Ohm's Law:

Every conversion of energy from one form to another can be related to the following equations

$$\boxed{\text{Effect} = \frac{\text{cause}}{\text{opposition}}}$$

In electric circuits, the effect we are trying to establish is <u>the flow of charge</u>, or <u>current</u>. The potential difference between two points is the cause ("pressure"), and the opposition is the resistance.

$\therefore$ $\text{current} = \dfrac{\text{potential difference}}{\text{resistance}}$

$\therefore$ $\boxed{I = \dfrac{E}{R}}$ (amperes, A) ------ Ohm's law

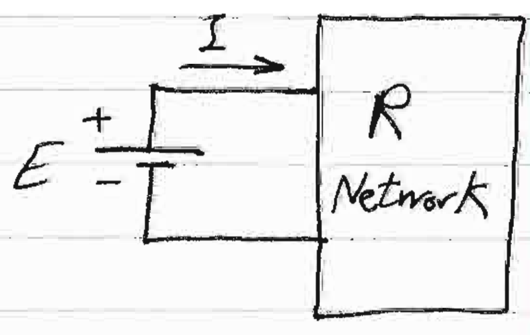
$\therefore$ The greater the voltage across a resistor, the more the current, and the more the resistance for the same voltage, the less the current.

$\therefore$ The current is proportional to the applied voltage ($I \propto E$) and inversely proportional to the resistance ($I \propto \frac{1}{R}$).

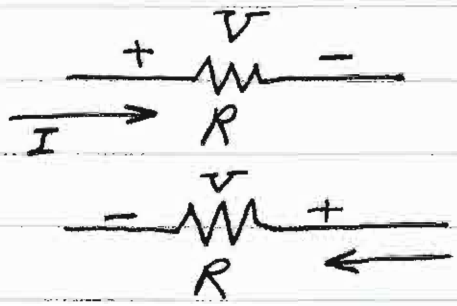
From Ohm's law:

$\boxed{E = R\,I}$ (volts, V)

$\boxed{R = \dfrac{E}{I}}$ (ohms, Ω)

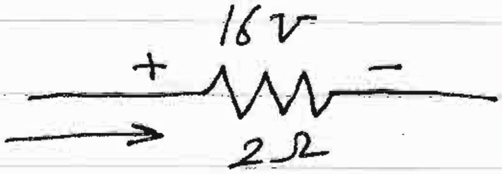


For resistors, the polarity of the voltage drop is as shown:



In general, the flow of charge is from a high (+) to a low (−) potential.

Example: Calculate the current through the $2\,\Omega$ resistor of the following figure if the voltage drop across it is $16\,V$.



Sol:)

$\because I = \frac{V}{R} = \frac{16}{2} = 8\ A$

Example: A charge of 800mC is transferred each second when a potential is applied across the terminal of a lamp having a resistance of $150\,\Omega$. Calculate the voltage across it.

Sol:) $\because V = RI \Rightarrow I = \frac{Q}{t} = \frac{800\ mC}{1\ s} = 800\ mA$

