elements with only one source of voltage.

The solution of this type of circuits requires that the resistance "seen" by the source (R_T) be determined by combining series and parallel elements until a single equivalent resistance remains. We can then find the source current (I_T) and all the other currents.

Example: For the following cct., find I_T, I_B, I_C, V_A, V_B and V_C.

Sol.:



$R_B = R_2 \parallel R_3$

$= \frac{R}{N} = \frac{4}{2} = 2\Omega$

$R_C = R_4 + R_5 = 0.5 + 1.5 = 2\Omega$

$\therefore R_{B\parallel C} = \frac{R}{N} = \frac{2}{2} = 1\Omega$

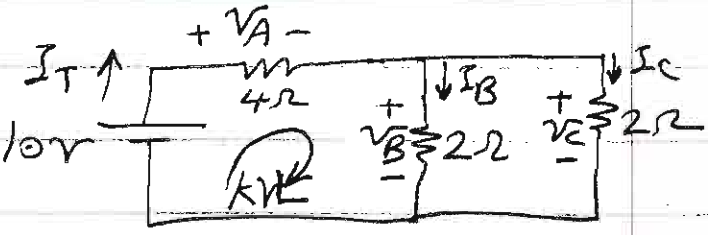
$\therefore R_T = R_1 + R_{B\parallel C} = 4 + 1 = 5\Omega$

$\therefore I_T = \frac{E}{R_T} = \frac{10V}{5\Omega} = 2A$

$\therefore I_T = I_A = 2A$

$I_B = I_C = \frac{I_T}{2} = 1A$

From the previous fig.



$I_{R_2} = I_{R_3} = 0.5A$

$V_A = 2A(4\Omega) = 8V$

$V_B = 1A(2\Omega) = 2V$

$V_C = V_B = 2V$

If we apply KVL:

$E - V_A - V_B = 0 \Rightarrow E = V_A + V_B$

$10V = 8V + 2V$

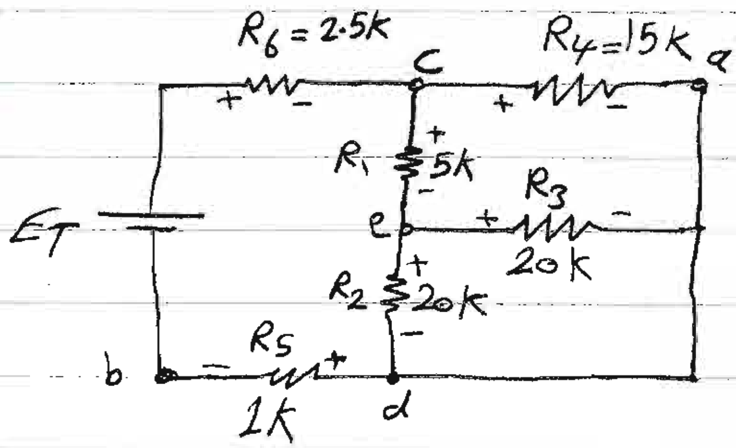
$\therefore 10V = 10V$ (checks)

Example: For the following cct., $I_2 = 2mA$. Calculate:

a) all the remaining currents in all resistors.

b) Supply voltage E_T.

c) Voltage V_{ab}.



Sol:)

a) $\because R_2 // R_3$ and $R_2 = R_3$

$\therefore I_2 = I_3 = 2mA$

By KCL at node e:

$I_1 = I_2 + I_3 = 4mA$

$\because R_A = (R_1 + R_2 // R_3) = 5k\Omega + 20k\Omega // 20k\Omega = 15k\Omega$

$\because R_A // R_4$ and $R_A = R_4$

$\therefore I_A = I_4 = 4mA$

By KCL at node C:

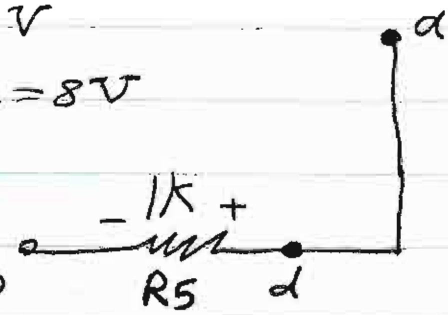
$I_6 = I_A + I_4 = 4mA + 4mA = 8mA = I_5 = I_T$

