Moment Diagrams by Parts

The moment-area method of finding the deflection of a beam will demand the accurate computation of the area of a moment diagram, as well as the moment of such area about any axis. To pave its way, the following deal on how to draw moment diagrams by parts and to calculate the moment of such diagrams about a specified axis.

Basic Principles

1. The bending moment caused by all forces to the left or to the right of any section is equal to the respective algebraic sum of the bending moments at that section caused by each load acting separately.

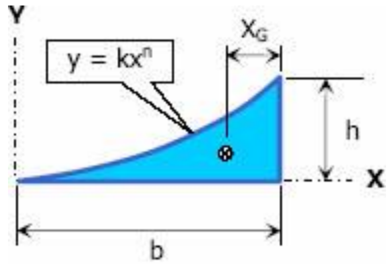
   $$M = (\Sigma M\,)_L = (\Sigma M\,)_R$$

2. The moment of a load about a specified axis is always defined by the equation of a spandrel

   $$y = kx^n$$

   where n is the degree of power of x.

The graph of the above equation is as shown below:



Area and centroid of moment diagram (spandrel)

and the area and location of centroid are defined as follows.

$$A = \frac{1}{n+1} bh$$

$$X_G = \frac{1}{n+2} b$$

Cantilever Loadings

A= area of moment diagram

Mx = moment about a section of distance x

barred x = location of centoid

Degree = degree power of the moment diagram

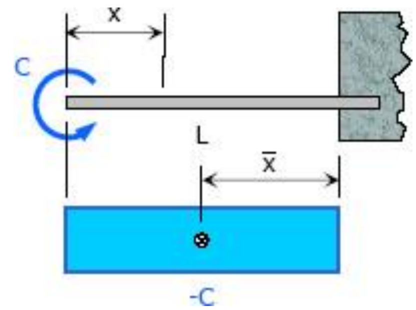
Couple or Moment Load

