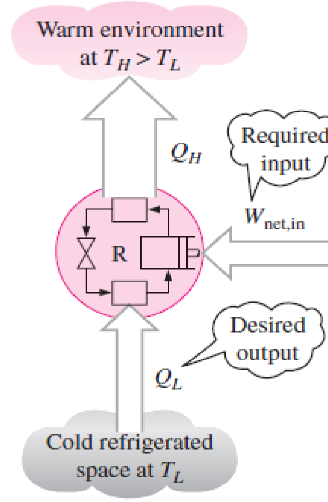
Refrigerators and Heat Pumps



The heat is transferred from high-temperature mediums to low-temperature ones without requiring any devices. The transfer of heat from a low-temperature medium to a high-temperature one requires special devices called **refrigerators.**
The objective of a refrigerator is to maintain the refrigerated space at a low temperature by removing heat from it

Q_L =the magnitude of the heat removed from the refrigerated space at temperature T_L.
Q_H= the magnitude of the heat rejected to the warm environment at temperature T_H.
$W_{\text{net,in}}$ = the net work input to the refrigerator.

Coefficient of Performance

The *efficiency* of a refrigerator is expressed in terms of the **coefficient of performance** (COP), denoted by COP_R which is defined as the desired output (Q_L) to required input ($W_{\text{net,in}}$):

$$COP_R = \frac{Q_L}{W_{net,in}}$$

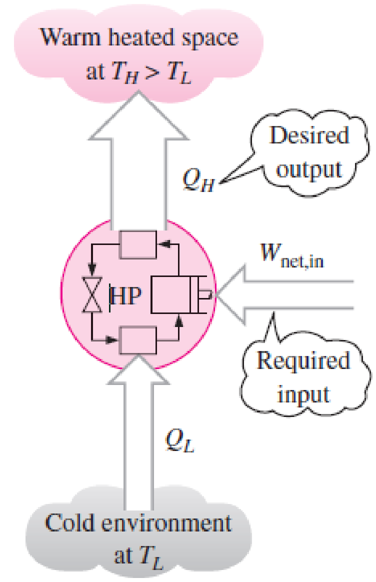
And $W_{net,in} = Q_H - Q_L$

$$COP_R = \frac{Q_L}{Q_H - Q_L}$$

$$COP_R = \frac{1}{\frac{Q_H}{Q_L} - 1}$$

Notice that the value of COP_R can be greater than unity (Q_L can be > $W_{\text{net,in}}$), while the thermal efficiency of heat engine never be greater than 1.

Heat Pumps



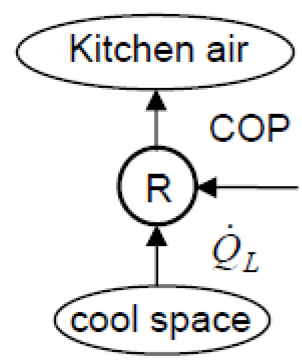
The objective of a heat pump, however, is to maintain a heated space at a high temperature. This is accomplished by absorbing heat from a low-temperature source, such as cold outside air in winter, and supplying this heat to the high-temperature medium such as a house.
The measure of performance of a heat pump is also expressed in terms of the coefficient of performance COP_{HP}, defined as:

$$COP_{HP} = \frac{\textbf{\textit{Desired Output}}}{\textbf{\textit{Required Input}}}$$

$$COP_{HP} = \frac{Q_H}{W_{net,in}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

$$COP_{HP} = COP_R + 1$$

Ex: A household refrigerator with a COP of 1.2 removes heat from the refrigerated space at a rate of 60 kJ/min. Determine (*a*) the electric power consumed by the refrigerator and
(*b*) the rate of heat transfer to the kitchen air. Also, determine the coefficient of performance if this device is used as heat pump?



Sol:

$COP_R=1.2$, $\dot{Q}_L = 60\, kJ/\min$

$\dot{W}_{net,in} = ?$, $\dot{Q}_H = ?$, $COP_{HP}=?$

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{net,in}}$$

$$1.2 = \frac{60}{\dot{W}_{net,in}}, \quad \dot{W}_{net,in} = 50\ kJ/\min = \frac{50}{60} = 0.833\ kW$$

$$\dot{Q}_H = \dot{W}_{net,in} + \dot{Q}_L = 50 + 60 = 110\ kJ/\min$$

$$COP_{HP} = \frac{\dot{Q}_H}{\dot{W}_{net,in}} = \frac{110}{50} = 2.2$$

$$Or \;\; COP_{HP} = COP_R + 1 = 1.2 + 1 = 2.2$$

<u>Ex:</u> A household refrigerator that has a power input of 450 W and a COP of 2.5 is to cool five large watermelons, 10 kg each, to 8°C. If the watermelons are initially at 20°C, determine how long it will take for the refrigerator to cool them. The watermelons can be treated as water whose specific heat is 4.2 kJ/kg.