b) ∴ $R_T = (R_A // R_4) + R_6 + R_5$

$R_T = 15k // 15k + 2.5k + 1k = 11 k\Omega$

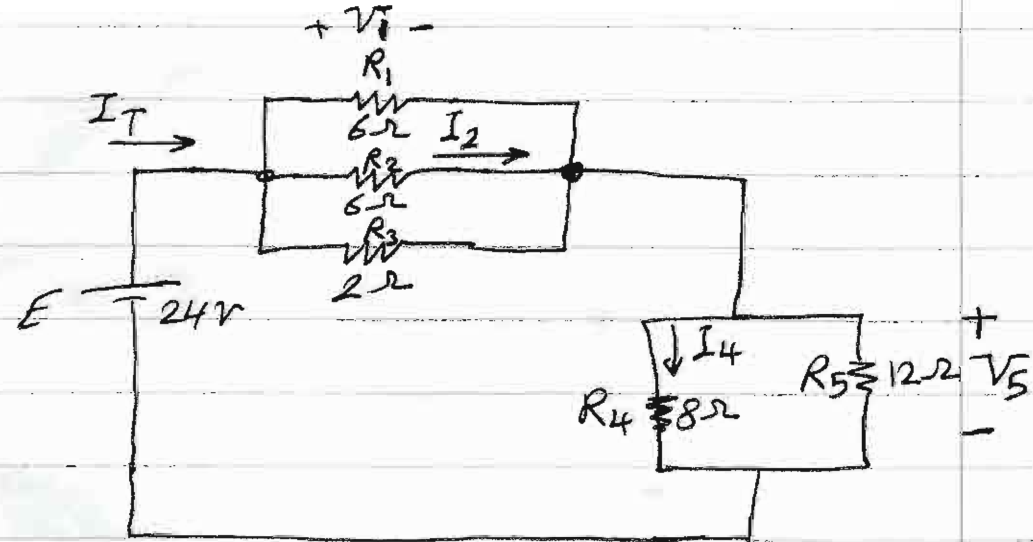
$\therefore E_T = I_T * R_T = 8mA * 11 k\Omega = 88 V$

c) $V_{ab} = V_{db} = V_{R_5} = 1k * I_5 = 1k * 8m = 8V$



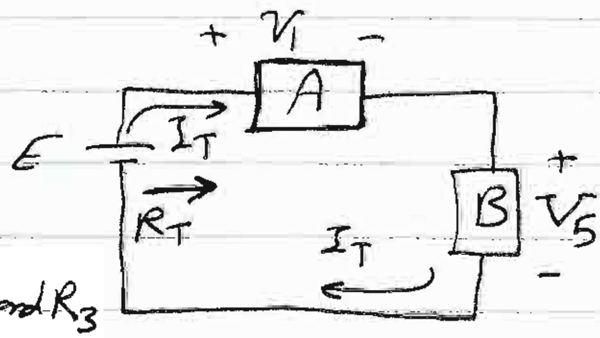
Example 7.5: [page 153]

Find the indicated currents and voltages for the following network.



We can see that A and B are in series.

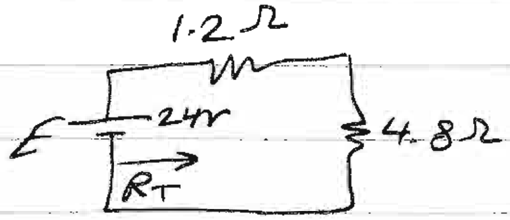
The voltage V_1 will be the same across the three parallel branches of R_1, R_2 and R_3 and V_5 will be the same across R_4 and R_5. Once V_1 and V_5 are known, the required currents can be found using Ohm's law.



$$R_{1\|2} = \frac{R}{N} = \frac{6}{2} = 3\,\Omega$$

$$R_A = R_{1\|2\|3} = \frac{(3)(2)}{3+2} = \frac{6}{5} = 1.2\,\Omega$$

$$R_B = R_{4\|5} = \frac{(8)(12)}{8+12} = 4.8\,\Omega$$



$$\therefore R_T = R_{1\|2\|3} + R_{4\|5}$$

$$R_T = 1.2 + 4.8 = 6\,\Omega$$

$$\therefore I_T = \frac{E}{R_T} = \frac{24}{6} = 4A$$

$$V_1 = I_T R_{1\|2\|3} = (4)(1.2) = 4.8\ V = V_{R_1} = V_{R_2} = V_{R_3}$$

$$V_5 = I_T R_{4\|5} = (4)(4.8) = 19.2\ V = V_{R_4} = V_{R_5}$$

By applying Ohm's Law:

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8V}{6\,\Omega} = 0.8\ A$$

$$I_4 = \frac{V_5}{R_4} = \frac{19.2\ V}{8\,\Omega} = 2.4\ A$$

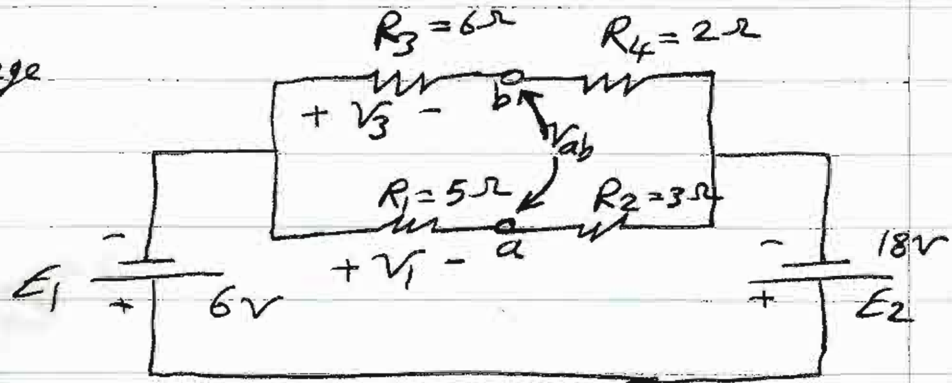
Example 7.6: [page 154] For the following cct.,