$A = -CL$

$M_x = -C$

$\bar{x} = \frac{1}{2}L$

Degree: zero



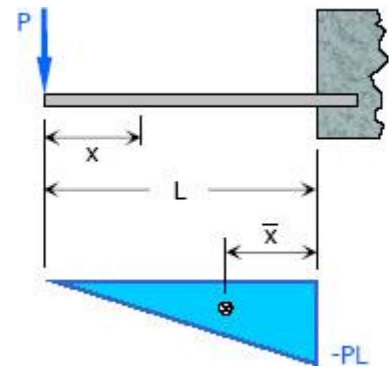
Concentrated Load

$A = -\frac{1}{2}PL^2$

$M_x = -Px$

$\bar{x} = \frac{1}{3}L$

Degree: first



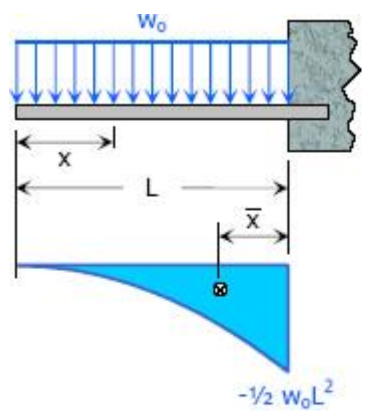
Uniformly Distributed Load

$A = -\frac{1}{6}w_oL^3$

$M_x = -\frac{1}{2}w_ox^2$

$\bar{x} = \frac{1}{4}L$

Degree: second



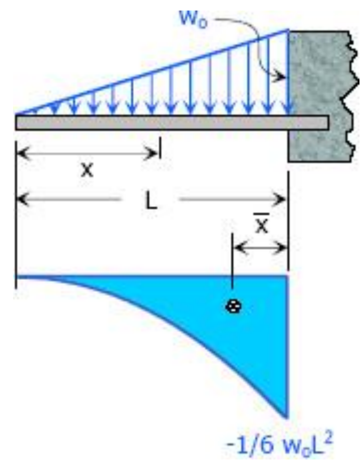
Uniformly Varying Load

$A = -\frac{1}{24}w_o L^3$

$M_x = -\frac{w_o}{6L}x^2$

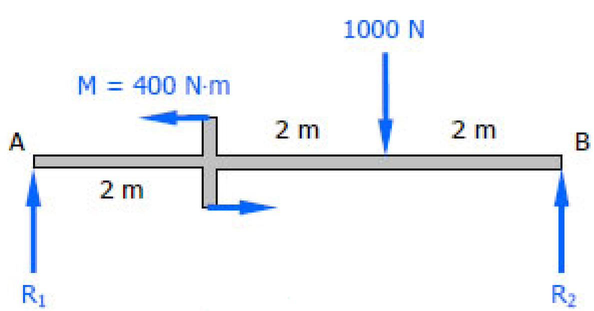
$x = \frac{1}{5}L$

Degree: third



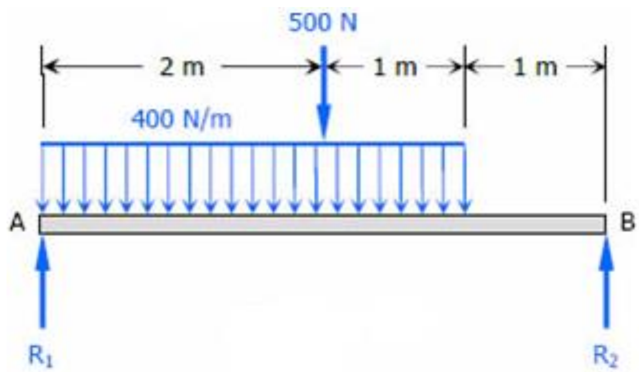
Problem1

For the beam loaded as shown, compute the moment of area of the M diagrams between the reactions about both the left and the right reaction.



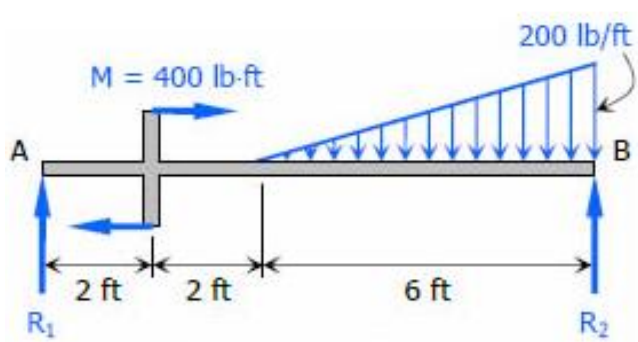
Problem2

For the beam loaded as shown, compute the moment of area of the M diagrams between the reactions about both the left and the right reaction. (Hint: Draw the moment diagram by parts from right to left.)



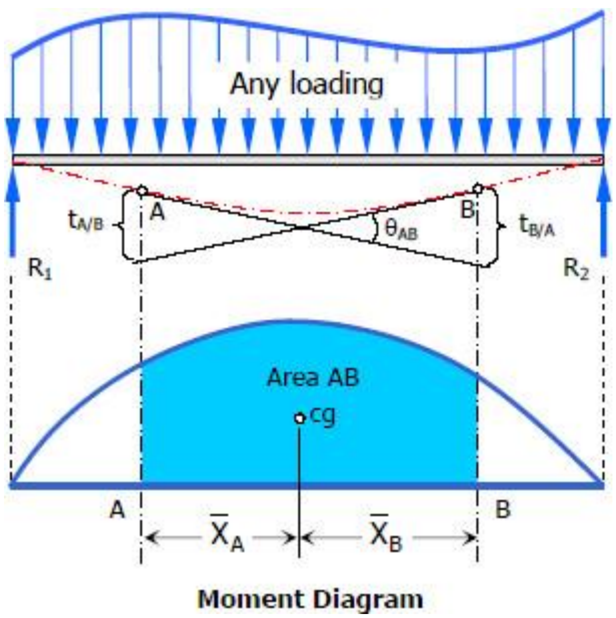
Problem3

For the beam loaded with uniformly varying load and a couple as shown, compute the moment of area of the M diagrams between the reactions about both the left and the right reaction.



