EXAMPLE 11–5 A ball bearing is run at four piecewise continuous steady loads as shown in the following table. Columns (1), (2), and (5) to (8) are given.

(1) Time Fraction	(2) Speed, rev/min	(3) Product, Column (1) × (2)	(4) Turns Fraction, (3)/∑(3)	(5) F_{ri}, lbf	(6) F_{ai}, lbf	(7) F_{ei}, lbf	(8) a_{fi}	(9) $a_{fi} F_{ei}$, lbf
0.1	2000	200	0.077	600	300	794	1.10	873
0.1	3000	300	0.115	300	300	626	1.25	795
0.3	3000	900	0.346	750	300	878	1.10	966
0.5	2400	1200	0.462	375	300	668	1.25	835
		2600	1.000					

Columns 1 and 2 are multiplied to obtain column 3. The column 3 entry is divided by the sum of column 3, 2600, to give column 4. Columns 5, 6, and 7 are the radial, axial, and equivalent loads respectively. Column 8 is the appropriate application factor. Column 9 is the product of columns 7 and 8.

Solution From Eq. (11–10), with $a = 3$, the equivalent radial load F_e is

Answer

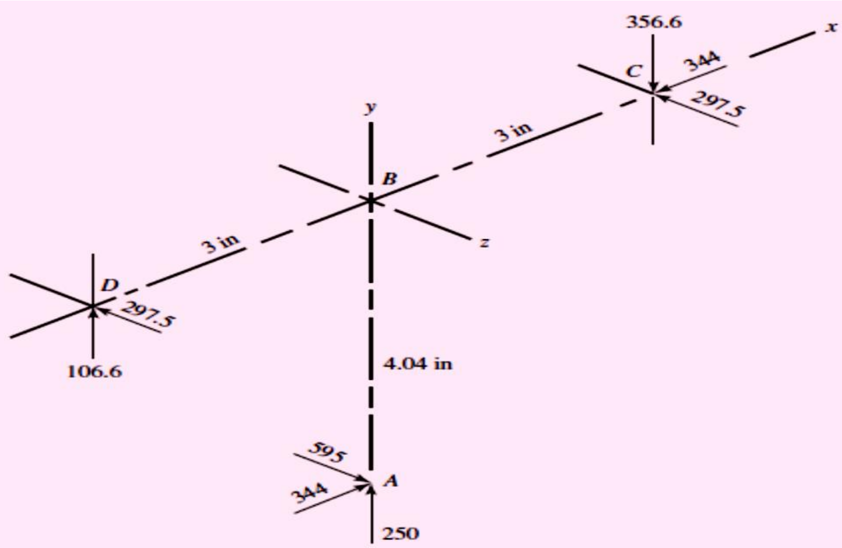
$$F_e = \left[0.077(873)^3 + 0.115(795)^3 + 0.346(966)^3 + 0.462(835)^3\right]^{1/3} = 884 \text{ lbf}$$

Selection of Ball and Straight Roller Bearings

EXAMPLE 11–7 The second shaft on a parallel-shaft 25-hp foundry crane speed reducer contains a helical gear with a pitch diameter of 8.08 in. Helical gears transmit components of force in the tangential, radial, and axial directions (see Chap. 13). The components of the gear force transmitted to the second shaft are shown in Fig. 11–12, at point *A*. The bearing reactions at *C* and *D*, assuming simple-supports, are also shown. A ball bearing is to be selected for location *C* to accept the thrust, and a cylindrical roller

Figure 11–12

Forces in pounds applied to the second shaft of the helical gear speed reducer of Ex. 11–7.



bearing is to be utilized at location D. The life goal of the speed reducer is 10 kh, with a reliability factor for the ensemble of all four bearings (both shafts) to equal or exceed 0.96 for the Weibull parameters of Ex. 11–3. The application factor is to be 1.2.

(*a*) Select the roller bearing for location D.

(*b*) Select the ball bearing (angular contact) for location C, assuming the inner ring rotates.

Solution The torque transmitted is $T = 595(4.04) = 2404$ lbf · in. The speed at the rated horsepower, given by Eq. (3–40), p. 138, is

$$n_D = \frac{63\,025H}{T} = \frac{63\,025(25)}{2404} = 655.4 \text{ rev/min}$$

The radial load at D is $\sqrt{106.6^2 + 297.5^2} = 316.0$ lbf, and the radial load at C is $\sqrt{356.6^2 + 297.5^2} = 464.4$ lbf. The individual bearing reliabilities, if equal, must be at least $\sqrt[4]{0.96} = 0.98985 \doteq 0.99$. The dimensionless design life for both bearings is

$$x_D = \frac{L}{L_{10}} = \frac{60L_D n_D}{60L_R n_R} = \frac{60(10\,000)655.4}{10^6} = 393.2$$

(*a*) From Eq. (11–7), the Weibull parameters of Ex. 11–3, an application factor of 1.2, and $a = 10/3$ for the roller bearing at D, the catalog rating should be equal to or greater than

$$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}}\right]^{1/a}$$

$$= 1.2(316.0)\left[\frac{393.2}{0.02 + 4.439(1 - 0.99)^{1/1.483}}\right]^{3/10} = 3591 \text{ lbf} = 16.0 \text{ kN}$$

1 Choose Y_2 from Table 11–1.
2 Find C_{10}.
3 Tentatively identify a suitable bearing from Table 11–2, note C_0.
4 Using F_a/C_0 enter Table 11–1 to obtain a new value of Y_2.
5 Find C_{10}.
6 If the same bearing is obtained, stop.
7 If not, take next bearing and go to step 4.

