Dr Dhirgaam A. Kadhim

College of Engineering

Kerbala University

SIGNAL-FLOW GRAPH MODELS AND MASON'S RULE

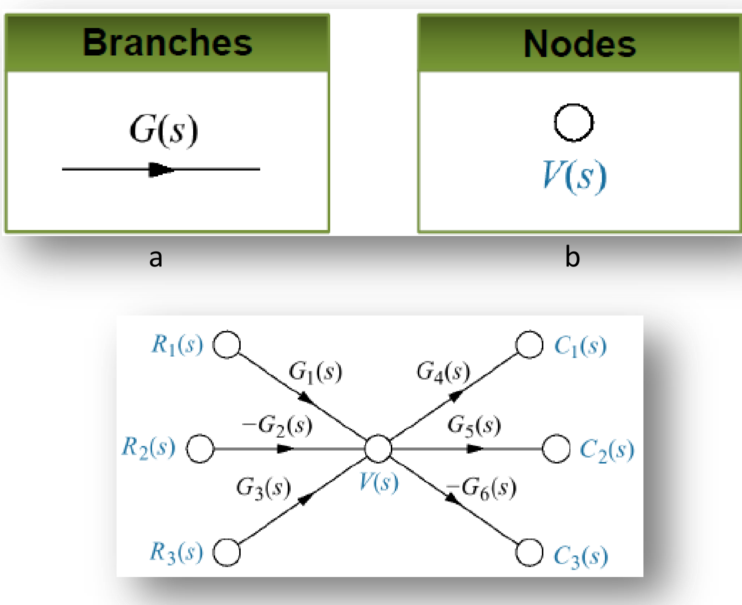
1. *Why we use the signal flow graph (SFG) method if we have the block diagram method?*

For a system with reasonably complex interrelationships, the ***block diagram*** reduction procedure is cumbersome and often quite difficult to complete, so ***signal flow graph (SFG)*** method can be used to determine the relationship between system variables that's based on a representation of the system by line segments.

2. *The advantage of the SFG*

Is the availability of a flow graph gain formula, which provides the relation between system variables without requiring any reduction procedure or manipulation of the flow graph.

A signal-flow graph (SFG) is a diagram consisting tow part: branches (systems), and nodes (signals) that are connected by several directed branches as shown in the following figure



c

Figure 1. Signal-flow graph component parts: (a) system; (b) signals; (c) interconnection of systems and signals.

3. ***Illustration of the figure 1***

✓ Figure 1a is representing a directed line segment joining two nodes. By this is line with an arrow showing the direction of signal flow through the system. Adjacent to the line we write the transfer function.

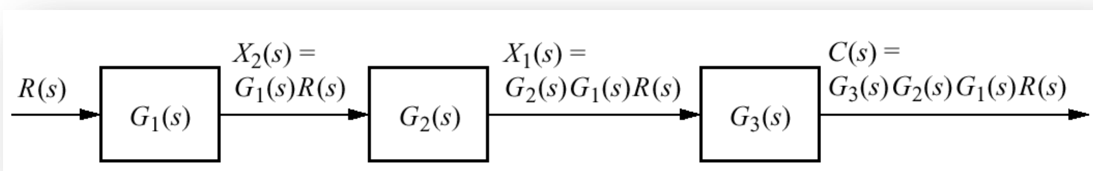
- ✓ In Figure 1b a signal is a node with the signal name written adjacent to the node. The node here having multi outgoing/ incoming branch, While in some problems node having only one outgoing/ incoming branch.
- ✓ Figure 1c shows the interconnection of the systems and the signals Each signal is the sum of signals flowing into it For example, in Figure 1c the signal $X(s)=R_1(s)G_1(s) - R_2(s)G_2(s)+R_3(s)G_3(s)$.The signal,$G3(s) = -X(s)\ G_6(s)= - R_1(s)G_1(s)G_6(s)+R_2(s)G_2(s)G6(s)-R_3(s)G_3(s)G_6(s)$.

