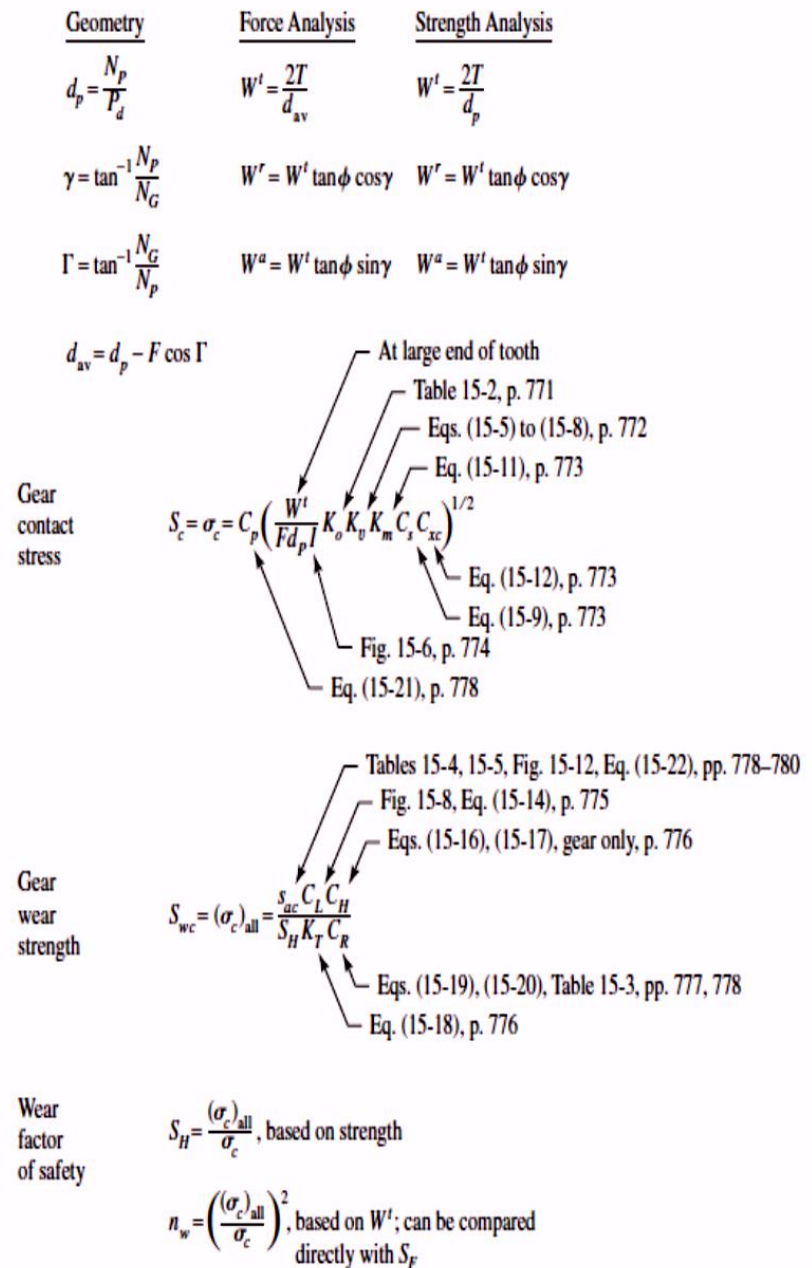


Figure 15-14

"Roadmap" summary of principal straight-bevel gear wear equations and their parameters.