As a first approximation, take the middle entry from Table 11–1:

$$X_2 = 0.56 \qquad Y_2 = 1.63.$$

From Eq. (11–8*b*), with $V = 1$,

$$\frac{F_e}{VF_r} = X + \frac{Y}{V}\frac{F_a}{F_r} = 0.56 + 1.63\frac{344}{(1)464.4} = 1.77$$

$$F_e = 1.77VF_r = 1.77(1)464.4 = 822 \text{ lbf} \qquad \text{or} \qquad 3.66 \text{ kN}$$

From Eq. (11–7), with $a = 3$,

$$C_{10} = 1.2(3.66)\left[\frac{393.2}{0.02 + 4.439(1 - 0.99)^{1/1.483}}\right]^{1/3} = 53.4 \text{ kN}$$

From Table 11–2, angular-contact bearing 02-60 mm has $C_{10} = 55.9$ kN. C_0 is 35.5 kN. Step 4 becomes, with F_a in kN,

$$\frac{F_a}{C_0} = \frac{344(4.45)10^{-3}}{35.5} = 0.0431$$

which makes e from Table 11–1 approximately 0.24. Now $F_a/[VF_r] = 344/[(1)\,464.4] = 0.74$, which is greater than 0.24, so we find Y_2 by interpolation:

F_a/C_0	Y_2	
0.042	1.85	
0.043	Y_2	from which $Y_2 = 1.84$
0.056	1.71	

From Eq. (11–8b),

$$\frac{F_e}{VF_r} = 0.56 + 1.84\frac{344}{464.4} = 1.92$$

$$F_e = 1.92VF_r = 1.92(1)464.4 = 892 \text{ lbf} \qquad \text{or} \qquad 3.97 \text{ kN}$$

The prior calculation for C_{10} changes only in F_e, so

$$C_{10} = \frac{3.97}{3.66}53.4 = 57.9 \text{ kN}$$

From Table 11–2 an angular contact bearing 02-65 mm has $C_{10} = 63.7$ kN and C_0 of 41.5 kN. Again,

$$\frac{F_a}{C_0} = \frac{344(4.45)10^{-3}}{41.5} = 0.0369$$

making e approximately 0.23. Now from before, $F_a/VF_r = 0.74$, which is greater than 0.23. We find Y_2 again by interpolation:

F_a/C_0	Y_2	
0.028	1.99	
0.0369	Y_2	from which $Y_2 = 1.90$
0.042	1.85	

From Eq. (11–8*b*),

$$\frac{F_e}{VF_r} = 0.56 + 1.90\frac{344}{464.4} = 1.967$$

$$F_e = 1.967VF_r = 1.967(1)464.4 = 913.5 \text{ lbf} \quad \text{or} \quad 4.065 \text{ kN}$$

The prior calculation for C_{10} changes only in F_e, so

$$C_{10} = \frac{4.07}{3.66}53.4 = 59.4 \text{ kN}$$

Answer From Table 11–2 an angular-contact 02-65 mm is still selected, so the iteration is complete.

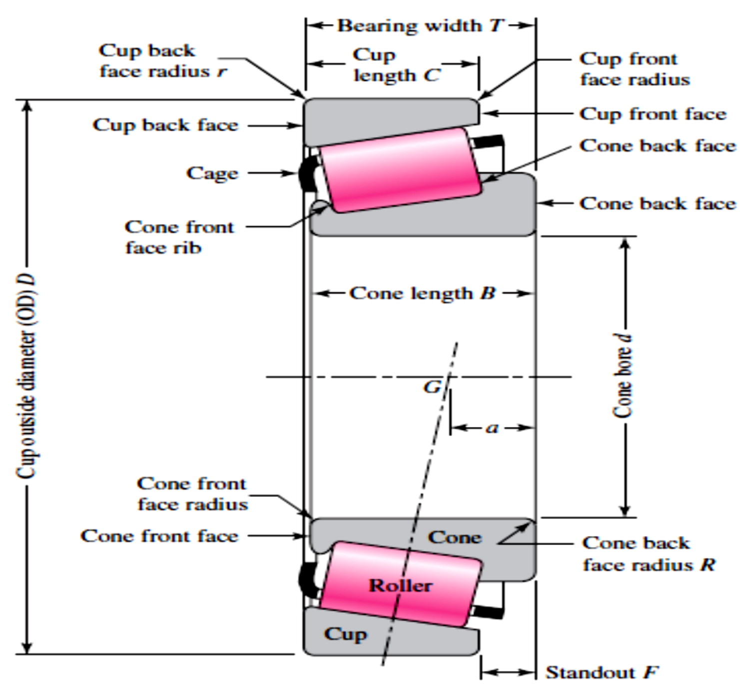
Selection of Tapered Roller Bearings

Tapered roller bearings are more complicated than ball and straight roller bearings.

- The four components of a tapered roller bearing are: cone (*inner ring*), cup (*outer ring*), tapered rollers and cage (*retainer*) see *fig. 11-13*.

Figure 11–13

Nomenclature of a tapered roller bearing. Point *G* is the location of the effective load center; use this point to estimate the radial bearing load. *(Courtesy of The Timken Company.)*



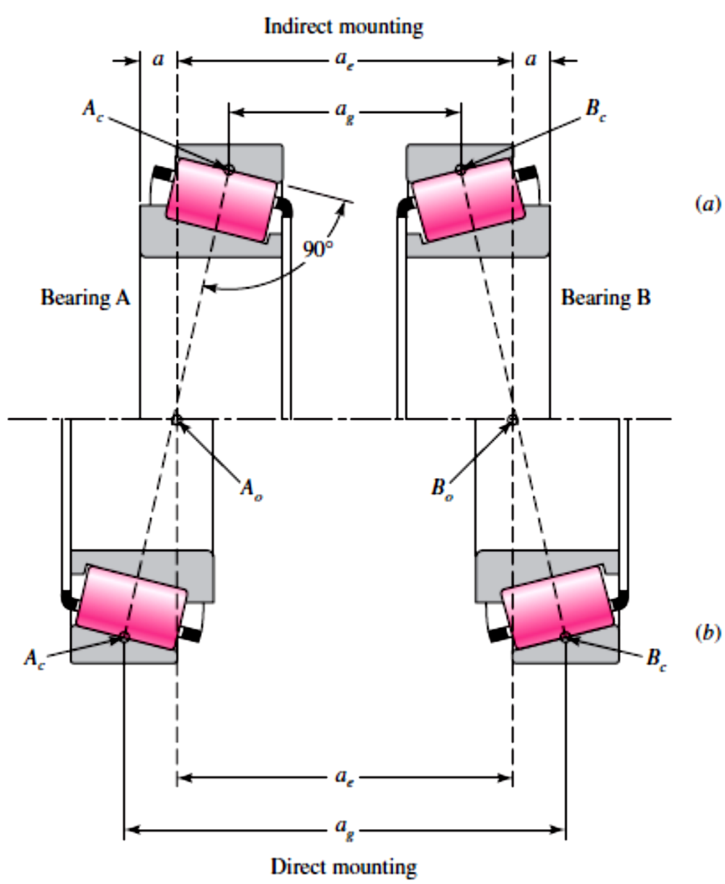
- The assembled bearing consists of two separate parts:

1. The cone assembly (*cone, rollers and cage*).
2. The cup.

- Tapered roller bearing can carry radial or thrust loads or any combination of the two.
- Even if the bearing is under radial load only, because of the taper, a thrust reaction will be induced which will separate the cone and cup assemblies.
- One way to overcome this problem is to use two tapered roller bearings in opposite orientation "*direct or indirect mounting*" see *fig. 11-14.*

Figure 11–14

Comparison of mounting stability between indirect and direct mountings. *(Courtesy of The Timken Company.)*



- The induced axial component can be found as:

$$F_i = \frac{0.47 F_r}{K} \qquad \text{(11–15)}$$

where, $K = 0.389 \cot \alpha$ and α is half the cup angle.