  Notice: The summing of negative signals is handled by associating the negative sign with the system and not with a summing junction as in the case of block diagrams.

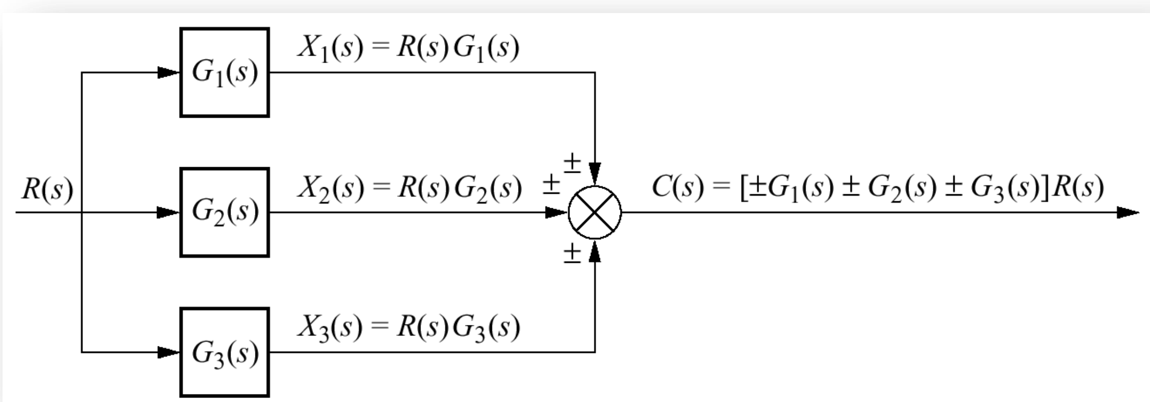
4. ***Relationship Between Block Diagrams and Signal-Flow Graphs***

To show the parallel between block diagrams and signal-flow graphs, we convert some block diagram forms to signal-flow graphs, In each case, we first convert the signals (system variables) to nodes and then interconnect the nodes with systems.

Example 1 Convert the cascaded, parallel, and feedback forms of the block diagrams shown in figures below, respectively, into signal-flow graphs.



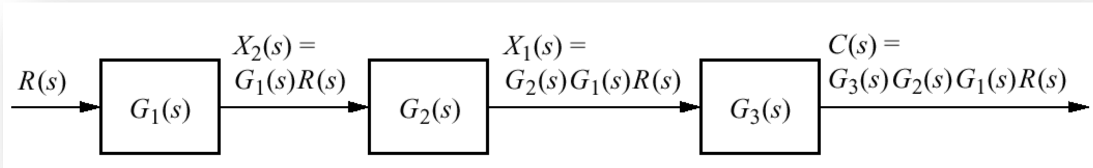
(a)



(b)

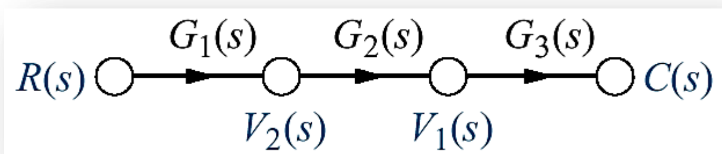
Solution 1:

