Bernoulli's Equation:

It is a special case of the (E.E.) in which $(\mathbf{h}_{\mathbf{L}_{1-2}} = 0)$, (frictionless flow) and no work exchange:

$$\textbf{Thus}: \left(\frac{\mathbf{P1}}{\boldsymbol{\gamma}} + \frac{\mathbf{V1}^2}{\mathbf{2g}} + \mathbf{Z1}\right) = \left(\frac{\mathbf{P2}}{\boldsymbol{\gamma}} + \frac{\mathbf{V2}^2}{\mathbf{2g}} + \mathbf{Z2}\right)$$

General Procedure for E.E. Application:

1. Section (1) is the upstream and (2) is the downstream.
2. Each term in the E.E. represents $\frac{energy}{weight} = \frac{N.m}{m} = (m)$ or it's called the head h(m).

3.
$$\left.\begin{array}{l} \frac{\mathbf{P}}{\boldsymbol{\gamma}} = \text{Flow energy head} \\ \frac{\mathbf{v}^2}{\mathbf{2g}} = \text{Kinetic energy head, } \mathrm{V} = \mathrm{V}_{\text{ave.}} \\ \mathrm{z} = \text{Potential head} \end{array}\right\} \text{Total Head}$$

$h_P = Pump\ Head$

$h_T = Turbine\ Head$

$h_L = Losses\ Head$

$$Total\ head = \frac{\mathrm{P}}{\gamma} + \frac{\mathrm{V}^2}{2\mathrm{g}} + \mathrm{Z}$$

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4. Flow direction $[\![h_{total}\text{High} \longrightarrow h_{total}\text{Low}]\!]$

5. Power $= P = \frac{E}{t} = \frac{E}{W}.\frac{W}{t} = \frac{E}{W}.\frac{m}{t}.g$

   Also, Power $= \dot{m}.g.h = \rho.Q.g.h = \gamma Qh$

6. Pump efficiency $\eta_P = \frac{output\ power}{Input\ power} = \frac{\gamma Qh_P}{I_P}$

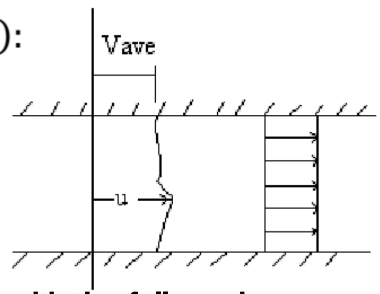
7. Turbine efficiency $\eta_T = \frac{output\ power}{Input\ power} = \frac{\text{out power}}{\gamma Qh_T}$

8. Kinetic energy correction factor (α):

   Where, $\frac{Vave^2}{2g} \neq \left(\frac{u^2}{2g}\right) ave$

   $\alpha \times \frac{1}{2}\dot{m}v_{ave}^2 = \int \frac{1}{2}d\dot{m}\,u^2$

   $\Longrightarrow \alpha = \frac{1}{A}\int (\frac{u}{V_{ave}})^3\,dA$



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3.5 Momentum Equation: (M.E.)

Let, B= mv and, $b = \frac{B}{m} = \frac{mv}{m} = v$

Thus:

$$\frac{d(mv)_{sys}}{dt} = \frac{\partial}{\partial t}\int_{C.V.} \rho.V.d\forall + \int_{C.S.} \rho.V.V.dA$$

But, we have by the Newton's 2^{nd} law,

$$\sum F = m.a = m.\frac{dv}{dt} = \frac{d(mv)_{sys}}{dt}$$

$$\Rightarrow \sum F = \frac{\partial}{\partial t}\int_{C.V.} \rho.V.d\forall + \int_{C.S.} \rho.V.V.dA \quad \text{This is the G.M.E}$$

Or,

$$\sum F = \frac{\partial}{\partial t}(mv)_{c.v.} + (\dot{m}v)_{out} - (\dot{m}v)_{in}$$

And for steady flow, $\frac{\partial}{\partial t}(mv)_{c.v.} = 0$

Thus:

$$\sum \mathbf{F} = (\dot{\mathbf{m}}\mathbf{v})_{\mathbf{out}} - (\dot{\mathbf{m}}\mathbf{v})_{\mathbf{in}} \qquad \textbf{Steady Flow M.E.}$$

And for one inlet and one outlet, or $(\dot{\mathbf{m}})_{\mathbf{out}} - (\dot{\mathbf{m}})_{\mathbf{in}}$

$$\Rightarrow \sum \mathbf{F} = \dot{\mathbf{m}}(\mathbf{v}_{\mathbf{out}} - \mathbf{v}_{\mathbf{in}})$$