- Before a particular bearing is selected, an estimated value of ***K*=1.5** is used.
- *Fig. 11-15* shows a catalog page for tapered roller bearing from *Timken Company* [*90×10^6= rev. life*].

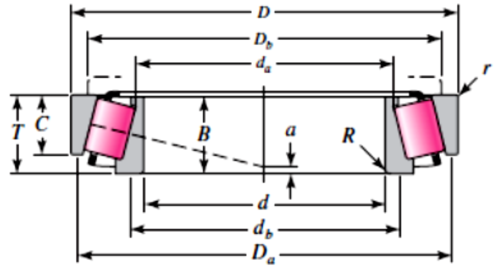
SINGLE-ROW STRAIGHT BORE

bore d	outside diameter D	width T	rating at 500 rpm for 3000 hours L_{10}: one-row radial N / lbf	rating: thrust N / lbf	factor K	eff. load center a②	part numbers: cone	part numbers: cup	cone: max shaft fillet radius R①	cone: width B	cone: backing shoulder diameters d_b	cone: backing shoulder diameters d_a	cup: max housing fillet radius r①	cup: width C	cup: backing shoulder diameters D_b	cup: backing shoulder diameters D_a
25.000 0.9843	52.000 2.0472	16.250 0.6398	8190 1840	5260 1180	1.56	−3.6 −0.14	◆30205	◆30205	1.0 0.04	15.000 0.5906	30.5 1.20	29.0 1.14	1.0 0.04	13.000 0.5118	46.0 1.81	48.5 1.91
25.000 0.9843	52.000 2.0472	19.250 0.7579	9520 2140	9510 2140	1.00	−3.0 −0.12	◆32205-B	◆32205-B	1.0 0.04	18.000 0.7087	34.0 1.34	31.0 1.22	1.0 0.04	15.000 0.5906	43.5 1.71	49.5 1.95
25.000 0.9843	52.000 2.0472	22.000 0.8661	13200 2980	7960 1790	1.66	−7.6 −0.30	◆33205	◆33205	1.0 0.04	22.000 0.8661	34.0 1.34	30.5 1.20	1.0 0.04	18.000 0.7087	44.5 1.75	49.0 1.93
25.000 0.9843	62.000 2.4409	18.250 0.7185	13000 2930	6680 1500	1.95	−5.1 −0.20	◆30305	◆30305	1.5 0.06	17.000 0.6693	32.5 1.28	30.0 1.18	1.5 0.06	15.000 0.5906	55.0 2.17	57.0 2.24
25.000 0.9843	62.000 2.4409	25.250 0.9941	17400 3910	8930 2010	1.95	−9.7 −0.38	◆32305	◆32305	1.5 0.06	24.000 0.9449	35.0 1.38	31.5 1.24	1.5 0.06	20.000 0.7874	54.0 2.13	57.0 2.24
25.159 0.9905	50.005 1.9687	13.495 0.5313	6990 1570	4810 1080	1.45	−2.8 −0.11	07096	07196	1.5 0.06	14.260 0.5614	31.5 1.24	29.5 1.16	1.0 0.04	9.525 0.3750	44.5 1.75	47.0 1.85
25.400 1.0000	50.005 1.9687	13.495 0.5313	6990 1570	4810 1080	1.45	−2.8 −0.11	07100	07196	1.0 0.04	14.260 0.5614	30.5 1.20	29.5 1.16	1.0 0.04	9.525 0.3750	44.5 1.75	47.0 1.85
25.400 1.0000	50.005 1.9687	13.495 0.5313	6990 1570	4810 1080	1.45	−2.8 −0.11	07100-S	07196	1.5 0.06	14.260 0.5614	31.5 1.24	29.5 1.16	1.0 0.04	9.525 0.3750	44.5 1.75	47.0 1.85
25.400 1.0000	50.292 1.9800	14.224 0.5600	7210 1620	4620 1040	1.56	−3.3 −0.13	L44642	L44610	3.5 0.14	14.732 0.5800	36.0 1.42	29.5 1.16	1.3 0.05	10.668 0.4200	44.5 1.75	47.0 1.85
25.400 1.0000	50.292 1.9800	14.224 0.5600	7210 1620	4620 1040	1.56	−3.3 −0.13	L44643	L44610	1.3 0.05	14.732 0.5800	31.5 1.24	29.5 1.16	1.3 0.05	10.668 0.4200	44.5 1.75	47.0 1.85
25.400 1.0000	51.994 2.0470	15.011 0.5910	6990 1570	4810 1080	1.45	−2.8 −0.11	07100	07204	1.0 0.04	14.260 0.5614	30.5 1.20	29.5 1.16	1.3 0.05	12.700 0.5000	45.0 1.77	48.0 1.89
25.400 1.0000	56.896 2.2400	19.368 0.7625	10900 2450	5740 1290	1.90	−6.9 −0.27	1780	1729	0.8 0.03	19.837 0.7810	30.5 1.20	30.0 1.18	1.3 0.05	15.875 0.6250	49.0 1.93	51.0 2.01
25.400 1.0000	57.150 2.2500	19.431 0.7650	11700 2620	10900 2450	1.07	−3.0 −0.12	M84548	M84510	1.5 0.06	19.431 0.7650	36.0 1.42	33.0 1.30	1.5 0.06	14.732 0.5800	48.5 1.91	54.0 2.13
25.400 1.0000	58.738 2.3125	19.050 0.7500	11600 2610	6560 1470	1.77	−5.8 −0.23	1986	1932	1.3 0.05	19.355 0.7620	32.5 1.28	30.5 1.20	1.3 0.05	15.080 0.5937	52.0 2.05	54.0 2.13
25.400 1.0000	59.530 2.3437	23.368 0.9200	13900 3140	13000 2930	1.07	−5.1 −0.20	M84249	M84210	0.8 0.03	23.114 0.9100	36.0 1.42	32.5 1.27	1.5 0.06	18.288 0.7200	49.5 1.95	56.0 2.20
25.400 1.0000	60.325 2.3750	19.842 0.7812	11000 2480	6550 1470	1.69	−5.1 −0.20	15578	15523	1.3 0.05	17.462 0.6875	32.5 1.28	30.5 1.20	1.5 0.06	15.875 0.6250	51.0 2.01	54.0 2.13
25.400 1.0000	61.912 2.4375	19.050 0.7500	12100 2730	7280 1640	1.67	−5.8 −0.23	15101	15243	0.8 0.03	20.638 0.8125	32.5 1.28	31.5 1.24	2.0 0.08	14.288 0.5625	54.0 2.13	58.0 2.28
25.400 1.0000	62.000 2.4409	19.050 0.7500	12100 2730	7280 1640	1.67	−5.8 −0.23	15100	15245	3.5 0.14	20.638 0.8125	38.0 1.50	31.5 1.24	1.3 0.05	14.288 0.5625	55.0 2.17	58.0 2.28
25.400 1.0000	62.000 2.4409	19.050 0.7500	12100 2730	7280 1640	1.67	−5.8 −0.23	15101	15245	0.8 0.03	20.638 0.8125	32.5 1.28	31.5 1.24	1.3 0.05	14.288 0.5625	55.0 2.17	58.0 2.28
25.400 1.0000	62.000 2.4409	19.050 0.7500	12100 2730	7280 1640	1.67	−5.8 −0.23	15102	15245	1.5 0.06	20.638 0.8125	34.0 1.34	31.5 1.24	1.3 0.05	14.288 0.5625	55.0 2.17	58.0 2.28
25.400 1.0000	62.000 2.4409	20.638 0.8125	12100 2730	7280 1640	1.67	−5.8 −0.23	15101	15244	0.8 0.03	20.638 0.8125	32.5 1.28	31.5 1.24	1.3 0.05	15.875 0.6250	55.0 2.17	58.0 2.28
25.400 1.0000	63.500 2.5000	20.638 0.8125	12100 2730	7280 1640	1.67	−5.8 −0.23	15101	15250	0.8 0.03	20.638 0.8125	32.5 1.28	31.5 1.24	1.3 0.05	15.875 0.6250	56.0 2.20	59.0 2.32
25.400 1.0000	63.500 2.5000	20.638 0.8125	12100 2730	7280 1640	1.67	−5.8 −0.23	15101	15250X	0.8 0.03	20.638 0.8125	32.5 1.28	31.5 1.24	1.5 0.06	15.875 0.6250	55.0 2.17	59.0 2.32
25.400 1.0000	64.292 2.5312	21.433 0.8438	14500 3250	13500 3040	1.07	−3.3 −0.13	M86643	M86610	1.5 0.06	21.433 0.8438	38.0 1.50	36.5 1.44	1.5 0.06	16.670 0.6563	54.0 2.13	61.0 2.40