a-

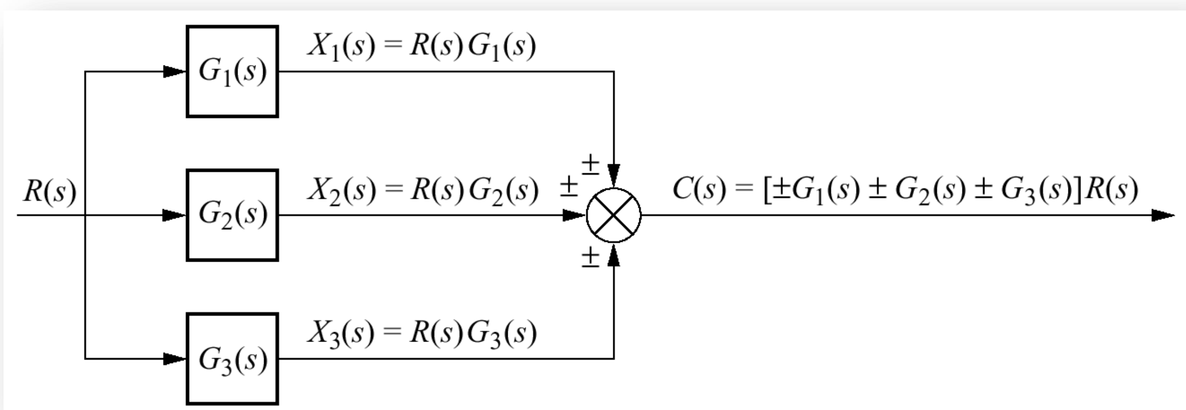


1. *Draw the signal nodes for the system:*

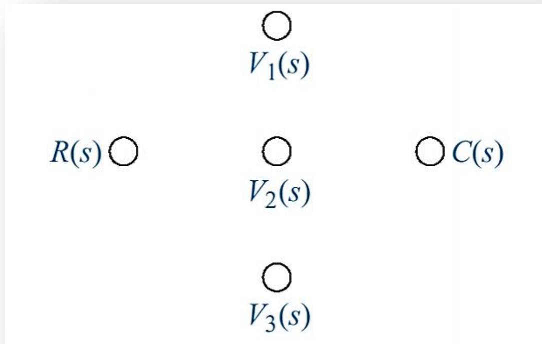
2. *Interconnect the signal codes with system branches :*



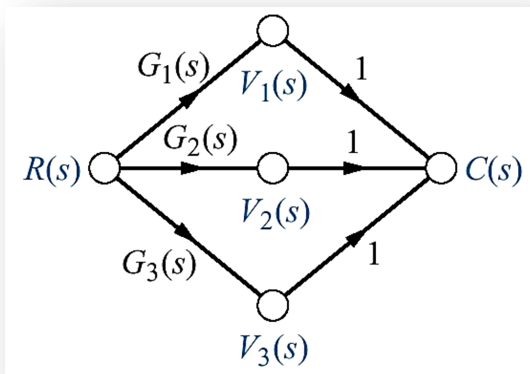
b-



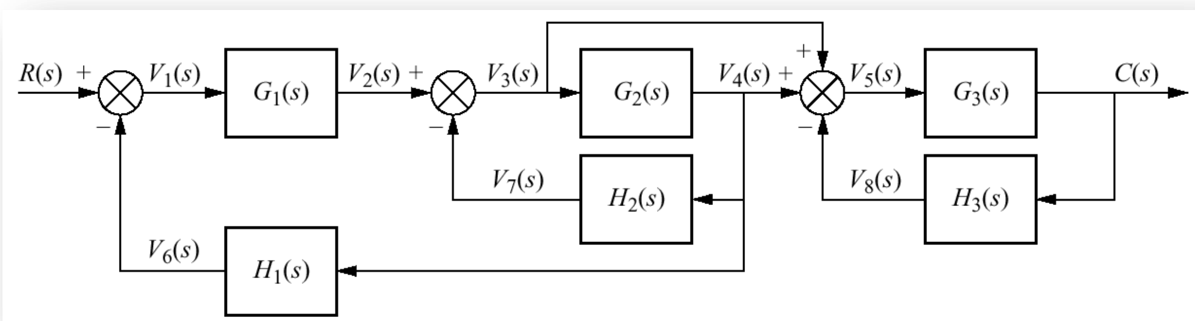
1. *Draw the signal nodes for the system:*



2. *Interconnect the signal codes with system branches :*

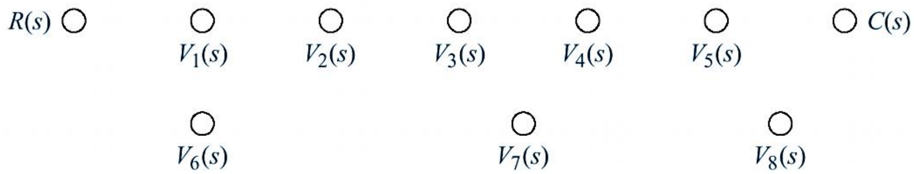


Example 2 Convert the block diagram (***Multiple-loop system***)of figure below to a signal-flow graph.