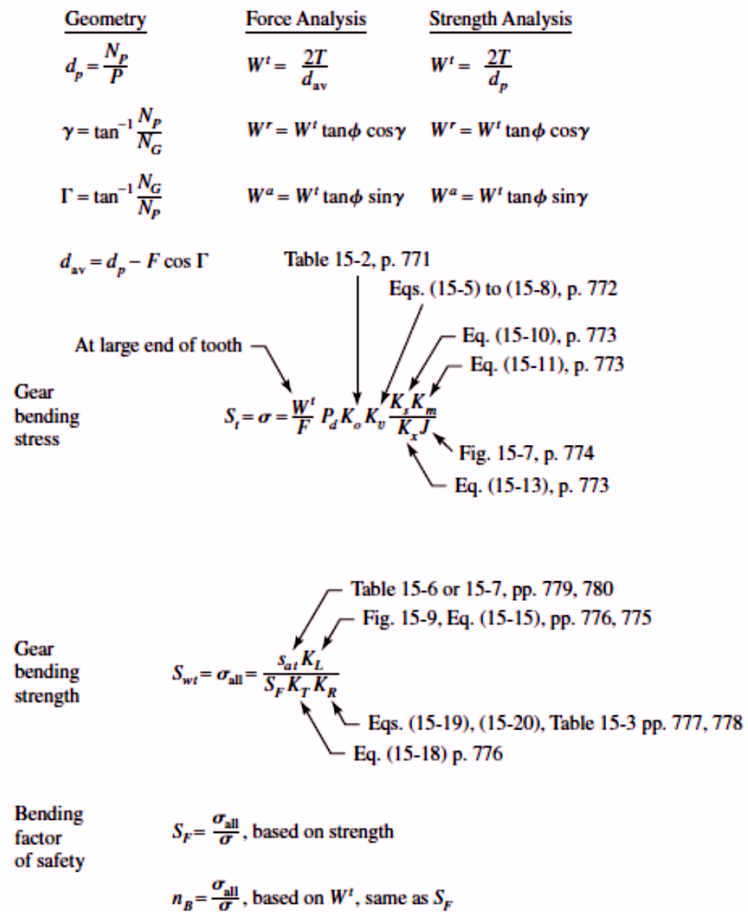
STRAIGHT-BEVEL GEAR WEAR


BASED ON ANSI /AGMA 2003-B97

Figure 15-15

"Roadmap" summary of principal straight-bevel gear bending equations and their parameters.

STRAIGHT-BEVEL GEAR BENDING



BASED ON ANSI /AGMA 2003-B97

EXAMPLE 15-1

A pair of identical straight-tooth miter gears listed in a catalog has a diametral pitch of 5 at the large end, 25 teeth, a 1.10-in face width, and a 20° normal pressure angle; the gears are grade 1 steel through-hardened with a core and case hardness of 180 Brinell. The gears are uncrowned and intended for general industrial use. They have a quality number of $Q_v = 7$. It is likely that the application intended will require outboard mounting of the gears. Use a safety factor of 1, a 10^7 cycle life, and a 0.99 reliability.

(a) For a speed of 600 rev/min find the power rating of this gearset based on AGMA bending strength.

(b) For the same conditions as in part (a) find the power rating of this gearset based on AGMA wear strength.

(c) For a reliability of 0.995, a gear life of 10^9 revolutions, and a safety factor of $S_F = S_H = 1.5$, find the power rating for this gearset using AGMA strengths.

Solution From Figs. 15–14 and 15–15,

$$d_P = N_P / P = 25 / 5 = 5.000 \text{ in}$$

$$v_t = \pi d_P n_P / 12 = \pi (5) 600 / 12 = 785.4 \text{ ft/min}$$

Overload factor: uniform-uniform loading, Table 15–2, $K_o = 1.00$.

Safety factor: $S_F = 1$, $S_H = 1$.

Dynamic factor K_v : from Eq. (15–6),

$$B = 0.25(12 - 7)^{2/3} = 0.731$$

$$A = 50 + 56(1 - 0.731) = 65.06$$

$$K_v = \left(\frac{65.06 + \sqrt{785.4}}{65.06} \right)^{0.731} = 1.299$$

From Eq. (15–8),

$$v_{t \max} = [65.06 + (7 - 3)]^2 = 4769 \text{ ft/min}$$

$v_t < v_{t \max}$, that is, $785.4 < 4769 \text{ ft/min}$, therefore K_v is valid. From Eq. (15–10),

$$K_s = 0.4867 + 0.2132/5 = 0.529$$

From Eq. (15–11),

$$K_{mb} = 1.25 \quad \text{and} \quad K_m = 1.25 + 0.0036(1.10)^2 = 1.254$$

From Eq. (15–13), $K_x = 1$. From Fig. 15–6, $I = 0.065$; from Fig. 15–7, $J_P = 0.216$, $J_G = 0.216$. From Eq. (15–15),