Figure 11–15

Catalog entry of single-row straight-bore Timken roller bearings, in part. *(Courtesy of The Timken Company.)*

SINGLE-ROW STRAIGHT BORE



bore d	outside diameter D	width T	rating at 500 rpm for 3000 hours L_{10} one-row radial N / lbf	rating at 500 rpm for 3000 hours L_{10} thrust N / lbf	factor K	eff. load center a②	part numbers: cone	part numbers: cup	cone: max shaft fillet radius R①	cone: width B	cone: backing shoulder diameters d_b	cone: backing shoulder diameters d_a	cup: max housing fillet radius r①	cup: width C	cup: backing shoulder diameters D_b	cup: backing shoulder diameters D_a
25.400 1.0000	65.088 2.5625	22.225 0.8750	13100 2950	16400 3690	0.80	−2.3 −0.09	23100	23256	1.5 0.06	21.463 0.8450	39.0 1.54	34.5 1.36	1.5 0.06	15.875 0.6250	53.0 2.09	63.0 2.48
25.400 1.0000	66.421 2.6150	23.812 0.9375	18400 4140	8000 1800	2.30	−9.4 −0.37	2687	2631	1.3 0.05	25.433 1.0013	33.5 1.32	31.5 1.24	1.3 0.05	19.050 0.7500	58.0 2.28	60.0 2.36
25.400 1.0000	68.262 2.6875	22.225 0.8750	15300 3440	10900 2450	1.40	−5.1 −0.20	02473	02420	0.8 0.03	22.225 0.8750	34.5 1.36	33.5 1.32	1.5 0.06	17.462 0.6875	59.0 2.32	63.0 2.48
25.400 1.0000	72.233 2.8438	25.400 1.0000	18400 4140	17200 3870	1.07	−4.6 −0.18	HM88630	HM88610	0.8 0.03	25.400 1.0000	39.5 1.56	39.5 1.56	2.3 0.09	19.842 0.7812	60.0 2.36	69.0 2.72
25.400 1.0000	72.626 2.8593	30.162 1.1875	22700 5110	13000 2910	1.76	−10.2 −0.40	3189	3120	0.8 0.03	29.997 1.1810	35.5 1.40	35.0 1.38	3.3 0.13	23.812 0.9375	61.0 2.40	67.0 2.64
26.157 1.0298	62.000 2.4409	19.050 0.7500	12100 2730	7280 1640	1.67	−5.8 −0.23	15103	15245	0.8 0.03	20.638 0.8125	33.0 1.30	32.5 1.28	1.3 0.05	14.288 0.5625	55.0 2.17	58.0 2.28
26.162 1.0300	63.100 2.4843	23.812 0.9375	18400 4140	8000 1800	2.30	−9.4 −0.37	2682	2630	1.5 0.06	25.433 1.0013	34.5 1.36	32.0 1.26	0.8 0.03	19.050 0.7500	57.0 2.24	59.0 2.32
26.162 1.0300	66.421 2.6150	23.812 0.9375	18400 4140	8000 1800	2.30	−9.4 −0.37	2682	2631	1.5 0.06	25.433 1.0013	34.5 1.36	32.0 1.26	1.3 0.05	19.050 0.7500	58.0 2.28	60.0 2.36
26.975 1.0620	58.738 2.3125	19.050 0.7500	11600 2610	6560 1470	1.77	−5.8 −0.23	1987	1932	0.8 0.03	19.355 0.7620	32.5 1.28	31.5 1.24	1.3 0.05	15.080 0.5937	52.0 2.05	54.0 2.13
† 26.988 † 1.0625	50.292 1.9800	14.224 0.5600	7210 1620	4620 1040	1.56	−3.3 −0.13	L44649	L44610	3.5 0.14	14.732 0.5800	37.5 1.48	31.0 1.22	1.3 0.05	10.668 0.4200	44.5 1.75	47.0 1.85
