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Fluid Mechanics

CHAPTER FOUR

DIMENSIONAL ANALYSIS AND DYNAMIC SIMILITUDE

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8. Establish the functional relation

$$f_1(\Pi_1,\Pi_2,\Pi_3, \ . \ . \ . \ ,\Pi_{n-m}) = 0$$

or solve for one of the Π's explicitly:

$$\Pi_2 = f(\Pi_1,\Pi_3, \ . \ . \ . \ ,\Pi_{n-m})$$

9. Recombine, if desired, to alter the forms of the Π-parameters, keeping the same number of independent parameters.

4.4. Discussion of Dimensionless Parameters. The five dimensionless parameters, pressure coefficient, Reynolds number, Froude number, Weber number, and Mach number, are of importance in correlating experimental data. They are discussed in this section, with particular emphasis placed on the relation of pressure coefficient to the other parameters.

Pressure Coefficient. The pressure coefficient $\Delta p/(\rho V^2/2)$ is the ratio of pressure to dynamic pressure. When multiplied by area it is the ratio of pressure force to inertial force, as $(\rho V^2/2)A$ would be the force needed to reduce the velocity to zero. It may also be written as $\Delta h/(V^2/2g)$ by division by γ. For pipe flow the Darcy-Weisbach equation relates losses h_l to length of pipe L, diameter D, and velocity V by a dimensionless friction factor[1] f

$$h_l = f\frac{L}{D}\frac{V^2}{2g}$$

or

$$\frac{fL}{D} = \frac{h_l}{V^2/2g} = f_2\left(\mathbf{R},\mathbf{F},\mathbf{W},\mathbf{M}, \frac{l}{l_1}, \frac{l}{l_2}\right)$$

as fL/D is shown to be equal to the pressure coefficient (see Example 4.4). In pipe flow, gravity has no influence on losses; therefore **F** may be dropped out. Similarly surface tension has no effect and **W** drops out. For steady liquid flow compressibility is not important and **M** is dropped. l may refer to D, l_1 to roughness height projection ϵ in the pipe wall, and l_2 to their spacing ϵ'; hence

$$\frac{FL}{D} = f_2\left(\mathbf{R}, \frac{\epsilon}{D}, \frac{\epsilon'}{D}\right) \qquad (4.4.1)$$

Pipe-flow problems are discussed in Chaps. 5, 6, and 10. If compressibility is important,

$$\frac{FL}{D} = f_2\left(\mathbf{R},\mathbf{M}, \frac{\epsilon}{D}, \frac{\epsilon'}{D}\right) \qquad (4.4.2)$$

With orifice flow, studied in Chap. 9, $V = C_v \sqrt{2gH}$,

$$\frac{H}{V^2/2g} = \frac{1}{C_v^2} = f_2\left(\mathbf{R},\mathbf{W},\mathbf{M}, \frac{l}{l_1}, \frac{l}{l_2}\right) \tag{4.4.3}$$

in which l may refer to orifice diameter and l_1 and l_2 to upstream dimensions. Viscosity and surface tension are unimportant for large orifices and low-viscosity fluids. Mach number effects may be very important for gas flow with large pressure drops, i.e., Mach numbers approaching unity.

In steady, uniform open-channel flow, discussed in Chap. 11, the Chézy formula relates average velocity V, slope of channel S, and hydraulic radius of cross section R (area of section divided by wetted perimeter) by

$$V = C\sqrt{RS} = C\sqrt{R\frac{\Delta h}{L}} \tag{4.4.4}$$

C is a coefficient depending upon size, shape, and roughness of channel. Then

$$\frac{\Delta h}{V^2/2g} = \frac{2gL}{R}\frac{1}{C^2} = f_2\left(\mathbf{F},\mathbf{R}, \frac{l}{l_1}, \frac{l}{l_2}\right) \tag{4.4.5}$$

since surface tension and compressible effects are usually unimportant.

The drag F on a body is expressed by $F = C_D A \rho V^2/2$, in which A is a typical area of the body, usually the projection of the body onto a plane normal to the flow. Then F/A is equivalent to Δp, and

$$\frac{F}{A\rho V^2/2} = C_D = f_2\left(\mathbf{R},\mathbf{F},\mathbf{M}, \frac{l}{l_1}, \frac{l}{l_2}\right) \tag{4.4.6}$$

The term $\mathbf{R}$ is related to *skin friction* drag due to viscous shear as well as to *form*, or *profile*, drag resulting from *separation* of the flow streamlines from the body; $\mathbf{F}$ is related to wave drag if there is a free surface; for large Mach numbers C_D may vary more markedly with $\mathbf{M}$ than with the other parameters; the length ratios may refer to shape or roughness of the surface.

Reynolds Number. Reynolds number $VD\rho/\mu$ is the ratio of inertial forces to viscous forces. It may also be viewed as a ratio of turbulent shear forces to viscous shear forces (Sec. 5.3). A "critical" Reynolds number distinguishes among flow regimes, such as laminar or turbulent flow in pipes, in the boundary layer, or around immersed objects. The particular value depends upon the situation. In compressible flow, the Mach number is generally more significant than the Reynolds number.

Froude Number. The Froude number V^2/gl, when multiplied and divided by ρA, is a ratio of dynamic (or inertial force) to weight. With

free liquid surface flow the nature of the flow (rapid[1] or tranquil) depends upon whether the Froude number is greater or less than unity. It is useful in calculations of hydraulic jump, in design of hydraulic structures, and in ship design.

Weber Number. The Weber number $V^2 l\rho/\sigma$ is the ratio of inertial forces to surface-tension forces (evident when numerator and denominator are multiplied by l). It is important at gas-liquid or liquid-liquid interfaces and also where these interfaces are in contact with a boundary. Surface tension causes small (capillary) waves and droplet formation and has an effect on discharge of orifices and weirs at very small heads.

Mach Number. The speed of sound in a liquid is written $\sqrt{K/\rho}$, if K is the bulk modulus of elasticity (Secs. 1.7 and 6.2) or $c = \sqrt{kRT}$ (k is the specific heat ratio and T the absolute temperature, for a perfect gas). V/c or $V/\sqrt{K/\rho}$ is the Mach number. It is a measure of the ratio of inertial forces to elastic forces. By squaring V/c and multiplying by $\rho A/2$ in numerator and denominator, the numerator is the dynamic force and the denominator is the dynamic force at sonic flow. It may also be shown to be a measure of the ratio of kinetic energy of the flow to internal energy of the fluid. It is the most important correlating parameter when velocities are near or above local sonic velocities.

4.5. Similitude—Model Studies. Model studies of proposed hydraulic structures and machines are frequently undertaken as an aid to the designer. They permit visual observation of the flow and make possible the obtaining of certain numerical data, e.g., calibrations of weirs and gates, depths of flow, velocity distributions, forces on gates, efficiencies and capacities of pumps and turbines, pressure distributions, and losses.

If accurate quantitative data are to be obtained from a model study there must be dynamic similitude between model and prototype. This similitude requires (*a*) that there be exact geometric similitude, and (*b*) that the ratio of dynamic pressures at corresponding points be a constant. Part *b* may also be expressed as a kinematic similitude; i.e., the streamlines must be geometrically similar.

Geometric similitude extends to the actual surface roughness of model and prototype. If the model is one-tenth the size of the prototype in every linear dimension, then the height of roughness projections must be in the same ratio. For dynamic pressures to be in the same ratio at corresponding points in model and prototype, the ratios of the various types of forces must be the same at corresponding points. Hence, for strict dynamic similitude, the Mach, Reynolds, Froude, and Weber numbers must be the same in both model and prototype.