Or:

$$\sum \mathbf{F_x} = \dot{\mathbf{m}}\left(\mathbf{v}_{\mathbf{x_{out}}} - \mathbf{v}_{\mathbf{x_{in}}}\right) \quad \text{and} \quad \sum \mathbf{F_y} = \dot{\mathbf{m}}\left(\mathbf{v}_{\mathbf{y_{out}}} - \mathbf{v}_{\mathbf{y_{in}}}\right)$$

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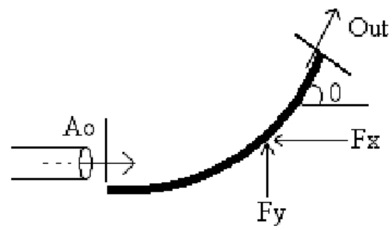
3.5.1: Application of M.E.

I. Forces on Blades (Vanes)

A. Fixed Vanes



Assumptions:

- Steady
- Incompressible
- Frictionless
- $Z_{in} = Z_{out}$

Now, apply B.E. from **in** to **out**

$$\frac{P1}{\gamma} + \frac{Vin^2}{2g} + Z1 = \frac{P2}{\gamma} + \frac{Vout^2}{2g} + Z2$$

$But, P1 = P2 = 0 \text{ Atm.}$

$and, Z1 = Z2, \text{Horizontal Blade}$

$\Rightarrow \text{Vin} = \text{Vout} = V$

Then, by applying M.E.

$$\overset{\rightarrow +}{\sum} Fx = \dot{m}\left(v_{x_{out}} - v_{x_{in}}\right) \implies -Fx = \rho VA(V\cos\theta - V)$$

$$\mathbf{Fx = \rho AV^2(1 - \cos\theta)}$$

$$\overset{\uparrow +}{\sum} \mathbf{Fy} = \dot{m}\left(v_{y_{out}} - v_{y_{in}}\right) \Rightarrow \ Fy = \rho VA(V\sin\theta - 0)$$

$$\mathbf{Fy = \rho AV^2 Sin\theta}$$

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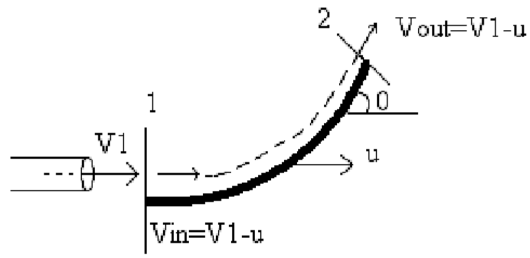
B. Moving Vanes



- Single moving vanes:

$$\sum F = \dot{m}_r(Vr_{out} - Vr_{in})$$

Where, Vr is the relative velocity $= V1 - u$

And, $\dot{m}_r = \rho Q_r = \rho . A . Vr = \rho . A . (V1 - u)$

$$\Rightarrow \sum \mathbf{Fx} = \dot{\mathbf{m}}_\mathbf{r}\left(\mathbf{Vr}_{\mathbf{x}_{\mathbf{out}}} - \mathbf{Vr}_{\mathbf{x}_{\mathbf{in}}}\right) \quad \textit{and,}$$

$$\sum \mathbf{Fy} = \dot{\mathbf{m}}_\mathbf{r}\left(\mathbf{Vr}_{\mathbf{y}_{\mathbf{out}}} - \mathbf{Vr}_{\mathbf{y}_{\mathbf{in}}}\right)$$

Or,

$$\mathbf{F_x = \rho A(V1 - u)^2(1 - cos\theta)} \quad \textit{and,}$$

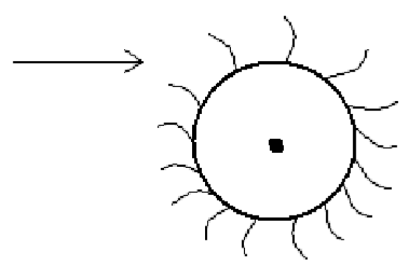
$$\mathbf{F_y = \rho A(V1 - u)^2 sin\theta}$$