Beam Deflections (Area-Moment Method)

Another method of determining the slopes and deflections in beams is the area-moment method, which involves the area of the moment diagram.



Theorem I

The change in slope between the tangents drawn to the elastic curve at any two points A and B is equal to the product of 1/EI multiplied by the area of the moment diagram between these two points.

$$\theta_{AB} = \frac{1}{EI}(Area_{AB})$$

Theorem II

The deviation of any point B relative to the tangent drawn to the elastic curve at

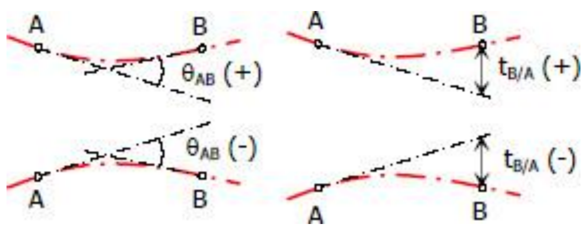
any other point A, in a direction perpendicular to the original position of the beam, is equal to the product of 1/EI multiplied by the moment of an area about B of that part of the moment diagram between points A and B.

$$t_{B/A} = \frac{1}{EI}(Area_{AB}) \cdot \bar{X}_B$$

and

$$t_{A/B} = \frac{1}{EI}(Area_{AB}) \cdot \bar{X}_A$$

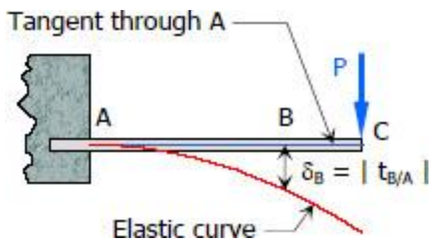
Rules of Sign



The deviation at any point is positive if the point lies above the tangent, negative if the point is below the tangent. Measured from left tangent, if θ is counterclockwise, the change of slope is positive, negative if θ is clockwise.

Deflection of Cantilever Beams | Area-Moment Method

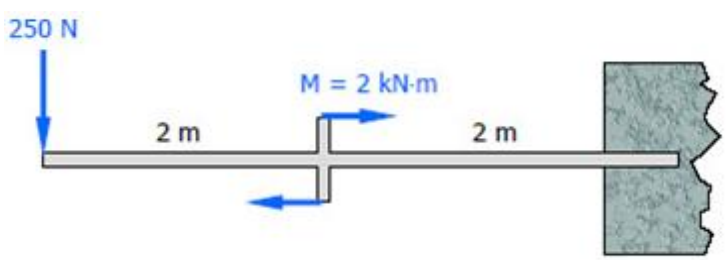
Generally, the tangential deviation t is not equal to the beam deflection. In cantilever beams, however, the tangent drawn to the elastic curve at the wall is horizontal and coincidence therefore with the neutral axis of the beam. The tangential deviation in this case is equal to the deflection of the beam as shown below.



From the figure above, the deflection at B denoted as δB is equal to the deviation of B from a tangent line through A denoted as $t_{B/A}$. This is because the tangent line through A lies with the neutral axis of the beam.

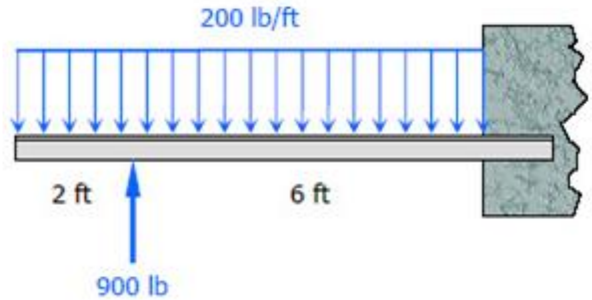
Problem4

For the cantilever beam shown, determine the value of EIδ at the left end. Is this deflection upward or downward?