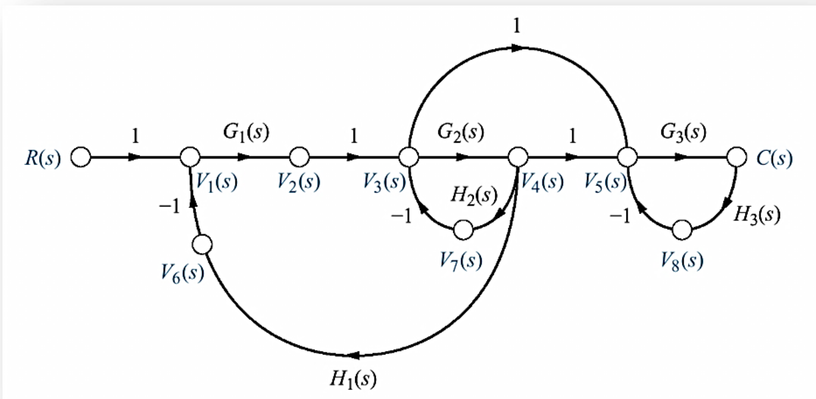


Solution 2:

1. *Draw the signal nodes for the system:*



2. *Interconnect the signal codes with system branches :*



5. Mason's Rule

Mason's rule is a method can be used for finding the overall transfer function of a system through inspection of a linear signal-flow graph (***SFG)*** using Mason’s gain formula ***(MGF)*** given by :

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

$$\Delta = 1 - \sum_{i=1}^{n} L_i + \sum_{\substack{i,j \\ nontouching}} L_i L_{ij} - \sum_{\substack{j,i,p \\ nontouching}} L_i L_j L_p + \cdots,$$

Where

- G= overall gain of the system.
- R = input-node variable.
- C= output-node variable.
- Δ = determinant of SFG.
- k = number of forward path.
- T_k = the kth forward path gain.
- Δ = 1 – (∑ loop gains) + (∑ non-touching loop gains taken two at a time) – (∑non-touching loop gains taken three at a time).
- Δ_k = 1 – (loop-gain which does not touch the forward path).

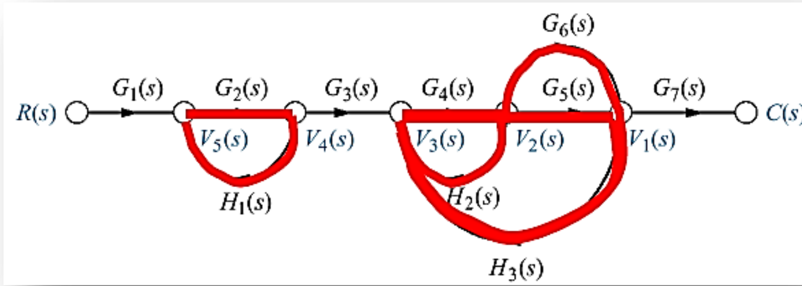
*5.1 **Procedure***

To use this technique,

1. Make a list of all forward paths, and their gains, and label these T_k.
2. Make a list of all the loops and their gains. Make a list of all pairs of non-touching loops,. Make a list of all pairwise non-touching loops taken three at a time, then four at a time, and so forth, until there are no more.
3. Compute the determinant Δ and cofactors Δ_k.
4. Apply the formula.

*5.2 **Mason's Rule – definitions***

- ***Loop gain*** **–** the product of branch gains found by traversing a path that starts at a node and ends at the same node.