† 26.988 † 1.0625	60.325 2.3750	19.842 0.7812	11000 2480	6550 1470	1.69	−5.1 −0.20	15580	15523	3.5 0.14	17.462 0.6875	38.5 1.52	32.0 1.26	1.5 0.06	15.875 0.6250	51.0 2.01	54.0 2.13
† 26.988 † 1.0625	62.000 2.4409	19.050 0.7500	12100 2730	7280 1640	1.67	−5.8 −0.23	15106	15245	0.8 0.03	20.638 0.8125	33.5 1.32	33.0 1.30	1.3 0.05	14.288 0.5625	55.0 2.17	58.0 2.28
† 26.988 † 1.0625	66.421 2.6150	23.812 0.9375	18400 4140	8000 1800	2.30	−9.4 −0.37	2688	2631	1.5 0.06	25.433 1.0013	35.0 1.38	33.0 1.30	1.3 0.05	19.050 0.7500	58.0 2.28	60.0 2.36
28.575 1.1250	56.896 2.2400	19.845 0.7813	11600 2610	6560 1470	1.77	−5.8 −0.23	1985	1930	0.8 0.03	19.355 0.7620	34.0 1.34	33.5 1.32	0.8 0.03	15.875 0.6250	51.0 2.01	54.0 2.11
28.575 1.1250	57.150 2.2500	17.462 0.6875	11000 2480	6550 1470	1.69	−5.1 −0.20	15590	15520	3.5 0.14	17.462 0.6875	39.5 1.56	33.5 1.32	1.5 0.06	13.495 0.5313	51.0 2.01	53.0 2.09
28.575 1.1250	58.738 2.3125	19.050 0.7500	11600 2610	6560 1470	1.77	−5.8 −0.23	1985	1932	0.8 0.03	19.355 0.7620	34.0 1.34	33.5 1.32	1.3 0.05	15.080 0.5937	52.0 2.05	54.0 2.13
28.575 1.1250	58.738 2.3125	19.050 0.7500	11600 2610	6560 1470	1.77	−5.8 −0.23	1988	1932	3.5 0.14	19.355 0.7620	39.5 1.56	33.5 1.32	1.3 0.05	15.080 0.5937	52.0 2.05	54.0 2.13
28.575 1.1250	60.325 2.3750	19.842 0.7812	11000 2480	6550 1470	1.69	−5.1 −0.20	15590	15523	3.5 0.14	17.462 0.6875	39.5 1.56	33.5 1.32	1.5 0.06	15.875 0.6250	51.0 2.01	54.0 2.13
28.575 1.1250	60.325 2.3750	19.845 0.7813	11600 2610	6560 1470	1.77	−5.8 −0.23	1985	1931	0.5 0.03	19.355 0.7620	34.0 1.34	33.5 1.32	1.3 0.05	15.875 0.6250	52.0 2.05	55.0 2.17

① These maximum fillet radii will be cleared by the bearing corners.
② Minus value indicates center is inside cone backface.
† For standard class **ONLY**, the maximum metric size is a whole millimetre value.
* For "J" part tolerances—see metric tolerances, page 73, and fitting practice, page 65.
◆ ISO cone and cup combinations are designated with a common part number and should be purchased as an assembly.
For ISO bearing tolerances—see metric tolerances, page 73, and fitting practice, page 65.

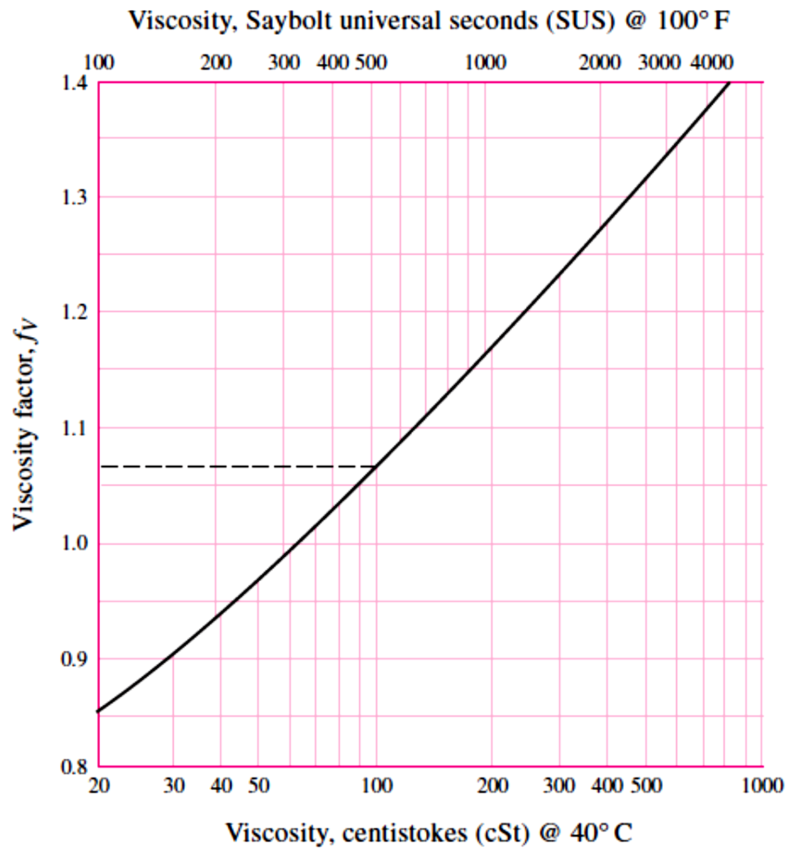
Figure 11–15

(Continued)

• To determine the equivalent design load for each bearing, first we need to identify the bearing that carries the external thrust load (*if any is present*) and label that bearing as *Bearing A* and the other one will be named *Bearing B* (see *fig. 11-17*).