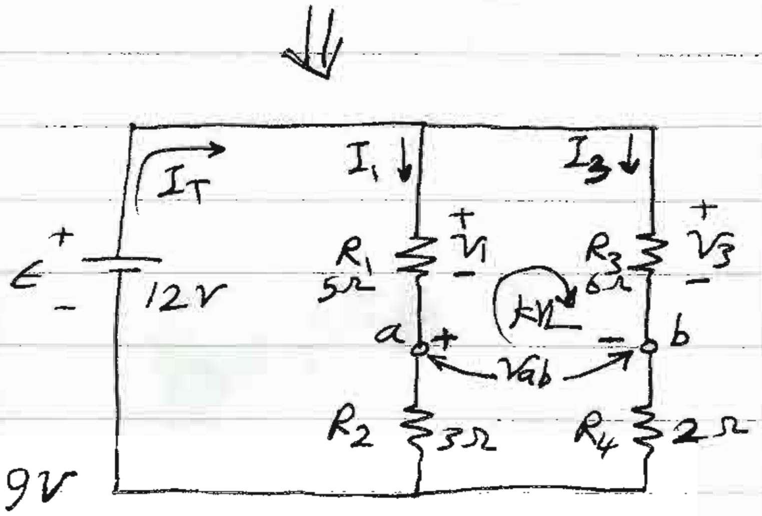
a) Find V_1, V_3, and V_{ab}.

b) Calculate the source current I_T.

Sol.)

The net source voltage is the difference between the two sources with the polarity of the larger.





By applying the VDR:

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{5(12)}{5+3}$$

$$\therefore V_1 = 7.5V$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{6(12)}{6+2} = 9V$$

The open-circuit voltage V_{ab} is found by KVL in the CW direction:

$$+V_1 - V_3 + V_{ab} = 0 \Longrightarrow V_{ab} = V_3 - V_1 = 9V - 7.5V = 1.5V$$

b) From Ohm's Law:

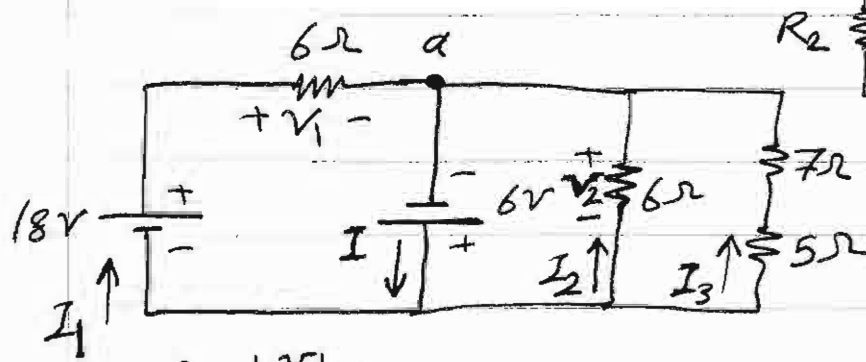
$$I_1 = \frac{V_1}{R_1} = \frac{7.5V}{5\Omega} = 1.5A$$

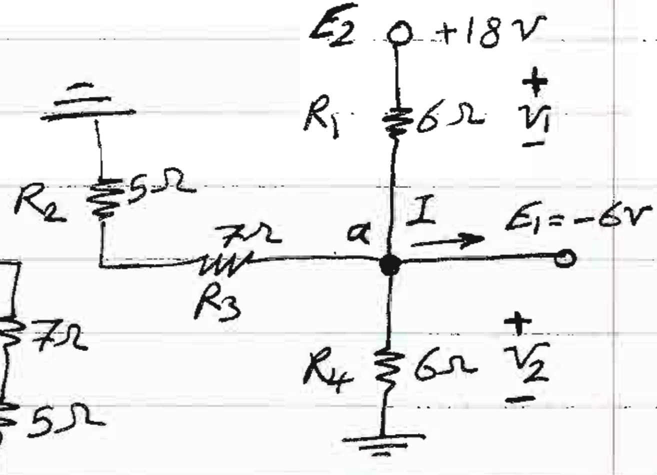
$$I_3 = \frac{V_3}{R_3} = \frac{9V}{6\Omega} = 1.5A$$

By KCL:

$$I_T = I_1 + I_3 = 1.5 + 1.5 = 3A$$

Example 7.7: [Page 155]





By KVL:

$-V_2 - 6V = 0 \Rightarrow V_2 = -6V$

The minus sign simply indicates that the chosen polarity for V_2 in the original fig. is opposite to the actual voltage.

By KVL: $+18 - V_1 + 6 = 0 \Rightarrow V_1 = 18 + 6 = 24V$

By KCL:

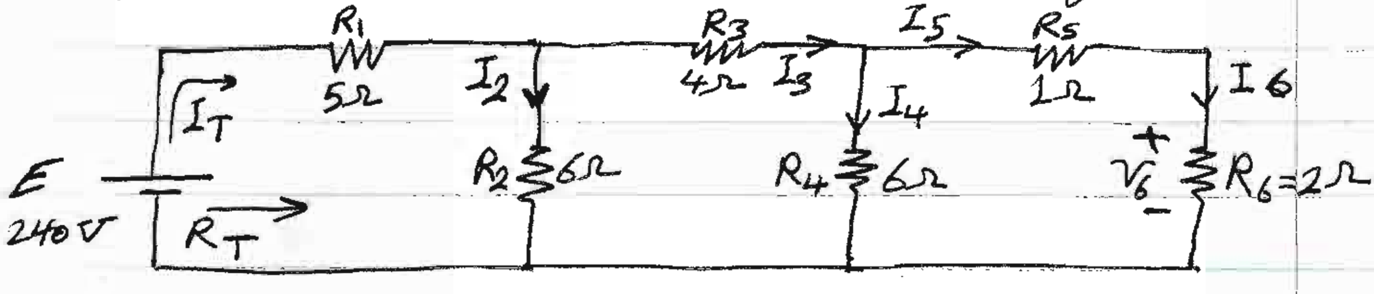
$I = I_1 + I_2 + I_3 = \frac{V_1}{6\Omega} + \frac{V_2}{6\Omega} + \frac{V_2}{12\Omega} = \frac{24}{6} + \frac{6}{6} + \frac{6}{12}$