- Power delivered by the Vane (P):

$$\mathbf{P = F_x . u = \gamma Q_r \frac{V_1^2 - V_2^2}{2g}}$$

- Kinetic Energy remaining in jet= $\mathbf{\gamma Q_r \frac{V_2^2}{2g}}$

- Series of moving vanes:



$$\sum F = \dot{m}_{abs}(Vr_{out} - Vr_{in})$$

$$P = F_x . u = \gamma Q_{abs} \frac{V_1^2 - V_2^2}{2g}$$

$$\text{K.E. remaining in jet} = \gamma Q_{abs} \frac{V_2^2}{2g}$$

II. Forces on Pipe Bends

Given: P1,V1,A1,Z,A2,θ, h_{l1-2}, W_b *and*, $\forall_b$

Find: Fx and Fy

Ans:

Apply C.E. to get: A1V1=A2V2 $\Longrightarrow$ V2

Apply E.E.1-2 to get:

$$\frac{P1}{\gamma} + \frac{Vin^2}{2g} + Z1 = \frac{P2}{\gamma} + \frac{Vout^2}{2g} + Z2 + h_{l1-2} \quad \Longrightarrow P2,$$

where $Z1 = Z2$

Then, apply M.E. to get:

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$$\sum^{\rightarrow +} Fx = \dot{m}\left(v_{x_{out}} - v_{x_{in}}\right)$$

$$-Fx + P1A1 - P2A2\cos\theta = \dot{m}(V_2 Cos\theta - V_1) \ \Longrightarrow Fx$$

$$\sum^{\uparrow +} Fy = \dot{m}\left(v_{y_{out}} - v_{y_{in}}\right)$$

$$F_y - W_b - W_{fluid} - P2A2Sin\theta = \rho V1A1(V2Sin\theta - 0) \Longrightarrow Fy$$

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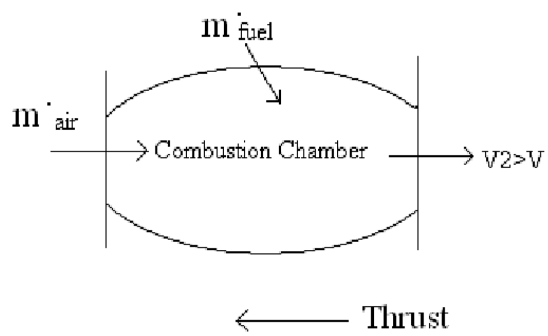
III. Jet Engine:

$$\mathbf{F_{th}} = (\dot{m}v)_{out} - (\dot{m}v)_{in}$$
$$= (\dot{m}_{air} + \dot{m}_{fuel})V2 - \dot{m}_{air}V1$$
$$= \dot{m}_{air}\left[\left(1 + \frac{\dot{m}_{fuel}}{\dot{m}_{air}}\right)V2 - V1\right]$$



V2=Exit velocity

V1=Aircraft velocity

And, $\frac{\dot{m}_{fuel}}{\dot{m}_{air}} =$ f = Fuel consumption ratio

IIII. Rocket Machine:

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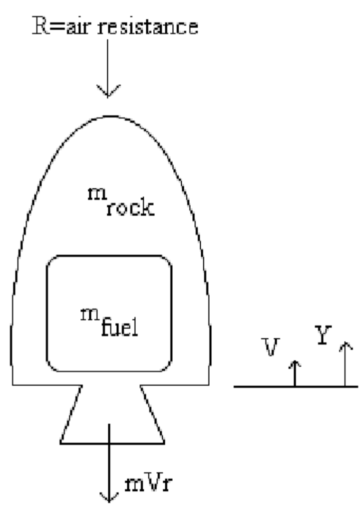
$$\sum F = \frac{\partial}{\partial t}(mv)_{c.v.} + (\dot{m}v)_{out} - (\dot{m}v)_{in}$$



But, $\dot{m}_{in} = 0$

So,

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$$-R-(m_{rock}+m_{fuel})g \;=\frac{\partial}{\partial t}\,(m_{rock}+m_{fuel})V1+(-\dot{m}V_r)$$

Since (V) is a function of (t) only, the equation can be written as a total differential equation:

$$\frac{dV}{dt}=\frac{dV1}{dt}=\frac{\dot{m}V_r-R-(m_{rock}+m_{fuel})g}{(m_{rock}+m_{fuel})}$$

which represents the acceleratio

where:

m_{rock} = Rocket mass

m_{fuel} = Fuel mass = $m_{f_o}-\dot{m}\;t$

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m_{f_o} = intial mass of fuel, $\dot{m}$ = rate of burning

$\dot{m}V_r$ = Thrust of rocket (F_{th})