Figure 11–17

Viscosity factor f_V as a function of oil viscosity. (This graph applies to petroleum oil with a viscosity index of approximately 90.) *(Courtesy of The Timken Company.)*



- Then, the equivalent design loads for each of the two bearings can be calculated as:

> ➢ **If** $F_{iA} \leq (F_{iB} + F_{ae})$ ➔ $F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae})$
> $F_{eB} = F_{rB}$
>
> ➢ **If** $F_{iA} > (F_{iB} + F_{ae})$ ➔ $F_{eB} = 0.4F_{rB} + K_B(F_{iA} - F_{ae})$
> $F_{eA} = F_{rA}$

where,

F_{eA} & F_{eB}: are the equivalent radial loads for bearings A & B.
F_{rA} & F_{rB}: are the direct radial loads acting on bearings A & B.
F_{iA} & F_{iB}: are the induced axial loads on bearings A & B.
F_{ae}: is the external axial load.

EXAMPLE 11–8 The shaft depicted in Fig. 11–18*a* carries a helical gear with a tangential force of 3980 N, a separating force of 1770 N, and a thrust force of 1690 N at the pitch cylinder with directions shown. The pitch diameter of the gear is 200 mm. The shaft runs at a speed of 1050 rev/min, and the span (effective spread) between the direct-mount bearings is 150 mm. The design life is to be 5000 h and an application factor of 1 is appropriate. The lubricant will be ISO VG 68 (68 cSt at 40°C) oil with an estimated operating temperature of 55°C. If the reliability of the bearing set is to be 0.99, select suitable single-row tapered-roller Timken bearings.

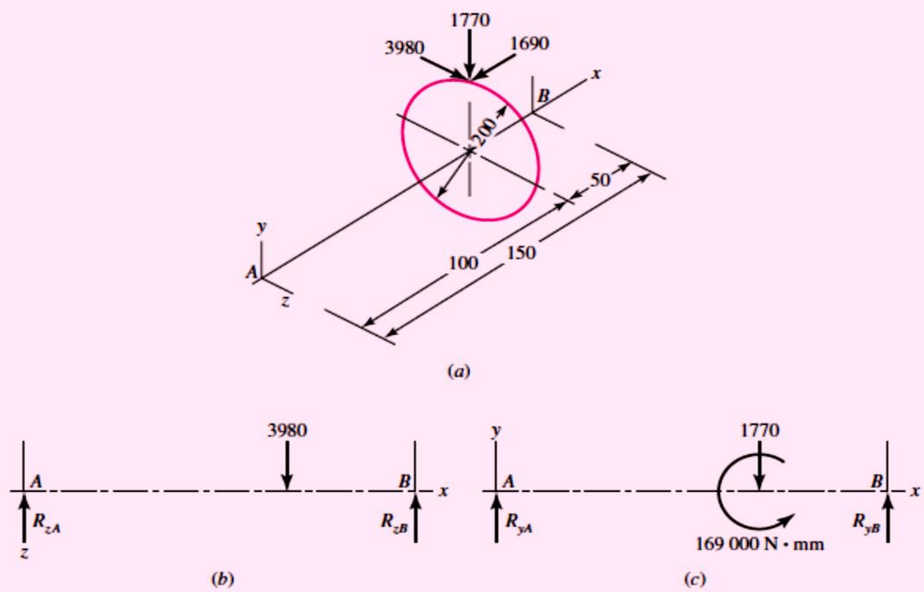
Solution The reactions in the xz plane from Fig. 11–18*b* are

$$R_{zA} = \frac{3980(50)}{150} = 1327\,\text{N}$$

$$R_{zB} = \frac{3980(100)}{150} = 2653\,\text{N}$$

Figure 11–18

Essential geometry of helical gear and shaft. Length dimensions in mm, loads in N, couple in N · mm. (*a*) Sketch (not to scale) showing thrust, radial, and tangential forces. (*b*) Forces in *xz* plane. (*c*) Forces in *xy* plane.



The reactions in the xy plane from Fig. 11–18*c* are

$$R_{yA} = \frac{1770(50)}{150} + \frac{169\,000}{150} = 1716.7 = 1717\,\text{N}$$

$$R_{yB} = \frac{1770(100)}{150} - \frac{169\,000}{150} = 53.3\,\text{N}$$

The radial loads F_{rA} and F_{rB} are the vector additions of R_{yA} and R_{zA}, and R_{yB} and R_{zB}, respectively:

$$F_{rA} = \left(R_{zA}^2 + R_{yA}^2\right)^{1/2} = (1327^2 + 1717^2)^{1/2} = 2170\,\text{N}$$

$$F_{rB} = \left(R_{zB}^2 + R_{yB}^2\right)^{1/2} = (2653^2 + 53.3^2)^{1/2} = 2654\,\text{N}$$

Trial 1: We will use $K_A = K_B = 1.5$ to start. From Table 11–6, noting that $m = +1$ for direct mounting and F_{ae} to the right is positive, we write

$$\frac{0.47F_{rA}}{K_A} <?> \frac{0.47F_{rB}}{K_B} - mF_{ae}$$

$$\frac{0.47(2170)}{1.5} <?> \left[\frac{0.47(2654)}{1.5} - (+1)(-1690)\right]$$

$$680 < 2522$$

We use the upper set of equations in Table 11–6 to find the thrust loads:

$$F_{aA} = \frac{0.47F_{rB}}{K_B} - mF_{ae} = \frac{0.47(2654)}{1.5} - (+1)(-1690) = 2522\text{ N}$$

$$F_{aB} = \frac{0.47F_{rB}}{K_B} = \frac{0.47(2654)}{1.5} = 832\text{ N}$$

The dynamic equivalent loads P_A and P_B are

$$P_A = 0.4F_{rA} + K_A F_{aA} = 0.4(2170) + 1.5(2522) = 4651\text{ N}$$

$$P_B = F_{rB} = 2654\text{ N}$$

From Fig. 11–16 for 1050 rev/min at 55°C, $f_T = 1.31$. From Fig. 11–17, $f_v = 1.01$. For use in Eq. (11–16), $a_{3l} = f_T f_v = 1.31(1.01) = 1.32$. The catalog basic load rating corresponding to the load–life–reliability goals is given by Eq. (11–17). Estimate R_D as $\sqrt{0.99} = 0.995$ for each bearing. For bearing A, from Eq. (11–17) the catalog entry C_{10} should equal or exceed

$$C_{10} = (1)(4651)\left[\frac{5000(1050)60}{(4.48)1.32(1-0.995)^{2/3}90(10^6)}\right]^{3/10} = 11\,466\text{ N}$$

From Fig. 11–14, tentatively select type TS 15100 cone and 15245 cup, which will work: $K_A = 1.67$, $C_{10} = 12\,100$ N.