$$K_L = 1.683(10^7)^{-0.0323} = 0.99996 \doteq 1$$

From Eq. (15–14),

$$C_L = 3.4822(10^7)^{-0.0602} = 1.32$$

Since $H_{BP}/H_{BG} = 1$, then from Fig. 15–10, $C_H = 1$. From Eqs. (15–13) and (15–18), $K_x = 1$ and $K_T = 1$, respectively. From Eq. (15–20),

$$K_R = 0.70 - 0.15 \log(1 - 0.99) = 1, \quad C_R = \sqrt{K_R} = \sqrt{1} = 1$$

(a) *Bending*: From Eq. (15–23),

$$s_{at} = 44(180) + 2100 = 10\,020 \text{ psi}$$

From Eq. (15–3),

$$\begin{aligned} s_t = \sigma &= \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J} = \frac{W^t}{1.10} (5)(1) 1.299 \frac{0.529(1.254)}{(1)0.216} \\ &= 18.13 W^t \end{aligned}$$

From Eq. (15-4),

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R} = \frac{10\,020(1)}{(1)(1)(1)} = 10\,020 \text{ psi}$$

Equating s_t and s_{wt} ,

$$18.13 W^t = 10\,020 \quad W^t = 552.6 \text{ lbf}$$

Answer

$$H = \frac{W^t v_t}{33\,000} = \frac{552.6(785.4)}{33\,000} = 13.2 \text{ hp}$$

(b) *Wear*: From Fig. 15-12,

$$s_{ac} = 341(180) + 23\,620 = 85\,000 \text{ psi}$$

From Eq. (15-2),

$$\sigma_{c,all} = \frac{s_{ac} C_L C_H}{S_H K_T C_R} = \frac{85\,000(1.32)(1)}{(1)(1)(1)} = 112\,200 \text{ psi}$$

Now $C_p = 2290\sqrt{\text{psi}}$ from definitions following Eq. (15-21). From Eq. (15-9),

$$C_s = 0.125(1.1) + 0.4375 = 0.575$$

From Eq. (15-12), $C_{xc} = 2$. Substituting in Eq. (15-1) gives

$$\begin{aligned} \sigma_c &= C_p \left(\frac{W^t}{F d_p I} K_o K_v K_m C_s C_{xc} \right)^{1/2} \\ &= 2290 \left[\frac{W^t}{1.10(5)0.065} (1) 1.299 (1.254) 0.575 (2) \right]^{1/2} = 5242\sqrt{W^t} \end{aligned}$$

Equating σ_c and $\sigma_{c,all}$ gives

$$5242\sqrt{W^t} = 112\,200, \quad W^t = 458.1 \text{ lbf}$$

$$H = \frac{458.1(785.4)}{33\,000} = 10.9 \text{ hp}$$

Rated power for the gearset is

Answer

$$H = \min(12.9, 10.9) = 10.9 \text{ hp}$$

(c) Life goal 10^9 cycles, $R = 0.995$, $S_F = S_H = 1.5$, and from Eq. (15-15),

$$K_L = 1.683(10^9)^{-0.0323} = 0.8618$$

From Eq. (15-19),

$$K_R = 0.50 - 0.25 \log(1 - 0.995) = 1.075, \quad C_R = \sqrt{K_R} = \sqrt{1.075} = 1.037$$

From Eq. (15-14),

$$C_L = 3.4822(10^9)^{-0.0602} = 1$$

Bending: From Eq. (15-23) and part (a), $s_{at} = 10\,020$ psi. From Eq. (15-3),

$$s_t = \sigma = \frac{W^t}{1.10} 5(1) 1.299 \frac{0.529(1.254)}{(1)0.216} = 18.13 W^t$$

From Eq. (15-4),

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R} = \frac{10\,020(0.8618)}{1.5(1)1.075} = 5355 \text{ psi}$$

Equating s_t to s_{wt} gives

$$18.13 W^t = 5355 \quad W^t = 295.4 \text{ lbf}$$

$$H = \frac{295.4(785.4)}{33\,000} = 7.0 \text{ hp}$$

Wear: From Eq. (15-22), and part (b), $s_{ac} = 85\,000$ psi.

