Using Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial KE}{\partial \dot{q}_i}\right) - \frac{\partial KE}{\partial q_i} + \frac{\partial PE}{\partial q_i} = 0$$

$$KE = \frac{1}{2} m (l \cos\theta_1)^2 \dot{\theta}_1^2 + \frac{1}{2} m (l \cos\theta_2)^2 \dot{\theta}_2^2$$

$$KE = \frac{1}{2} m l^2 \dot{\theta}_1^2 + \frac{1}{2} m l^2 \dot{\theta}_2^2$$

$$PE = \frac{1}{2} K (a \sin(\theta_2 - \theta_1))^2 + mgl(1-\cos\theta_1) + mgl(1-\cos\theta_2)$$

$$PE = \frac{1}{2} K a^2 (\theta_2 - \theta_1)^2 + mgl(1-\cos\theta_1) + mgl(1-\cos\theta_2)$$

let $q_1 = \theta_1$

$$\frac{\partial KE}{\partial \dot{\theta}_1} = \frac{1}{2} m l^2 (2\dot{\theta}_1) = m l^2 \dot{\theta}_1$$

$$\frac{d}{dt}\left(\frac{\partial KE}{\partial \dot{\theta}_1}\right) = m l^2 \ddot{\theta}_1$$

$$\frac{\partial KE}{\partial \theta_1} = \text{zero}$$

$$\frac{\partial PE}{\partial \theta_1} = \frac{1}{2} K a^2 * 2 (\theta_2 - \theta_1) * -1 + mgl \sin\theta_1$$

$$= K a^2 (\theta_1 - \theta_2) + mgl\theta_1$$

$$\therefore m l^2 \ddot{\theta}_1 + K a^2 \theta_1 - K a^2 \theta_2 + mgl\theta_1 = 0$$

$$m l^2 \ddot{\theta}_1 + \theta_1 (K a^2 + mgl) - K a^2 \theta_2 = 0 \rightarrow$$ The same as in Eq. 1

let $q_2 = \theta_2$

$$\frac{\partial KE}{\partial \dot{\theta}_2} = \frac{1}{2} m l^2 (2\dot{\theta}_2) = m l^2 \dot{\theta}_2$$

$$\frac{d}{dt}\left(\frac{\partial KE}{\partial \dot{\theta}_2}\right) = m l^2 \ddot{\theta}_2$$

$$\frac{\partial KE}{\partial \theta_2} = \text{Zero}$$

$$\frac{\partial PE}{\partial \theta_2} = \frac{1}{2} K a^2 \times 2(\theta_2 - \theta_1) \times 1 + mgl \sin\theta_2$$

$$= Ka^2(\theta_2 - \theta_1) + mgl\,\theta_2$$

$$\therefore ml^2\ddot{\theta}_2 + Ka^2(\theta_2 - \theta_1) + mgl\,\theta_2 = 0$$

$$ml^2\ddot{\theta}_2 - Ka^2\theta_1 + \theta_2(Ka^2 + mgl) = 0$$

The same as in eq. (2)

Arranging Eqs. (1) and (2) in matrix form

$$\begin{bmatrix} ml^2 & 0 \\ 0 & ml^2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} mgl + Ka^2 & -Ka^2 \\ -Ka^2 & mgl + Ka^2 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$ml^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} mgl + Ka^2 & -Ka^2 \\ -Ka^2 & mgl + Ka^2 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Assuming harmonic motion $\theta_1 = \Theta_1 e^{i\omega t}$ $\theta_2 = \Theta_2 e^{i\omega t}$

$$\begin{bmatrix} mgl + Ka^2 - ml^2\omega^2 & -Ka^2 \\ -Ka^2 & mgl + Ka^2 - ml^2\omega^2 \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{--- (3)}$$

For non-trivial Solution

$$\begin{vmatrix} mgl + Ka^2 - ml^2\omega^2 & -Ka^2 \\ -Ka^2 & mgl + Ka^2 - ml^2\omega^2 \end{vmatrix} = 0$$

$$(mgl + Ka^2 - ml^2\omega^2)^2 - K^2a^4 = 0$$

$$(mgl + Ka^2 - ml^2\omega^2)^2 = K^2a^4$$