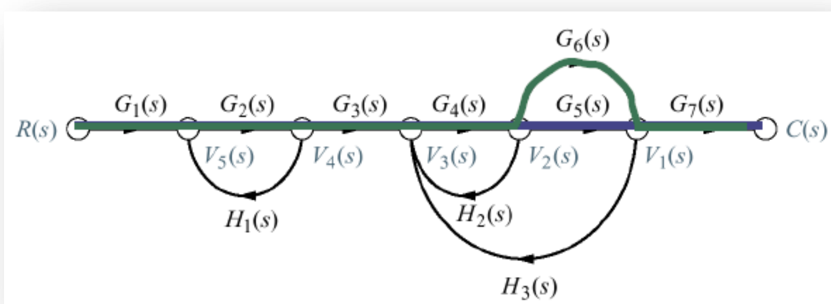
L1. $G_2(s)H_1(s)$

L2. $G_4(s)H_2(s)$

L3. $G_4(s)G_5(s)H_3(s)$

L4. $G_4(s)G_6(s)H_3(s)$

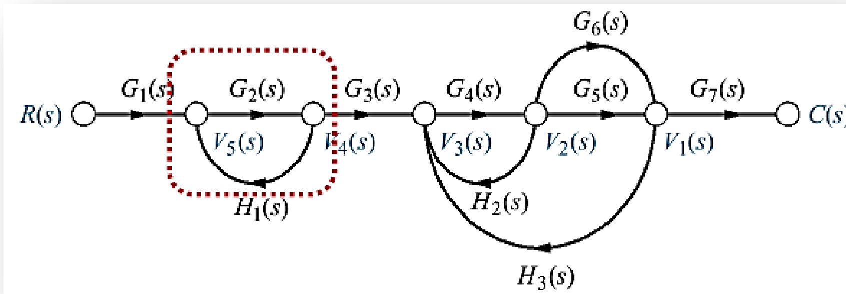
- ***Forward path*** – the product of gains found by traversing a path from the input node to the output node of the signal-flow graph



F1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$

F2. $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$

- ***Non-touching loop*** – Loops that do not have any nodes in common.
- ***Non-touching loop gain*** – The product of loop gains from non-touching loops taken two, three, four, or more at a time.



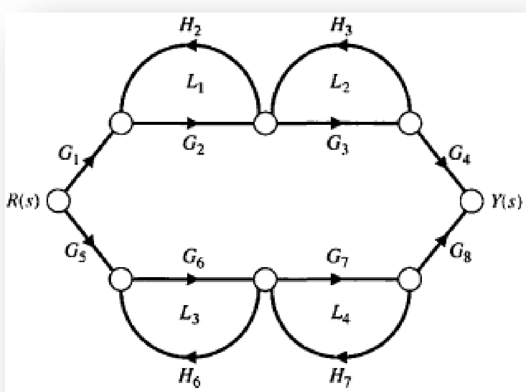
NT1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$

NT2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$

NT3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

Example 3 Find the transfer function of an interacting system A two-path signal-flow graph is shown in Figure a and the corresponding block diagram is shown in Figure b. An example of a control system with multiple signal paths is a multilegged robot. The paths connecting the input R(s) and output Y(s) are

$$T_1 = G_1G_2G_3G_4 \text{ (path 1)} \quad \text{and} \quad T_2 = G_5G_6G_7G_8 \text{ (path2)}$$



(a)

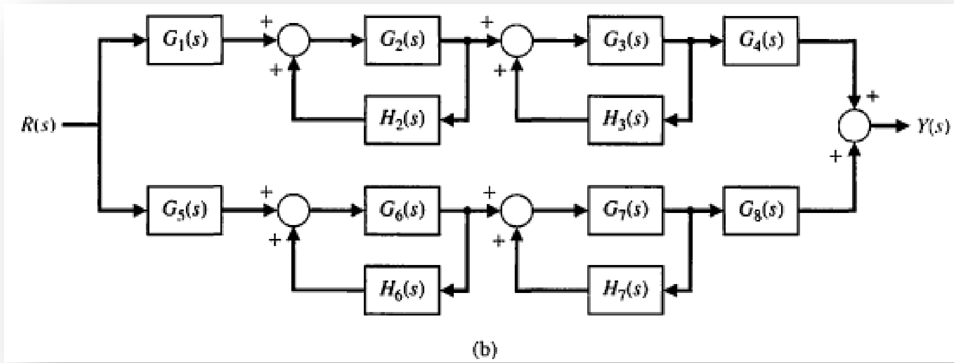


Figure 2. Two-path interacting system, (a) Signal-flow graph, (b) Block diagram.