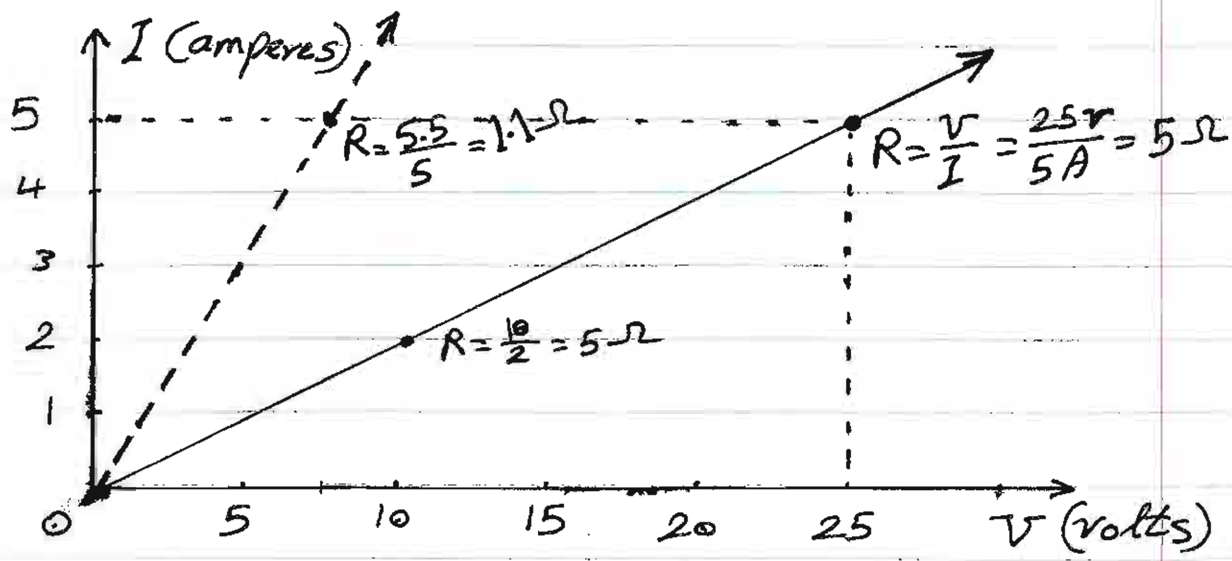
$\therefore V = 150 * 800 * 10^{-3} = 120\ V$

Example: A potential of $2.5\,V$ produces a current of $5\,mA$ in a resistor. Find a) The resistance b) The conductance.

Sol:) a) $\because R = \frac{V}{I} = \frac{2.5\ V}{5 * 10^{-3} A} = 500\ \Omega$

b) $\because G = \frac{1}{R} = \frac{1}{500\,\Omega} = 2\ mS$

Plotting Ohm's Law :



The linear (straight-line) reveals that the resistance is not changing with current or voltage level; rather it is a fixed quantity throughout.

At 25 V $\Rightarrow I = \frac{V}{R} = \frac{25}{5} = 5A$

At 10 V $\Rightarrow I = \frac{V}{R} = \frac{10}{5} = 2A$

Note that the less the resistance (see the 1.1 Ω line), the steeper the slope (closer to the vertical axis).
Using the straight-line equation:

$$I = \frac{1}{R} \cdot E + 0$$

$$y = m \cdot x + b \Rightarrow \therefore m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta I}{\Delta V} = \frac{1}{R}$$

Where Δ signifies a small finite change in the variable.
∴ The greater the resistance, the less the slope (as shown in the figure).

From the previous figure, we can conclude that the relationship between voltage and current is linear and R is a linear resistor.

In general, all resistive circuit elements are linear. Commercial resistors are relatively linear within certain bounds of voltage and current. If these bounds are exceeded, resistance increases and the element is nonlinear.
In addition, temperature affects the linearity of most resistive materials.

Power:-

Power is an indication of how much work can be accomplished in a specific amount of time.
In other words, power is the rate of doing work.

$$P = \frac{W}{t} = \frac{\text{total work done (in Joules)}}{\text{total time taken (in seconds)}} \quad \text{J/s or watts (W)}$$

$1 \text{ watt (W)} = 1 \text{ J/s}$

In addition, the power can be measured in horsepower (hp). The hp and the Watt are related as:

$$1 \text{ horsepower} \cong 746 \text{ watts}$$

The power delivered to, or absorbed by, an electrical system can be found in terms of the current and voltage:

$$\because V = \frac{W}{Q} \Longrightarrow W = VQ$$

$$\because I = \frac{Q}{t} \Longrightarrow Q = It$$

$$\because P = \frac{W}{t} = \frac{VQ}{t} = \frac{VI\cancel{t}}{\cancel{t}} = V \cdot I = R \cdot I^2 = V \cdot \frac{V}{R} = \frac{V^2}{R}$$

$$\therefore \boxed{P = V \cdot I = I^2 \cdot R = \frac{V^2}{R}} \quad \text{watts (W)}$$

Example: A resistor dissipates 500 J of energy in 5 min. Find the power dissipated by the resistor.

Sol:)

$$\because P = \frac{W}{t} = \frac{500\text{ J}}{5\,\cancel{\text{min}}\left(\frac{60\text{ s}}{1\,\cancel{\text{min}}}\right)} = \frac{500\text{ J}}{300\text{ s}} = 1.6667\text{ W}$$

Example: When a current of 5A flows in a resistor connected to 12V supply for a particular period, it dissipates 800 jouls. How long will it take to dissipate this quantity of energy.