For bearing B, from Eq. (11–17), the catalog entry C_{10} should equal or exceed

$$C_{10} = (1)2654\left[\frac{5000(1050)60}{(4.48)1.32(1-0.995)^{2/3}90(10^6)}\right]^{3/10} = 6543\text{ N}$$

Tentatively select the bearing identical to bearing A, which will work: $K_B = 1.67$, $C_{10} = 12\,100$ N.

Trial 2: Use $K_A = K_B = 1.67$ from tentative bearing selection. The sense of the previous inequality $680 < 2521$ is still the same, so the same equations apply:

$$F_{aA} = \frac{0.47F_{rB}}{K_B} - mF_{ae} = \frac{0.47(2654)}{1.67} - (+1)(-1690) = 2437 \text{ N}$$

$$F_{aB} = \frac{0.47F_{rB}}{K_B} = \frac{0.47(2654)}{1.67} = 747 \text{ N}$$

$$P_A = 0.4F_{rA} + K_A F_{aA} = 0.4(2170) + 1.67(2437) = 4938 \text{ N}$$

$$P_B = F_{rB} = 2654 \text{ N}$$

For bearing A, from Eq. (11–17) the corrected catalog entry C_{10} should equal or exceed

$$C_{10} = (1)(4938)\left[\frac{5000(1050)60}{(4.48)1.32(1-0.995)^{2/3}90(10^6)}\right]^{3/10} = 12\,174 \text{ N}$$

Although this catalog entry exceeds slightly the tentative selection for bearing A, we will keep it since the reliability of bearing B exceeds 0.995. In the next section we will quantitatively show that the combined reliability of bearing A and B will exceed the reliability goal of 0.99.

For bearing B, $P_B = F_{rB} = 2654$ N. From Eq. (11–17),

$$C_{10} = (1)2654\left[\frac{5000(1050)60}{(4.48)1.32(1-0.995)^{2/3}90(10^6)}\right]^{3/10} = 6543 \text{ N}$$

Select cone and cup 15100 and 15245, respectively, for both bearing A and B. Note from Fig. 11–14 the effective load center is located at $a = -5.8$ mm, that is, 5.8 mm into the cup from the back. Thus the shoulder-to-shoulder dimension should be $150 - 2(5.8) = 138.4$ mm. Note, also, the calculation for the second bearing C_{10} contains the same bracketed expression as for the first. For example, on the first trial C_{10} for bearing A is 11 466 N. C_{10} for bearing B can be easily calculated by

$$(C_{10})_B = \frac{(C_{10})_A}{P_A}P_B = \frac{11\,466}{4651}2654 = 6543 \text{ N}$$

The computational effort can be simplified only after this is understood, and not until then.

Design Assessment for Selected Rolling-Contact Bearings

When we design a machine, each component (*e.g., gears, shafts, bearings, etc.*) is designed separately. However, the components interact and influence each other.

It is always a good check to do a design assessment after all elements have been designed (or selected) to make sure that all elements will perform as they are assumed to do.

- For example if the machine has several bearings we can do design assessment to check the reliability of each of them and the total reliability for all.
- For ball and roller bearings, solving for the reliability we get:

Equation (11–7) can likewise be solved for R_D:

$$R = 1 - \left[\frac{x_D \left(\frac{a_f F_D}{C_{10}} \right)^a - x_0}{\theta - x_0} \right]^b \qquad (11\text{–}19)$$

For $R \geq 0.9$

EXAMPLE 11–9 In Ex. 11–3, the minimum required load rating for 99 percent reliability, at $x_D = L/L_{10} = 540$, is $C_{10} = 6671$ lbf $= 29.7$ kN. From Table 11–2 a 02-40 mm deep-groove ball bearing would satisfy the requirement. If the bore in the application had to be 70 mm or larger (selecting a 02-70 mm deep-groove ball bearing), what is the resulting reliability?

Solution From Table 11–2, for a 02-70 mm deep-groove ball bearing, $C_{10} = 61.8$ kN $= 13\,888$ lbf. Using Eq. (11–19), recalling from Ex. 11–3 that $a_f = 1.2$, $F_D = 413$ lbf, $x_0 = 0.02$, $(\theta - x_0) = 4.439$, and $b = 1.489$, we can write

Answer

$$R \doteq 1 - \left\{ \frac{\left[540 \left[\frac{1.2(413)}{13\,888} \right]^3 - 0.02 \right]}{4.439} \right\}^{1.489} = 0.999\,965$$

which, as expected, is much higher than 0.99 from Ex. 11–3.

In tapered roller bearings, or other bearings for a two-parameter Weibull distribution, Eq. (11–18) becomes, for $x_0 = 0$, $\theta = 4.48$, $b = \frac{3}{2}$,

$$R = \exp\left\{ -\left[\frac{x_D}{\theta (C_{10}/[a_f F_D])^a} \right]^b \right\}$$

$$= \exp\left\{ -\left[\frac{x_D}{4.48 f_T f_v (C_{10}/[a_f F_D])^{10/3}} \right]^{3/2} \right\} \qquad \text{(11–20)}$$

and Eq. (11–19) becomes

$$R \doteq 1 - \left\{ \frac{x_D}{\theta [C_{10}/(a_f F_D)]^a} \right\}^b = 1 - \left\{ \frac{x_D}{4.48 f_T f_v [C_{10}/(a_f F_D)]^{10/3}} \right\}^{3/2} \qquad \text{(11–21)}$$

EXAMPLE 11–10 In Ex. 11–8 bearings A and B (cone 15100 and cup 15245) have $C_{10} = 12\,100$ N. What is the reliability of the pair of bearings A and B?