$\therefore I = 4 + 1 + 0.5$

$\therefore I = 5.5 A$

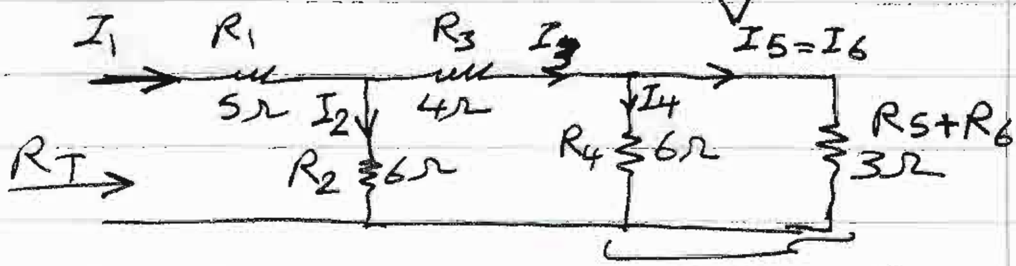
Ladder Network:

Example: Determine V_6, I_2, I_4 for the following network.

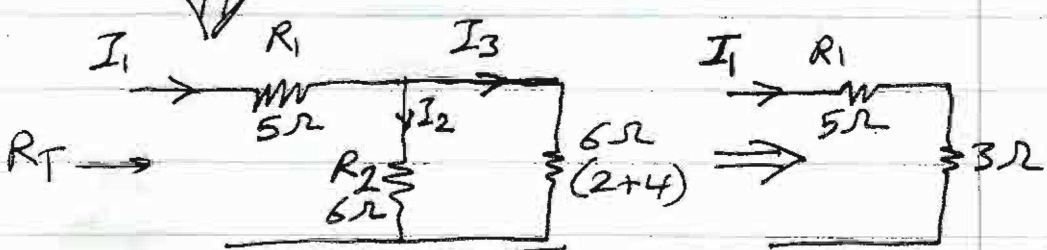


Sol:)

① Find the total resistance (R_T)



$\frac{6*3}{6+3} = 2\,\Omega$



$\therefore R_T = 5+3 = 8\,\Omega$ $\qquad \frac{6(6)}{12} = 3\,\Omega$

② Find the total current (I_T)

$I_T = \frac{E}{R_T} = \frac{240}{8} = 30\,A$

③ Work back through the ladder until the desired voltage or current is obtained.

$I_1 = I_T = 30\,A$

$I_3 = I_T * \frac{R_2}{R_2+6} = 15\,A \Rightarrow I_2 = I_1 - I_3 = 30-15 = 15\,A$

$I_6 = I_3 * \frac{R_4}{R_4+3} = 15 * \frac{6}{9} = 10\,A$

$\therefore V_6 = I_6 * R_6 = 10A * 2\Omega = 20V$

$I_4 = I_3 - I_6 = 15 - 10 = 5A$

Home Work:

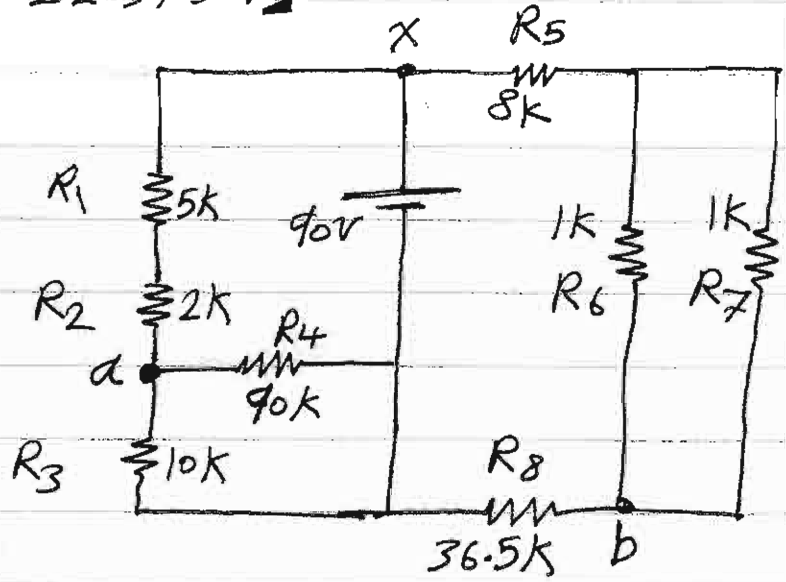
Q1 For the following circuit, calculate the:

a) Current in resistor R_1 through R_8.

b) Voltage between points x and a, V_{xa}.

c) " " " a " b, V_{ab}.