Sol:)

$$\because P = V \cdot I = 5A * 12V = 60\ W$$

$$\because P = \frac{W}{t} \Rightarrow t = \frac{W}{P} = \frac{800\,\cancel{J}}{60\ W(\text{or } \cancel{J}/s)} = 13.333\ \text{sec.}$$

Example: A transformer winding has 900 W loss, After 6 h of operation. Calculate heat energy in
a) Joules b) kilowatt-~~hour~~

Sol:)

$$a) \because P = \frac{W}{t} \Rightarrow W = P \cdot t = 900\,\frac{J}{\cancel{S}}\ 6\cancel{h}\left(\frac{60\ \cancel{min}}{1\cancel{h}}\right)\left(\frac{60\ s}{1\ \cancel{min}}\right)$$

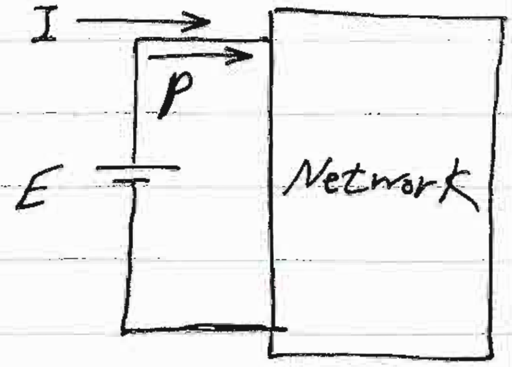
$$\therefore W = 1944 * 10^4 J$$

$$\therefore W = 19.44\ MJ$$

$$b)\ W = P \cdot t = 900\,\cancel{W}\left(\frac{kW}{1000\,\cancel{W}}\right) \cdot 6\ h = 5.4\ kW\text{-}h$$

The power delivered by an energy source is given by:

$$P = E\,I \dashrightarrow \text{watts}$$



where: E: is the source voltage

I: is the current drain from the source.

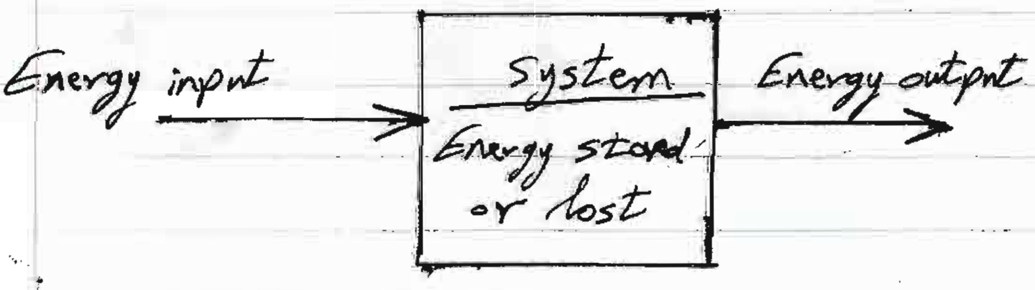
Sometimes the power is given and the current or voltage must be determined.

$$\because P = I^2 R \Longrightarrow I^2 = \frac{P}{R} \Longrightarrow I = \sqrt{\frac{P}{R}} \quad (A)$$

$$\because P = \frac{V^2}{R} \Longrightarrow V^2 = PR \Longrightarrow V = \sqrt{PR} \quad (V)$$

Efficiency:

Any electrical system that converts energy from one form to another can be represented by the following block diagram:



Conservation of energy requires that:

Energy input = Energy output + Energy lost or stored in the system

$$W_{in} = W_{out} + W_{losses}$$

Dividing both sides by t gives:

$$\frac{W_{in}}{t} = \frac{W_{out}}{t} + \frac{W_{losses}}{t}$$

since $P = W/t$, we have the following

$$\boxed{P_{in} = P_{out} + P_{losses}} \text{ ---- watts}$$

The efficiency (η) of any system is determined as:

$$\eta = \frac{\text{power output}}{\text{power input}} = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{losses}} = \frac{P_{in} - P_{losses}}{P_{in}}$$