Solution The desired life x_D was $5000(1050)60/[90(10^6)] = 3.5$ rating lives. Using Eq. (11–21) for bearing A, where from Ex. 11–8, $F_D = P_A = 4938$ N, $f_T f_v = 1.32$, and $a_f = 1$, gives

$$R_A \doteq 1 - \left\{ \frac{3.5}{4.48(1.32)\,[12\ 100/\,(1 \times 4938)]^{10/3}} \right\}^{3/2} = 0.994\ 846$$

which is less than 0.995, as expected. Using Eq. (11–21) for bearing B with $F_D = P_B = 2654$ N gives

$$R_B \doteq 1 - \left\{ \frac{3.5}{4.48(1.32)\,[12\ 100/\,(1 \times 2654)]^{10/3}} \right\}^{3/2} = 0.999\ 769$$

Answer The reliability of the bearing pair is

$$R = R_A R_B = 0.994\ 846(0.999\ 769) = 0.994\ 616$$

which is greater than the overall reliability goal of 0.99. When two bearings are made identical for simplicity, or reducing the number of spares, or other stipulation, and the loading is not the same, both can be made smaller and still meet a reliability goal. If the loading is disparate, then the more heavily loaded bearing can be chosen for a reliability goal just slightly larger than the overall goal.

An additional example is useful to show what happens in cases of pure thrust loading.

EXAMPLE 11–11 Consider a constrained housing as depicted in Fig. 11–19 with two direct-mount tapered roller bearings resisting an external thrust F_{ae} of 8000 N. The shaft speed is 950 rev/min, the desired life is 10 000 h, the expected shaft diameter is approximately 1 in. The lubricant is ISO VG 150 (150 cSt at 40°C) oil with an estimated bearing operating temperature of 80°C. The reliability goal is 0.95. The application factor is appropriately $a_f = 1$.
(*a*) Choose a suitable tapered roller bearing for *A*.
(*b*) Choose a suitable tapered roller bearing for *B*.
(*c*) Find the reliabilities R_A, R_B, and *R*.

Solution (*a*) The bearing reactions at *A* are

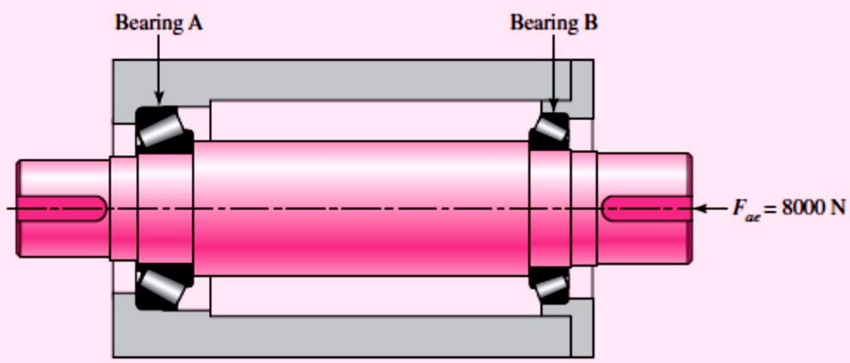
$$F_{rA} = F_{rB} = 0$$

$$F_{aA} = F_{ae} = 8000 \text{ N}$$

Since bearing *B* is unloaded, we will start with $R = R_A = 0.95$. From Table 11–6,

$$\frac{0.47F_{rA}}{K_A} <?> \frac{0.47F_{rB}}{K_B} - mF_{ae}$$

Figure 11–19
The constrained housing of Ex. 11–11.



Noting that F_{ae} to the right is positive (Table 11–6), with direct mounting $m = +1$, we can write

$$\frac{0(0)}{K_A} <?> \left[\frac{0.47(0)}{K_B} - (+1)(-8000)\right]$$

$$0 < 8000 \text{ N}$$

The top set of equations in Table 11–6 applies, so

$$F_{aA} = \frac{0.47(0)}{K_B} - (+1)(-8000) = 8000 \text{ N}$$

$$F_{aB} = \frac{0.47(0)}{K_B} = 0$$

If we set $K_A = 1$, we can find C_{10} in the thrust column and avoid iteration:

$$P_A = 0.4F_{rA} + K_A F_{aA} = 0.4(0) + (1)8000 = 8000 \text{ N}$$

$$P_B = F_{rB} = 0$$

The required life is

$$L_D = 10\,000(950)60 = 570(10^6)\text{ rev}$$

Under the given conditions, $f_T = 0.76$ from Fig. 11–16, and $f_v = 1.12$ from Fig. 11–17. This gives $f_T f_v = 0.76(1.12) = 0.85$. Then, from Eq. (11–17), for bearing A

$$C_{10} = a_f P \left[\frac{L_D}{4.48 f_T f_v (1 - R_D)^{2/3} 90(10^6)} \right]^{3/10}$$

$$= (1)8000 \left[\frac{570(10^6)}{4.48(0.85)(1 - 0.95)^{2/3}\, 90(10^6)} \right]^{3/10} = 16\,970\text{ N}$$

Answer Figure 11–15 presents one possibility in the 1-in bore (25.4-mm) size: cone, HM88630, cup HM88610 with a thrust rating $(C_{10})_a = 17\,200$ N.

Answer (*b*) Bearing B experiences no load, and the cheapest bearing of this bore size will do, including a ball or roller bearing.

(*c*) For Eq. (11–21), $x_D = L_D/L_{10} = 570(10^6)/90(10^6) = 6.333$. Thus the actual reliability of bearing A, from Eq. (11–21), is

Answer

$$R \doteq 1 - \left\{ \frac{x_D}{4.48 f_T f_v [C_{10}/(a_f F_D)]^{10/3}} \right\}^{3/2}$$

$$\doteq 1 - \left\{ \frac{6.333}{4.48(0.85)\,[17\,200/(1 \times 8000)]^{10/3}} \right\}^{3/2} = 0.953$$

which is greater than 0.95, as one would expect. For bearing B,

Answer

$$F_D = P_B = 0$$

$$R_B \doteq 1 - \left[\frac{6.333}{0.85(4.48)(17\,200/0)^{10/3}} \right]^{3/2} = 1 - 0 = 1$$

as one would expect. The combined reliability of bearings A and B as a pair is

Answer

$$R = R_A R_B = 0.953(1) = 0.953$$

which is greater than the reliability goal of 0.95, as one would expect.