[Ans: b 39.375 V, c −22.375 V]

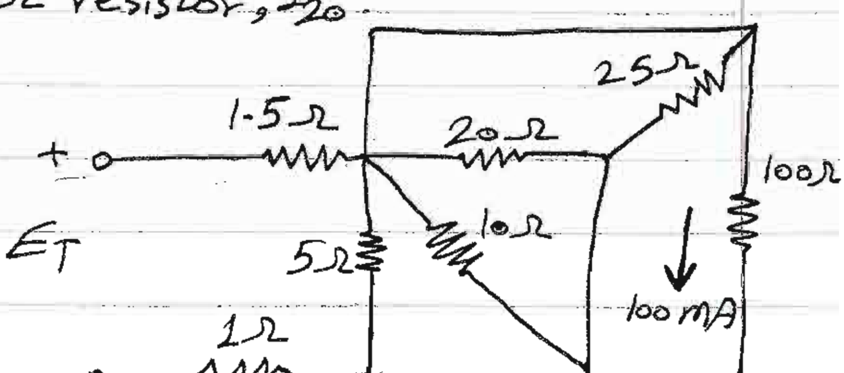


Q2 For the following circuit, calculate:

a) The supply voltage, E_T.

b) The current in 20Ω resistor, I_{20}.

[Ans: 20 V, 0.5 A]

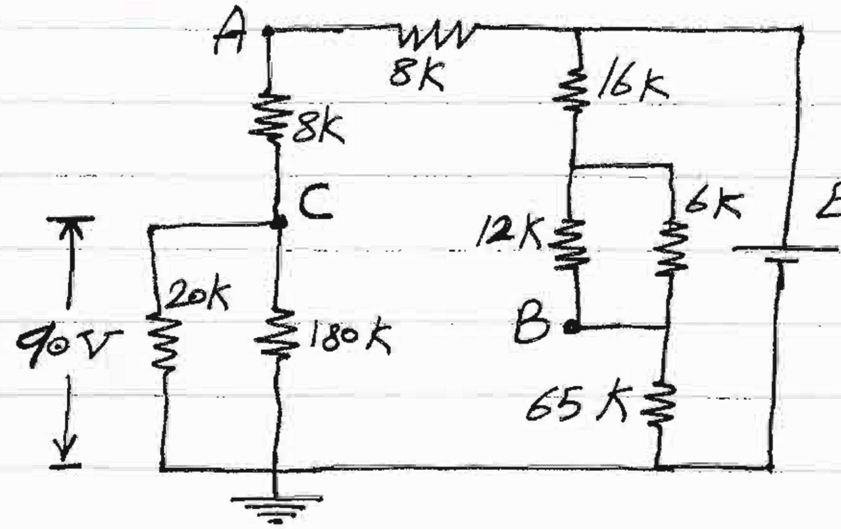


Q3 For the circuit shown, calculate:

a) E_T b) V_A c) V_B d) V_{AB}

e) V_{BC} f) Power dissipated by 6 kΩ.

[Ans: 170 V, 130 V, 130 V, 0 V, 40 V, 10.66 mW]

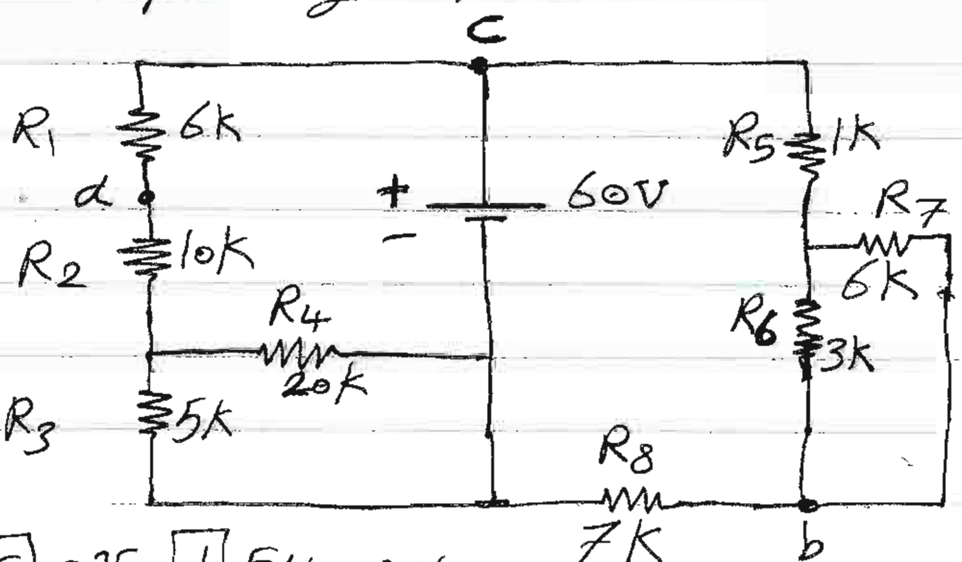


Q4 For the circuit shown,

a) Complete solution, showing all voltages and all currents for all resistors.

b) V_{cb} c) V_{ab} d) Total power drawn from the supply.

e) Total power dissipated by R_4.



[Ans: b 18V, c 0V, d 540 mW, e 7.2 mW].