We can re-write in terms of energy:

$$\eta = \frac{W_{out}/\cancel{t}}{W_{in}/\cancel{t}} = \frac{W_{out}}{W_{in}} = \frac{W_{out}}{W_{out} + W_{losses}} = \frac{W_{in} - W_{losses}}{W_{in}}$$

When η is expressed as a percentage:

$$\eta = \frac{P_{out}}{P_{in}} * 100\%$$

or $$\eta = \frac{W_{out}}{W_{in}} * 100\%$$

The maximum possible efficiency is 100%, which occurs when $P_{out} = P_{in}$, or when the power lost or stored in the system is zero. Clearly, the greater the internal losses of the system in generating the necessary output power or energy, the lower the net efficiency.

Example: A 2-hp motor operates at an efficiency of 75%. What is the power input in watts? If the input current is 9.05 A, what is the input voltage?

Sol:)

$$\because \eta = \frac{P_{out}}{P_{in}} \Longrightarrow P_{in} = \frac{P_{out}}{\eta} = \frac{2\,hp\left(\frac{746\,W}{1\,hp}\right)}{0.75}$$

$$\therefore P_{in} = 1989.333\ W$$

$$\because P_{in} = V_{in} \cdot I_{in} \Longrightarrow V_{in} = \frac{P_{in}}{I_{in}} = \frac{1989.333\ W}{9.05\ A} = 219.82 \cong 220V$$

Example: An electrical motor connected to a 120 V d.c. supply draws 10 A and develops an output power of 1.35 hp. Calculate:

a) Input power in watts b) Output power in watts

c) Motor efficiency d) Motor losses in watt

Sol:)

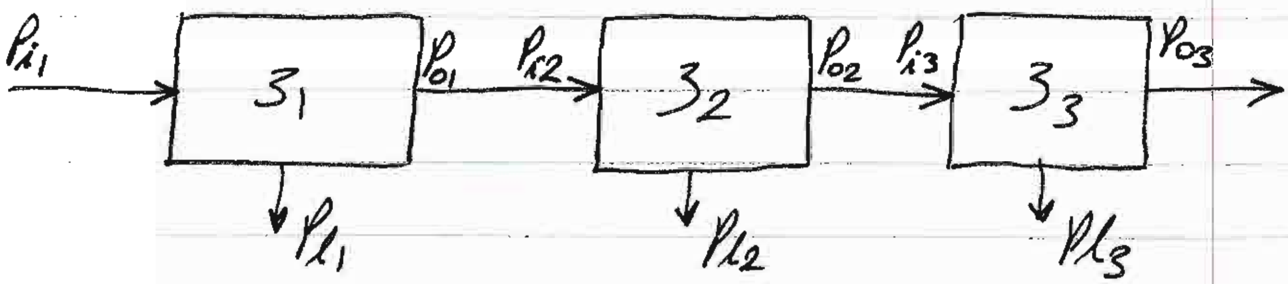
a) $P_{in} = V I = 120 * 10 = 1200\ W$

b) $P_{out} = 1.35\,hp\left(\frac{746\,W}{1\,hp}\right) = 1007.1\ W$

c) $\eta = \frac{P_{out}}{P_{in}} = \frac{1007.1\ W}{1200\ W} = 0.839 * \overbrace{\left(\frac{100}{100}\right)}^{=1} = 83.9\ \%$

d) $P_{losses} = P_{in} - P_{out} = 1200\,W - 1007.1\,W = 192.9\ W$

Consider the following system which has three basic components:



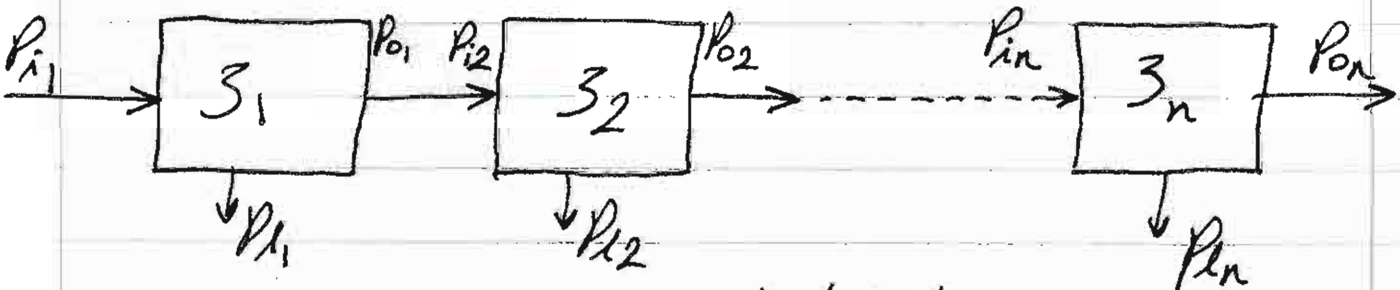
where $\eta_1 = \dfrac{P_{o1}}{P_{i1}}$, $\eta_2 = \dfrac{P_{o2}}{P_{i2}}$, $\eta_3 = \dfrac{P_{o3}}{P_{i3}}$

$$\eta_{total} = \eta_1 * \eta_2 * \eta_3 = \frac{\cancel{P_{o1}}}{P_{i1}} * \frac{\cancel{P_{o2}}}{\cancel{P_{i2}}} * \frac{P_{o3}}{\cancel{P_{i3}}}$$

But $P_{i2} = P_{o1}$ & $P_{i3} = P_{o2}$

$$\therefore \eta_{total} = \frac{P_{o3}}{P_{i1}}$$

In general, for the cascaded system (i.e. the output of one is fed as an input to the next) in this figure:



«cascaded system»

$$\eta_{total} = \eta_1 * \eta_2 * \text{------} * \eta_n$$

Notes :⇨

* The more components cascaded (in series), the lower the overall efficiency.
* If one of the system components has a considerably lower efficiency than the others, it reduces the overall system efficiency tremendously.

Example: For a nine cascaded transducers each of which has an efficiency of 0.7, calculate the overall system efficiency.

Sol:) $\eta_{total} = (0.7)^9 = 0.04 * \frac{100}{100} = 4\ \%$

Example: Now assume that one of the previous transducers has an efficiency of 0.25 and each of the others has an efficiency of 0.7. Find the overall system efficiency.

Sol:)

$$\eta_{total} = (0.7)^8 * 0.25 = 0.0144 * \frac{100}{100} = 1.44\ \%$$

Energy :-

In order for power to produce an energy conversion of any form, it must be used over a period of time. Therefore, the energy lost or gained by any system is determined by:

$$W = P * t \text{ ---- (wattseconds, Ws, or joules)}$$

The wattsecond, however, is too small a quantity for most purposes, so the watthour (Wh) and kilowatthour (kWh) were defined as:

$$Energy\,(Wh) = power\,(W) \times time\,(h)$$

$$\& \quad Energy\,(kWh) = \frac{power\,(W) \times time\,(h)}{1000}$$

Example: How much energy (in kilowatthours) is required to light a 60-W bulb continuously for 1 year (365 days)?

Sol:

$$\because W = P.t = 60\cancel{W}\left(\frac{kW}{1000\cancel{W}}\right) * 1\,\cancel{year}\left(\frac{365\,\cancel{day}}{1\,\cancel{year}}\right)\left(\frac{24\,h}{1\,\cancel{day}}\right)$$

$$\therefore W = 525.6\ kWh$$

Example: How long must a 3kW microwave oven remain on before consuming 5kJ of energy?

Sol:)

$$\because P = \frac{W}{t} \Rightarrow t = \frac{W}{P} = \frac{5kJ}{3kW(\text{or } J/s)} = 1.666 \text{ sec.}$$

Example: If the energy taken from a 220V supply by a transmitter in 15 min is 1.5 MJ, calculate the
a) current b) charge in coulombs transferred per minute
c) Energy consumed after 100 h of operation.