Substituting into Eq. (15-2) gives

$$\sigma_{c,\text{all}} = \frac{s_{ac} C_L C_H}{S_H K_T C_R} = \frac{85\,000(1)(1)}{1.5(1)1.037} = 54\,640 \text{ psi}$$

Substituting into Eq. (15-1) gives, from part (b), $\sigma_c = 5242\sqrt{W^t}$.

Equating σ_c to $\sigma_{c,\text{all}}$ gives

$$\sigma_c = \sigma_{c,\text{all}} = 54\,640 = 5242\sqrt{W^t} \quad W^t = 108.6 \text{ lbf}$$

The wear power is

$$H = \frac{108.6(785.4)}{33\,000} = 2.58 \text{ hp}$$

Answer The mesh rated power is $H = \min(7.0, 2.58) = 2.6 \text{ hp}$.

Design of straight Bevel Gears

The decision set can be classified in two groups:

Priori decisions	$\left\{ \begin{array}{l} \text{Function} \\ \text{Design factor} \\ \text{Tooth system} \\ \text{Gear ratio and tooth count} \end{array} \right.$
Design decisions	$\left\{ \begin{array}{l} \text{Diametral pitch and face width} \\ \text{Quality number} \\ \text{Gear material, core and case hardness} \\ \text{Pinion material, core and case hardness} \end{array} \right.$

Since there are limitations on the face width of bevel gears (due to teeth deflection), it is suggested that the face width is chosen such that:

In bevel gears the quality number is linked to the wear strength. The J factor for the gear can be smaller than for the pinion. Bending strength is not linear with face width, because added material is placed at the small end of the teeth. Consequently, face width is roughly prescribed as

$$F = \min(0.3A_0, 10/P_d) \quad (15-24)$$

where A_0 is the cone distance (see Fig. 13–20), given by

$$A_0 = \frac{d_P}{2 \sin \gamma} = \frac{d_G}{2 \sin \Gamma} \quad (15-25)$$

EXAMPLE 15-2 Design a straight-bevel gear mesh for shaft centerlines that intersect perpendicularly, to deliver 6.85 hp at 900 rev/min with a gear ratio of 3:1, temperature of 300°F, normal pressure angle of 20°, using a design factor of 2. The load is uniform-uniform. Although the minimum number of teeth on the pinion is 13, which will mesh with 31 or more teeth without interference, use a pinion of 20 teeth. The material is to be AGMA grade 1 and the teeth are to be crowned. The reliability goal is 0.995 with a pinion life of 10^9 revolutions.

Solution First we list the a priori decisions and their immediate consequences.

Function: 6.85 hp at 900 rev/min, gear ratio $m_G = 3$, 300°F environment, neither gear straddle-mounted, $K_{mb} = 1.25$ [Eq. (15–11)], $R = 0.995$ at 10^9 revolutions of the pinion,

$$\text{Eq. (15-14):} \quad (C_L)_G = 3.4822(10^9/3)^{-0.0602} = 1.068$$

$$(C_L)_P = 3.4822(10^9)^{-0.0602} = 1$$

$$\text{Eq. (15-15):} \quad (K_L)_G = 1.683(10^9/3)^{-0.0323} = 0.8929$$

$$(K_L)_P = 1.683(10^9)^{-0.0323} = 0.8618$$

$$\text{Eq. (15-19):} \quad K_R = 0.50 - 0.25 \log(1 - 0.995) = 1.075$$

$$C_R = \sqrt{K_R} = \sqrt{1.075} = 1.037$$

$$\text{Eq. (15-18):} \quad K_T = C_T = (460 + 300)/710 = 1.070$$

Design factor: $n_d = 2$, $S_F = 2$, $S_H = \sqrt{2} = 1.414$.