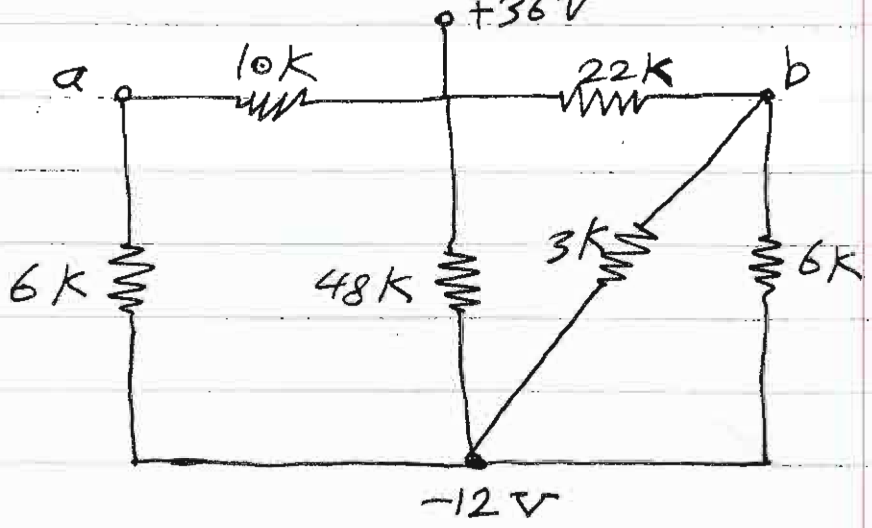
P5) For the circuit shown, calculate the:

a) Total current flowing through the two supplies.

b) Voltage at node a, V_a.

c) " " " b, V_b.

[Ans: 6 mA, 6 V, -8 V]



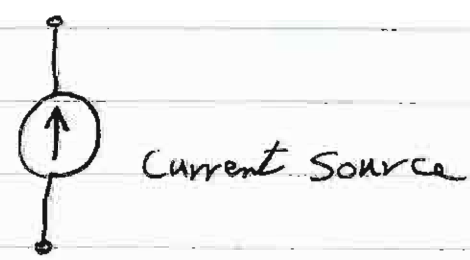
Methods of Analysis:

The circuits described in the previous chapters had only one source or two or more sources in series or parallel. The step-by-step procedure outlined in those chapters cannot be applied if two or more sources in the same network are not in series or parallel.
Methods of analysis ~~were~~ have been developed that allow us to ~~analys~~ deal with a network with any number of sources in any arrangement. These methods include branch-current analysis, mesh analysis, and nodal analysis.

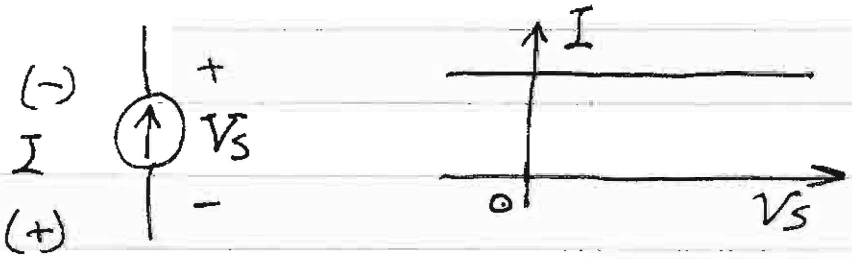
Current Sources:

While a battery (or a voltage source) supplies a fixed voltage and the source current can vary, the current source supplies a fixed current to the branch in which it is located, while its terminal voltage may vary as determined by the network to which it is applied.

The symbol of a current source is shown in this fig.



The terminal voltage is fixed at E volts for the range of current values. To the right of the E axis, the current will have one direction through the source, while to the left it is reversed. The current direction is determined by the other elements of the cct. In other words, as shown in the first fig., the terminal voltage of a source is unaffected by the direction of current through the source.

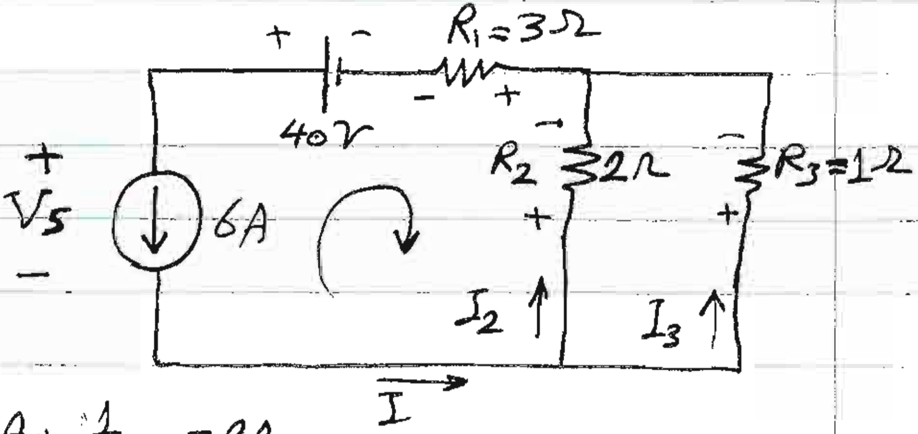


The current source will supply a fixed current even though the voltage across the source may vary in magnitude or reverse its polarity. The network to which the current source is connected will determine the magnitude and polarity of the voltage across the source.

Example 8.3: [Page: 175]

Find V_S for the following cct.