There are four self-loops:

$L_1 = G_2H_2$, $L_2 = H_3G_3$, $L_3 = G_6H_6$, and $L_4 = G_7H_7$.

Loops L_1 and L_2 do not touch L_3 and L_4. Therefore, the determinant is

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4).$$

The cofactor of the determinant along path 1 is evaluated by removing the loops that touch path 1 from A. Hence, we have

$$L_1 = L_2 = 0 \quad \text{and} \quad \Delta_1 = 1 - (L_3 + L_4).$$

Similarly, the cofactor for path 2 is

$$\Delta_2 = 1 - (L_1 + L_2).$$

Therefore, the transfer function of the system is

$$\frac{Y(s)}{R(s)} = G(s) = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4}$$

A similar analysis can be accomplished using block diagram reduction techniques. The block diagram shown in Figure 2(b) has four inner feedback loops within the overall block diagram. The block diagram reduction is simplified by first reducing the four inner feedback loops and then placing the resulting systems in series. Along the top path, the transfer function is

$$Y_1(s) = G_1(s)\left[\frac{G_2(s)}{1 - G_2(s)H_2(s)}\right]\left[\frac{G_3(s)}{1 - G_3(s)H_3(s)}\right]G_4(s)R(s)$$
$$= \left[\frac{G_1(s)G_2(s)G_3(s)G_4(s)}{(1 - G_2(s)H_2(s))(1 - G_3(s)H_3(s))}\right]R(s).$$

Similarly across the bottom path, the transfer function is

$$Y_2(s) = G_5(s)\left[\frac{G_6(s)}{1 - G_6(s)H_6(s)}\right]\left[\frac{G_7(s)}{1 - G_7(s)H_7(s)}\right]G_8(s)R(s)$$
$$= \left[\frac{G_5(s)G_6(s)G_7(s)G_8(s)}{(1 - G_6(s)H_6(s))(1 - G_7(s)H_7(s))}\right]R(s).$$

The total transfer function is then given by

$$Y(s) = Y_1(s) + Y_2(s) = \left[\frac{G_1(s)G_2(s)G_3(s)G_4(s)}{(1 - G_2(s)H_2(s))(1 - G_3(s)H_3(s))}\right.$$
$$\left. + \frac{G_5(s)G_6(s)G_7(s)G_8(s)}{(1 - G_6(s)H_6(s))(1 - G_7(s)H_7(s))}\right]R(s)$$

$$\frac{Y(s)}{R(s)} = G(s) = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4}$$

$$\frac{Y(s)}{R(s)} = G(s) = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} = \frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4}$$

$$\frac{Y(s)}{R(s)} = G(s) = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} =$$

$$\frac{G_1G_2G_3G_4(1 - L_3 - L_4) + G_5G_6G_7G_8(1 - L_1 - L_2)}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4}$$

$$\Delta = 1 - \sum_{i=1}^{n} L_i + \sum_{\substack{i,j \\ nontouching}} L_i L_{ij} - \sum_{\substack{j,i,p \\ nontouching}} L_i L_j L_p + \cdots,$$

$$\Delta = 1 - \sum_{i=1}^{n} L_i + \sum_{\substack{i,j \\ nontouching}} L_i L_{ij} - \sum_{\substack{j,i,p \\ nontouching}} L_i L_j L_p + \cdots,$$

$$\Delta = 1 - \sum_{n=1}^{N} L_n + \sum_{\substack{n,\, m \\ \text{nontouching}}} L_n L_m - \sum_{\substack{n,\, m,\, p \\ \text{nontouching}}} L_n L_m L_p + \cdots,$$