Tooth system: crowned, straight-bevel gears, normal pressure angle 20° ,

$$\text{Eq. (15-13):} \quad K_x = 1$$

$$\text{Eq. (15-12):} \quad C_{xc} = 1.5.$$

With $N_P = 20$ teeth, $N_G = (3)20 = 60$ teeth and from Fig. 15-14,

$$\gamma = \tan^{-1}(N_P/N_G) = \tan^{-1}(20/60) = 18.43^\circ \quad \Gamma = \tan^{-1}(60/20) = 71.57^\circ$$

From Figs. 15-6 and 15-7, $I = 0.0825$, $J_P = 0.248$, and $J_G = 0.202$. Note that $J_P > J_G$.

Decision 1: Trial diametral pitch, $P_d = 8$ teeth/in.

$$\text{Eq. (15-10):} \quad K_s = 0.4867 + 0.2132/8 = 0.5134$$

$$d_P = N_P/P_d = 20/8 = 2.5 \text{ in}$$

$$d_G = 2.5(3) = 7.5 \text{ in}$$

$$v_t = \pi d_P n_P / 12 = \pi(2.5)900/12 = 589.0 \text{ ft/min}$$

$$W^t = 33\,000 \text{ hp}/v_t = 33\,000(6.85)/589.0 = 383.8 \text{ lbf}$$

$$\text{Eq. (15-25):} \quad A_0 = d_P/(2 \sin \gamma) = 2.5/(2 \sin 18.43^\circ) = 3.954 \text{ in}$$

$$\text{Eq. (15-24):}$$

$$F = \min(0.3A_0, 10/P_d) = \min[0.3(3.954), 10/8] = \min(1.186, 1.25) = 1.186 \text{ in}$$

Decision 2: Let $F = 1.25$ in. Then,

$$\text{Eq. (15-9):} \quad C_s = 0.125(1.25) + 0.4375 = 0.5937$$

$$\text{Eq. (15-11):} \quad K_m = 1.25 + 0.0036(1.25)^2 = 1.256$$

Decision 3: Let the transmission accuracy number be 6. Then, from Eq. (15–6),

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (15–5):
$$K_v = \left(\frac{59.77 + \sqrt{589.0}}{59.77} \right)^{0.8255} = 1.325$$

Decision 4: Pinion and gear material and treatment. Carburize and case-harden grade ASTM 1320 to

Core 21 HRC (H_B is 229 Brinell)

Case 55-64 HRC (H_B is 515 Brinell)

From Table 15–4, $s_{ac} = 200\,000$ psi and from Table 15–6, $s_{at} = 30\,000$ psi.

Gear bending: From Eq. (15–3), the bending stress is

$$\begin{aligned} (s_t)_G &= \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J_G} = \frac{383.8}{1.25} 8(1) 1.325 \frac{0.5134(1.256)}{(1)0.202} \\ &= 10\,390 \text{ psi} \end{aligned}$$

The bending strength, from Eq. (15–4), is given by

$$(s_{wt})_G = \left(\frac{s_{at} K_L}{S_F K_T K_R} \right)_G = \frac{30\,000(0.8929)}{2(1.070)1.075} = 11\,640 \text{ psi}$$

The strength exceeds the stress by a factor of $11640/10390 = 1.12$, giving an actual factor of safety of $(S_F)_G = 2(1.12) = 2.24$.

Pinion bending: The bending stress can be found from

$$(s_t)_P = (s_t)_G \frac{J_G}{J_P} = 10\,390 \frac{0.202}{0.248} = 8463 \text{ psi}$$

The bending strength, again from Eq. (15–4), is given by

$$(s_{wt})_P = \left(\frac{s_{at} K_L}{S_F K_T K_R} \right)_P = \frac{30\,000(0.8618)}{2(1.070)1.075} = 11\,240 \text{ psi}$$

The strength exceeds the stress by a factor of $11\,240/8463 = 1.33$, giving an actual factor of safety of $(S_F)_P = 2(1.33) = 2.66$.