Sol:)



$$I_2 = I_T * \frac{R_3}{R_2 + R_3} = 6A * \frac{1}{2+1} = 2A$$

$\therefore V_3 = V_2 = I_2 R_2 = 2 * 2 = 4V$

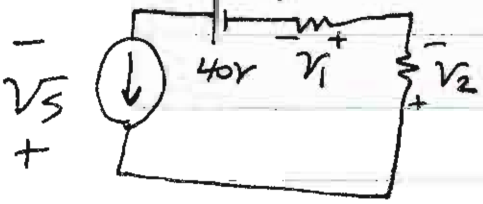
OR $V_3 = I_3 R_3 = (I_T - I_2) * R_3 = (6-2)*1 = 4V$

$V_1 = I R_1 = 6 * 3 = 18V$

To find V_{supply}, we can apply KVL:

$+V_S - 40 + V_1 + V_2 = 0 \Rightarrow V_S = 40 - 18 - 4 = 18V$ $\begin{matrix} + \\ V_S \\ - \end{matrix}$

If the polarity of V_S was given as:



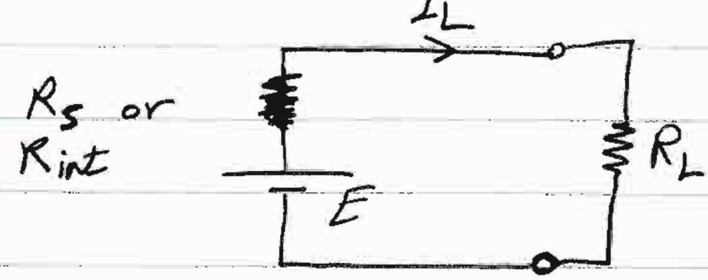
$\Rightarrow -V_S - 40 + 18 + 4 = 0$

$\therefore V_S = -40 + 18 + 4 = -18V$

which means that the correct polarity is $\begin{matrix} + \\ V_S \\ - \end{matrix}$

Source Conversions:

Consider the basic voltage source with its internal resistance, as shown in this fig.:



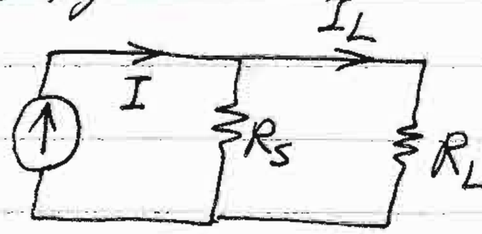
$$I_L = \frac{E}{R_S + R_L}$$

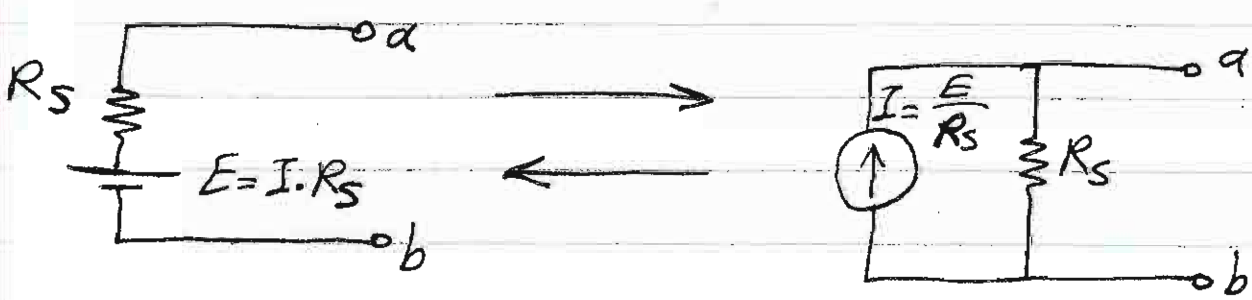
By multiplying the numerator by a factor $(1 = R_S/R_S)$

$$I_L = \frac{\frac{R_S}{R_S} \cdot E}{R_S + R_L} = \frac{R_S\,(E/R_S)}{R_S + R_L} = \frac{R_S\,(I)}{R_S + R_L}$$

The resulting equation is an application of the CDR to the network of the following fig.

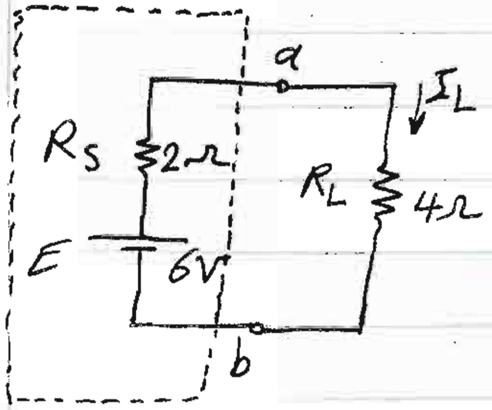
$$I = \frac{E}{R_S}$$





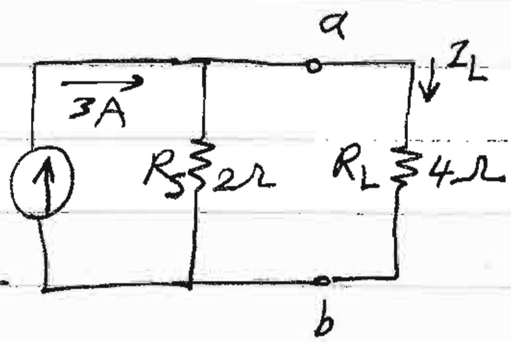
Note: To perform a conversion, the voltage source <u>must have a resistor in series with it,</u>
and a current source <u>must have a resistor in parallel.</u>

Example: Convert the voltage source of the following fig. to a current source and find I_L for each case.