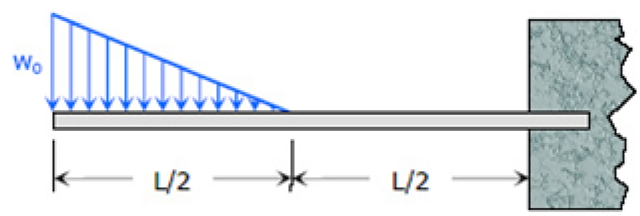
Problem5

The downward distributed load and an upward concentrated force act on the cantilever beam shown. Find the amount the free end deflects upward or downward if $E = 1.5 \times 10^6$ psi and $I = 60\ in^4$.



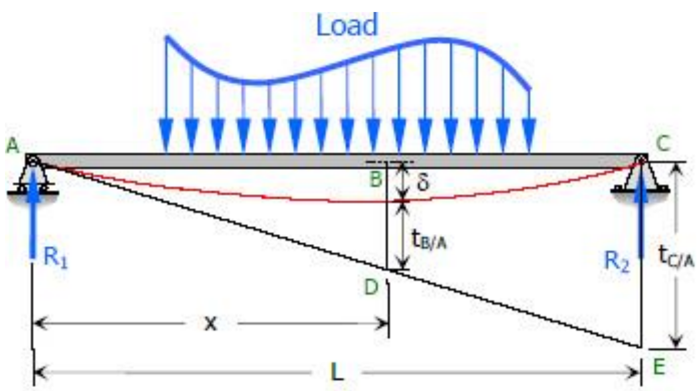
Problem6

Find the maximum value of EIδ for the beam shown.



Deflections in Simply Supported Beams | Area-Moment Method

The deflection δ at some point B of a simply supported beam can be obtained by the following steps:



Geometry of area-moment method for finding deformation δ in simply supported beam

1. Compute $t_{C/A} = \frac{1}{EI}(Area_{AC})\,\bar{X}_C$

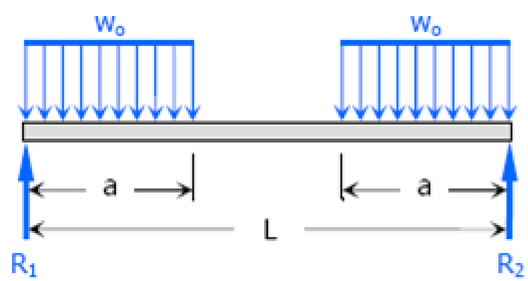
2. Compute $t_{B/A} = \frac{1}{EI}(Area_{AB})\,\bar{X}_B$

3. Solve δ by ratio and proportion (see figure above).

$$\frac{\delta + t_{B/A}}{x} = \frac{t_{C/A}}{L}$$

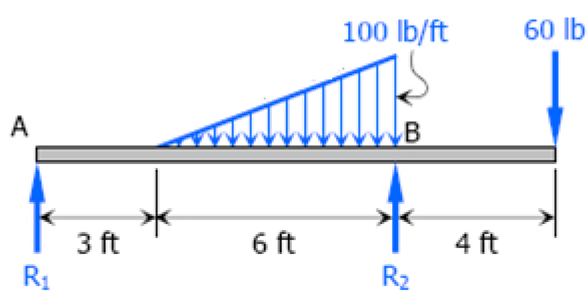
Problem7

Determine the maximum deflection of the beam shown.



Problem 8

Determine the value of EIδ at the right end of the overhanging beam shown. Is the deflection up or down?



Problem 9

Determine the value of EIδ at the left end of the overhanging beam shown.