Gear wear: The load-induced contact stress for the pinion and gear, from Eq. (15–1), is

$$\begin{aligned}
 s_c &= C_p \left(\frac{W^t}{F d_p I} K_o K_v K_m C_s C_{xc} \right)^{1/2} \\
 &= 2290 \left[\frac{383.8}{1.25(2.5)0.0825} (1) 1.325 (1.256) 0.5937 (1.5) \right]^{1/2} \\
 &= 107\,560 \text{ psi}
 \end{aligned}$$

From Eq. (15–2) the contact strength of the gear is

$$(s_{wc})_G = \left(\frac{s_{ac} C_L C_H}{S_H K_T C_R} \right)_G = \frac{200\,000(1.068)(1)}{\sqrt{2}(1.070)1.037} = 136\,120 \text{ psi}$$

The strength exceeds the stress by a factor of $136\,120/107\,560 = 1.266$, giving an actual factor of safety of $(S_H)_G^2 = 1.266^2(2) = 3.21$.

Pinion wear: From Eq. (15–2) the contact strength of the pinion is

$$(s_{wc})_P = \left(\frac{s_{ac} C_L C_H}{S_H K_T C_R} \right)_P = \frac{200\,000(1)(1)}{\sqrt{2}(1.070)1.037} = 127\,450 \text{ psi}$$

The strength exceeds the stress by a factor of $136\,120/127\,450 = 1.068$, giving an actual factor of safety of $(S_H)_P^2 = 1.068^2(2) = 2.28$.

The actual factors of safety are 2.24, 2.66, 3.21, and 2.28. Making a direct comparison of the factors, we note that the threat from gear bending and pinion wear are practically equal. We also note that three of the ratios are comparable. Our goal would be to make changes in the design decisions that drive the factors closer to 2. The next step would be to adjust the design variables. It is obvious that an iterative process is involved. We need a figure of merit to order the designs. A computer program clearly is desirable.

Table 15-1

 Symbols Used in Bevel Gear Rating Equations, ANSI/AGMA 2003-B97 Standard *Source: ANSI/AGMA 2003-B97*

AGMA Symbol	ISO Symbol	Description	Units
A_m	R_m	Mean cone distance	in (mm)
A_o	R_o	Outer cone distance	in (mm)
C_H	Z_W	Hardness ratio factor for pitting resistance	
C_I	Z_I	Inertia factor for pitting resistance	
C_L	Z_{NT}	Stress cycle factor for pitting resistance	
C_p	Z_E	Elastic coefficient	$[\text{lbf/in}^2]^{0.5}$ $[\text{N/mm}^2]^{0.5}$
C_R	Z_Z	Reliability factor for pitting	
C_{SF}		Service factor for pitting resistance	
C_S	Z_x	Size factor for pitting resistance	
C_{xc}	Z_{xc}	Crowning factor for pitting resistance	
D, d	d_{g2}, d_{g1}	Outer pitch diameters of gear and pinion, respectively	in (mm)
E_G, E_P	E_2, E_1	Young's modulus of elasticity for materials of gear and pinion, respectively	lbf/in^2 (N/mm^2)
e	e	Base of natural (Napierian) logarithms	
F	b	Net face width	in (mm)
F_{aG}, F_{aP}	b'_2, b'_1	Effective face widths of gear and pinion, respectively	in (mm)
f_p	R_{a1}	Pinion surface roughness	$\mu\text{in } (\mu\text{m})$
H_{BG}	H_{B2}	Minimum Brinell hardness number for gear material	HB
H_{BP}	H_{B1}	Minimum Brinell hardness number for pinion material	HB
h_c	$E_{H \min}$	Minimum total case depth at tooth middepth	in (mm)
h_e	h'_c	Minimum effective case depth	in (mm)
$h_{e \lim}$	$h'_{c \lim}$	Suggested maximum effective case depth limit at tooth middepth	in (mm)
I	Z_I	Geometry factor for pitting resistance	
J	Y_J	Geometry factor for bending strength	
J_G, J_P	Y_{J2}, Y_{J1}	Geometry factor for bending strength for gear and pinion, respectively	
K_F	Y_F	Stress correction and concentration factor	
K_I	Y_I	Inertia factor for bending strength	
K_L	Y_{NT}	Stress cycle factor for bending strength	
K_m	$K_{H\beta}$	Load distribution factor	
K_o	K_A	Overload factor	
K_R	Y_Z	Reliability factor for bending strength	
K_S	Y_X	Size factor for bending strength	
K_{SF}		Service factor for bending strength	
K_T	K_θ	Temperature factor	
K_v	K_v	Dynamic factor	
K_x	Y_β	Lengthwise curvature factor for bending strength	
	m_{at}	Outer transverse module	(mm)
	m_{mt}	Mean transverse module	(mm)
	m_{mn}	Mean normal module	(mm)
m_{NI}	ϵ_{NI}	Load sharing ratio, pitting	
m_{NJ}	ϵ_{NJ}	Load sharing ratio, bending	
N	Z_2	Number of gear teeth	
N_L	n_L	Number of load cycles	
n	Z_1	Number of pinion teeth	
n_P	n_1	Pinion speed	rev/min