Sol.)

a) $\because P = \frac{W}{t} = \frac{1.5 * 10^6 J}{15 \cancel{min}\left(\frac{60 s}{1 \cancel{min}}\right)} = 1.667 kW$

$\because P = VI \Rightarrow I = \frac{P}{V} = \frac{1.667 kW}{220 V} = 7.577 A$

b) $\because I = \frac{Q}{t} \Rightarrow Q = I.t = 7.577 A * 1 min$
$= 7.577 A * 60 s = 454.62 C$

c) Energy (kWh) $= 1.667 kW * 100 h = 166.7 kWh$

Example: A resistor is rated $10k\Omega$, 0.5W. Calculate the
a) highest voltage that may be applied to it without exceeding its ~~ap~~ ability to dissipate heat safely.
b) highest current it can safely carry.

Sol.) a) $\because P = \frac{V^2}{R} \Rightarrow V^2 = PR \Rightarrow V = \sqrt{PR} = \sqrt{0.5 * 10k} = 70.71 V$

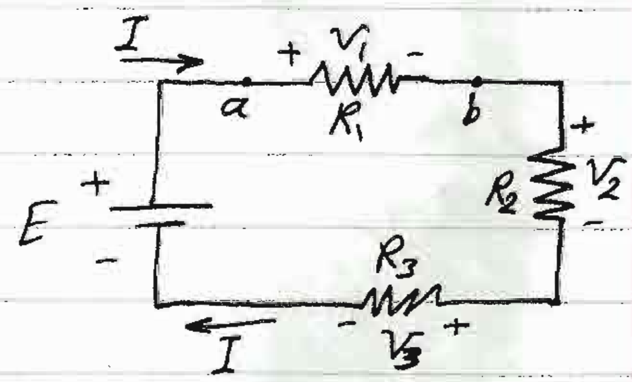
b) $\because P = VI \Rightarrow I = \frac{P}{V} = \frac{0.5}{70.71} = 7.071 mA$

D.C. Series Circuits:

There are two types of current. One is direct current (dc), in which ideally the flow of charge (current) does not charge in magnitude or direction. The other is sinusoidal alternating current (ac), in which the flow of charge is continually charging in magnitude and direction.

Components are connected in series when they carry the same current. The following fig. shows a voltage source connected to a series circuit which contains three resistors R_1, R_2 and R_3.

Note that:



* There is only one path for the current.
* The circuit elements are connected in cascade so that the output current of one element becomes the input current for the next.
* The same current **I** exists in each circuit element.

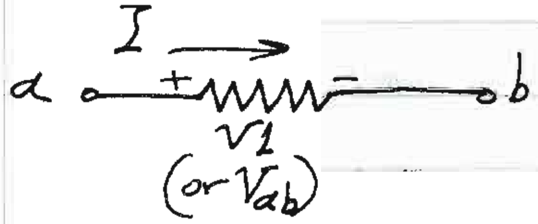
$$I \equiv I_1 \equiv I_2 \equiv I_3 \equiv --- \equiv I_n \quad \text{ampere (A)}$$

where I_1, I_2 and I_3 are the currents in resistors R_1, R_2 and R_3, respectively.

Notes:

① A rise of potential occurs in an active element (source) when going from (-) to (+) through the source, in the direction of current.

② A drop of potential occurs in a passive element (e.g. resistor) when going from (+) to (-) through the element in the direction of current.



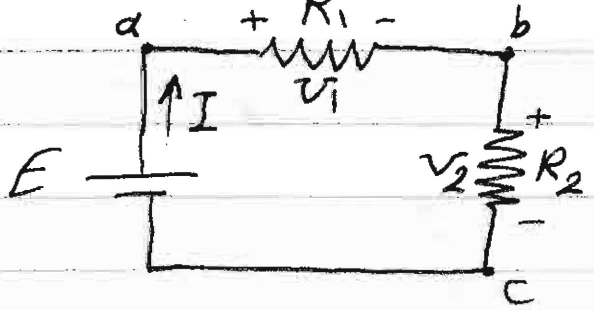
* The current is from point a to point b. This means that terminal a is more positive than terminal b (i.e. the potential at a is higher than that at b). If it were not, current would not flow from a to b but from b to a.

* V_1 is equal to V_{ab} (i.e. V_1 is the voltage measured from point a to point b). This voltage will be a positive voltage because it is the difference ~~between~~ in potential between the more positive point a and the less positive point b. But V_{ba} is recorded as a negative voltage since b is more negative than point a.