$I = \frac{E}{R_S} = \frac{6}{2}$

$I = 3A$

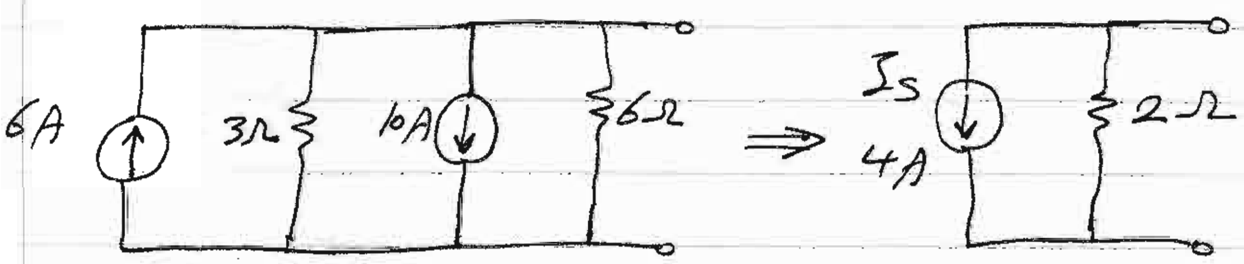


$$I_L = \frac{E}{R_S + R_L} = \frac{6}{2+4} = 1A \implies I_L = \frac{I_T \cdot R_S}{R_S + R_L} = \frac{3 \cdot 2}{2+4} = 1A$$

Current Sources in Parallel:

If two or more current sources are in parallel, they may all be replaced by one current source having the magnitude and direction of the resultant.

Example:

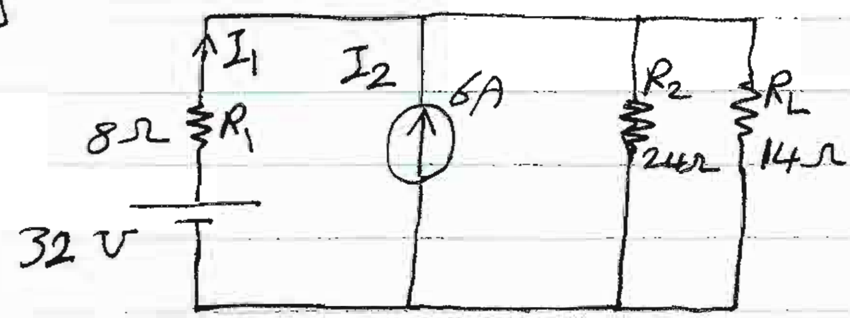


$I_S = 10 - 6 = 4A$

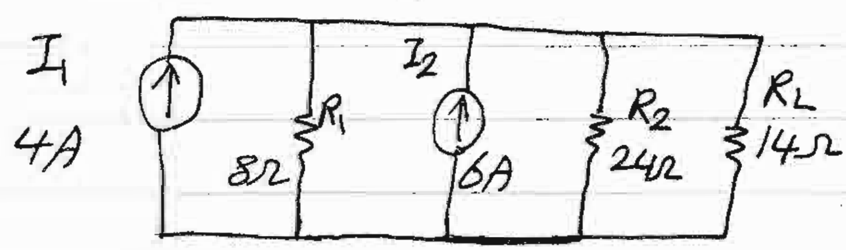
$R_S = 3\Omega \parallel 6\Omega = \frac{3 \cdot 6}{3+6} = 2\Omega$

Example: Calculate the current through R_L for the following cct.

Sol:

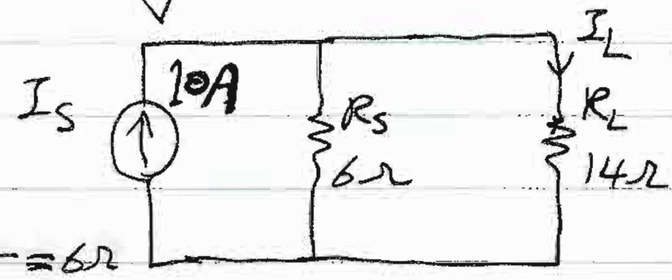


⇓



$$I_1 = \frac{E_1}{R_1} = \frac{32}{8} = 4A$$

⇓

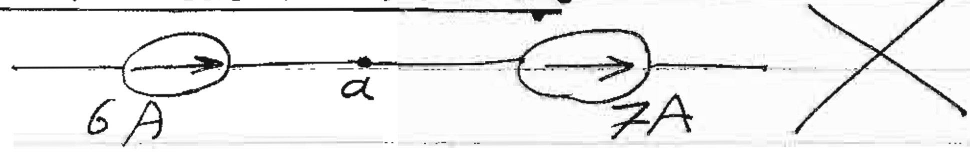


$$I_s = I_1 + I_2 = 4 + 6 = 10A$$

$$R_s = R_1 // R_2 = \frac{8 * 24}{8 + 24} = 6\Omega$$

$$\therefore I_L = \frac{I_s * R_s}{R_s + R_L} = \frac{10 * 6}{6 + 14} = 3A$$

Current Sources in Series:



Current sources of different current ratings are not connected in series, just as voltage sources of different voltage ratings are not connected in parallel.