Table 15-1

 Symbols Used in Gear Rating Equations, ANSI/AGMA 2003-B97 Standard (*Continued*)

AGMA Symbol	ISO Symbol	Description	Units
P	P	Design power through gear pair	hp (kW)
P_a	P_a	Allowable transmitted power	hp (kW)
P_{ac}	P_{az}	Allowable transmitted power for pitting resistance	hp (kW)
P_{acu}	P_{azu}	Allowable transmitted power for pitting resistance at unity service factor	hp (kW)
P_{at}	P_{ay}	Allowable transmitted power for bending strength	hp (kW)
P_{atu}	P_{ayu}	Allowable transmitted power for bending strength at unity service factor	hp (kW)
P_d		Outer transverse diametral pitch	in ⁻¹
P_m		Mean transverse diametral pitch	in ⁻¹
P_{mn}		Mean normal diametral pitch	in ⁻¹
Q_v	Q_v	Transmission accuracy number	
q	q	Exponent used in formula for lengthwise curvature factor	
R, r	r_{mpt2}, r_{mpt1}	Mean transverse pitch radii for gear and pinion, respectively	in (mm)
R_t, r_t	r_{mto2}, r_{mto1}	Mean transverse radii to point of load application for gear and pinion, respectively	in (mm)
r_c	r_{c0}	Cutter radius used for producing Zerol bevel and spiral bevel gears	in (mm)
s	g_c	Length of the instantaneous line of contact between mating tooth surfaces	in (mm)
s_{ac}	σ_{Hlim}	Allowable contact stress number	lbf/in ² (N/mm ²)
s_{at}	σ_{Flim}	Bending stress number (allowable)	lbf/in ² (N/mm ²)
s_c	σ_H	Calculated contact stress number	lbf/in ² (N/mm ²)
S_F	S_F	Bending safety factor	
S_H	S_H	Contact safety factor	
s_t	σ_F	Calculated bending stress number	lbf/in ² (N/mm ²)
s_{wc}	σ_{HFP}	Permissible contact stress number	lbf/in ² (N/mm ²)
s_{wt}	σ_{FFP}	Permissible bending stress number	lbf/in ² (N/mm ²)
T_p	T_1	Operating pinion torque	lbf in (Nm)
T_T	θ_T	Operating gear blank temperature	°F(°C)
t_0	s_{ai}	Normal tooth top land thickness at narrowest point	in (mm)
U_c	U_c	Core hardness coefficient for nitrided gear	lbf/in ² (N/mm ²)
U_H	U_H	Hardening process factor for steel	lbf/in ² (N/mm ²)
v_t	v_{et}	Pitch-line velocity at outer pitch circle	ft/min (m/s)
Y_{KG}, Y_{KP}	Y_{K2}, Y_{K1}	Tooth form factors including stress-concentration factor for gear and pinion, respectively	
μ_G, μ_P	ν_2, ν_1	Poisson's ratio for materials of gear and pinion, respectively	
ρ_0	ρ_{y0}	Relative radius of profile curvature at point of maximum contact stress between mating tooth surfaces	in (mm)
ϕ	α_n	Normal pressure angle at pitch surface	
ϕ_t	α_{wt}	Transverse pressure angle at pitch point	
ψ	β_m	Mean spiral angle at pitch surface	
ψ_b	β_{mb}	Mean base spiral angle	