Kirchhoff's Voltage Law (KVL):

States that the algebric sum of all voltage rises and drops taken around any closed path (or mesh or loop) in any circuit is zero.

$$\sum_{n=1}^{N} V_n = 0$$



or $V_1 + V_2 + V_3 + \cdots + V_N = 0$

From the fig., with CW direction:

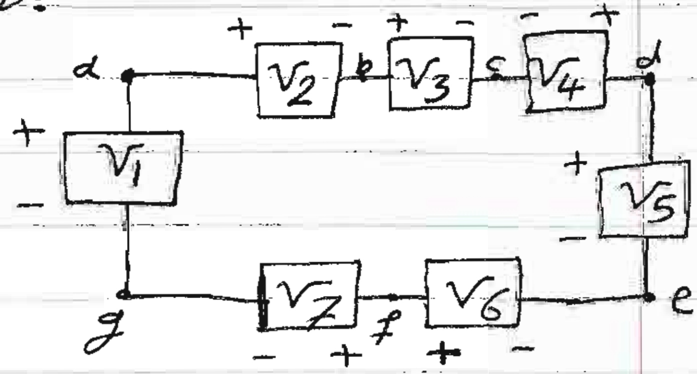
$+E - V_1 - V_2 = 0$

where E is the voltage rise, V_1 & V_2 are the voltage drops.

Notes:

* clockwise direction CW will be used for all applications of KVL.
* plus sign (+) is assigned to a potential rise (− to +).
* minus " (−) " " " " " drop (+ to −).

Example: For the following circuit:



a) Write the equation for all circuit elements, using KVL, starting at point a.

b) Find the elements that are passive for the given current.
c) " " " " " active " " " ".

Sol.:

a) $-V_2 - V_3 + V_4 - V_5 + V_6 - V_7 + V_1 = 0$

b) The passive elements are: V_2, V_3, V_5 and V_7.
c) " active " " : V_1, V_4 and V_6.

Series Circuit Relations:

Now, we are ready to summarize current, voltage, resistance and power relations for series circuits:
It is known that:

$I = I_1 = I_2 = I_3 = \cdots = I_N$ amperes (A)

From KVL:

$E = V_1 + V_2 + V_3 + \cdots + V_N$ volt (V)

By dividing:

$E/I = V_1/I_1 + V_2/I_2 + V_3/I_3 + \cdots + V_N/I_N$

Which from Ohm's law yields:

$R_T = R_1 + R_2 + R_3 + \cdots + R_N$ Ohms (Ω)

Finally,

$EI = V_1 I_1 + V_2 I_2 + V_3 I_3 + \cdots + V_N I_N$

OR $P_T = P_1 + P_2 + P_3 + \cdots + P_N$ watts (W)

Example: Three resistors (R_1, R_2 and R_3) having resistances of 20, 40 and 100 Ω, respectively, are connected in series to an ideal voltage source and draw 100 mA.
Calculate the:
a) Total resistance of the circuit.
b) Voltage drop across each resistor.
c) Total supply voltage E, by two methods.
d) Power dissipated by each resistor, by three methods.
e) Total power, by four methods.

Sol:)