Sol:

$$\dot{W} = 450W$$

$$COP_R = 2.5$$

$$Q_L = (m\,c\,\Delta T)_{watermilons} = 5*10*4.2*(20-8) = 2520\,kJ$$

$$COP_R = \frac{\dot{Q}_L}{\dot{W}}$$

$$\dot{Q}_L = COP_R\;\dot{W} = 2.5*450 = 1125\;W$$

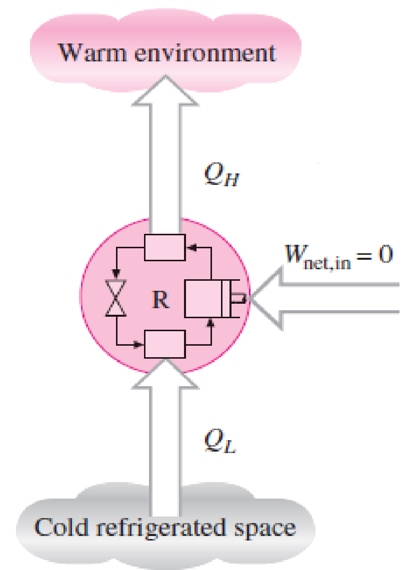
$$\dot{Q}_L = \frac{Q_L}{\Delta t}$$

$$\Delta t = \frac{Q_L}{\dot{Q}_L} = \frac{2520*1000}{1125} = 2240\,s = 37.33\,\text{min}$$

The Second Law of Thermodynamics (Clausius Statement)

"It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher-temperature body"

That is, the input work is required to transfer heat from a cold medium to a warmer one.



Impossible

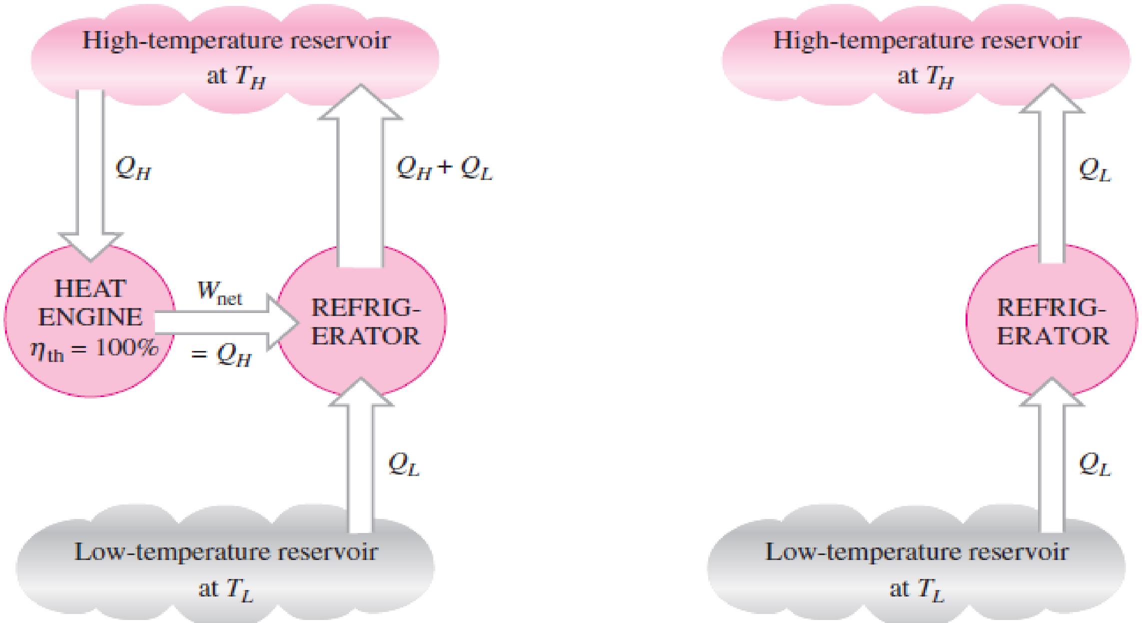
Equivalence of the Two Statements

The Kelvin–Planck and the Clausius statements are equivalent and any device that violates the Kelvin–Planck statement also violates the Clausius statement, and vice versa.

Consider the heat-engine-refrigerator combination shown in Fig. (a) operating between the same two reservoirs where:

- Violation Kelvin-Plank statement by (Q_L=0, W_{net}=Q_H and $\eta_{th} = 100\%$) and the work output is used to operate refrigerator.
- Refrigerator that removes heat in the amount of Q_L from the low-temperature reservoir and rejects heat in the amount of (Q_L+Q_H) to the high-temperature reservoir. During this process, the high-temperature reservoir receives a net amount of heat Q_L (the difference between Q_L+Q_H and Q_H)
- The combination of these two devices can be viewed as a refrigerator, as shown in Fig. (b), that transfers heat in an amount of Q_L from a cooler body to a warmer one without requiring any input from outside (the Clausius statement)

Therefore, a violation of the Kelvin–Planck statement results in the violation of the Clausius statement.



(*a*) A refrigerator that is powered by a 100 percent efficient heat engine

(*b*) The equivalent refrigerator

Exercise: Starting with violation of the Clausius statement and prove both statements are equivalent.