a) $R_T = R_1 + R_2 + R_3 = 20 + 40 + 100 = 160\,\Omega$

b) $V_1 = I R_1 = 100 * 10^{-3} * 20 = 2\,V$

$V_2 = I R_2 = 100 * 10^{-3} * 40 = 4\,V$

$V_3 = I R_3 = 100 * 10^{-3} * 100 = 10\,V$

c) $E = V_1 + V_2 + V_3 = 10 + 4 + 2 = 16\,V$

Check $E = I R_T = 100 * 10^{-3} * 160 = 16\,V$

d) $P_1 = V_1 I_1 = 2 * 100 * 10^{-3} = 0.2\,W$

$P_1 = V_1^2 / R_1 = 4/20 = 0.2\,W$

$P_1 = I^2 R_1 = (100 * 10^{-3})^2 * 20 = 0.2\,W$

$P_2 = V_2 I_2 = V_2^2 / R_2 = I^2 R_2 = 0.4\,W$

$P_3 = V_3 I_3 = V_3^2 / R_3 = I^2 R_3 = 1\,W$

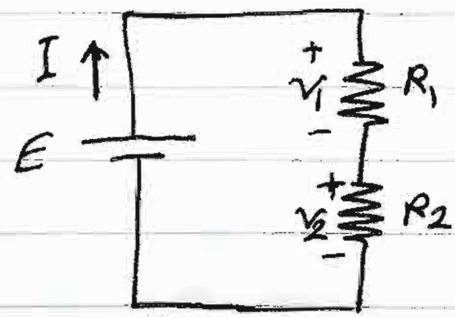
e) $P_T = P_1 + P_2 + P_3 = 0.2 + 0.4 + 1 = 1.6\,W$

$P_T = V_T I = 16 (100 * 10^{-3}) = 1.6\,W$

$P_T = V_T^2 / R_T = 16^2 / 160 = 1.6\,W$

$P_T = I^2 R_T = 1.6\,W$

Voltage Divider Rule :-



$\because R_T = R_1 + R_2$

& $I = \frac{E}{R_T}$

$$\left.\begin{aligned} \therefore V_1 &= I * R_1 = R_1 * \frac{E}{R_T} \\ \& \ V_2 &= I * R_2 = R_2 * \frac{E}{R_T} \end{aligned}\right\} \text{--- Voltage divider rule}$$

In general, the voltage across a single or combination of elements in series circuits, <u>without calculating the current</u> can be found by the voltage divider rule:

$$V_x = E * \frac{R_x}{R_T} \quad \text{---- General form}$$

where: V_x : voltage across R_x

E : supply voltage or the impressed voltage across the series elements.

R_T : total resistance.

Example: A $40 k\Omega$ and a $20 k\Omega$ are connected in series to $33V$-source. Without finding the current, calculate the :-

a) V_1 , V_2

b) P_1 , P_2

Sol:) a) $V_1 = E * \frac{R_1}{R_T} = 33 * \frac{40k}{60k} = 22V$

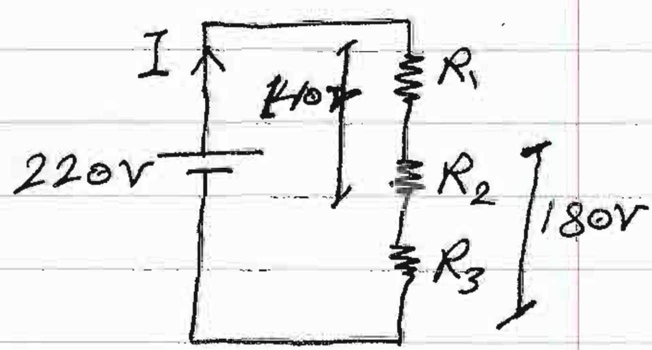
$$V_2 = E * \frac{R_2}{R_T} = 33 * \frac{20k}{60k} = 11\ V$$

b) $P_1 = V_1^2/R_1 = (22)^2/40k = 12.1\ mW$

$P_2 = V_2^2/R_2 = (11)^2/20k = 6.05\ mW$

Example:) Three resistors R_1, R_2 and R_3 are connected in series to 220V supply. The combined voltage drop across R_1 and R_2 is 140V and that across R_2 and R_3 is 180V. If the total resistance of the circuit is $11\ K\Omega$. calculate R_1, R_2 and R_3.

Sol:)



$$I = \frac{E}{R_T} = \frac{220V}{11K\Omega} = 20\ mA$$

$R_A = R_1 + R_2 \Rightarrow R_1 = R_A - R_2$

$R_A = \frac{140}{20mA} = 7K\Omega$

$R_B = R_2 + R_3 \Rightarrow R_3 = R_B - R_2$

$R_B = \frac{180}{20mA} = 9K\Omega$

$\because R_T = R_1 + R_2 + R_3$

$\therefore 11K\Omega = (7K - R_2) + R_2 + (9K - R_2)$

$\therefore 11K = 7K - R_2 + R_2 + 9K - R_2$

$\therefore R_2 = 16K\Omega - 11K\Omega = 5K\Omega$

$\therefore V_{R_2} = 140 * \frac{R_2}{R_A} = 140 * \frac{5k}{7k} = 100V$

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