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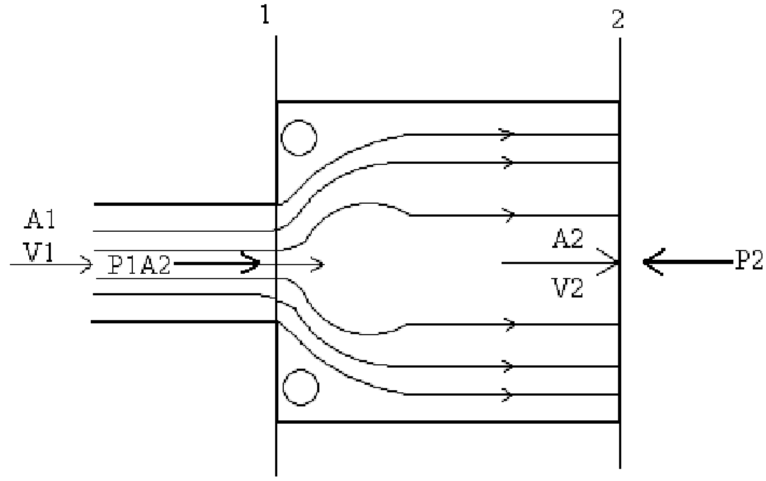
3.6 Losses due to Sudden Expansion



E.E. 1-2

$$\left(\frac{P1}{\gamma}+\frac{V1^2}{2g}+Z1\right)=\left(\frac{P2}{\gamma}+\frac{V2^2}{2g}+Z2\right)+h_e$$

For, Z1=Z2

$$h_e=\frac{P1-P2}{\gamma}+\frac{V1^2-V2^2}{2g} \quad \ldots\ldots\ldots(1)$$

Then, by applying C.E. so, A1V1=A2V2 ……….(2)

$$\text{Applying M.E.} \sum F_x=\dot{m}(V2-V1)$$

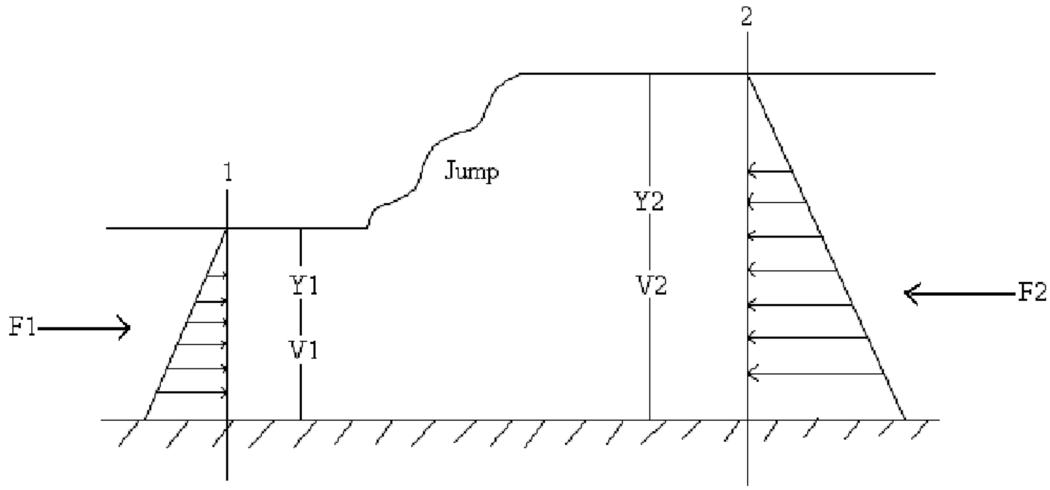
$$\Longrightarrow P1A2-P2A2=\rho A2V2^2-\rho A1V1^2 \quad \ldots\ldots\ldots(3)$$

By 1,2 and,3 Get:

$$\mathbf{h_e=\frac{(V1-V2)^2}{2g}=\frac{V1^2}{2g}\left(1-\frac{A1}{A2}\right)^2=\frac{V1^2}{2g}\left(1-\left(\frac{D1}{D2}\right)^2\right)^2}$$

Also, the power required to overcome the expansion losses is equal to: ($\boldsymbol{\gamma Q h_e}$)

3.7 Hydraulic Jump:



E.E. 1-2

$$\left(\frac{V1^2}{2g} + Y1\right) = \left(\frac{V2^2}{2g} + Y2\right) + h_J \qquad \ldots\ldots\ldots\ldots..(1)$$

$$C.E. \Longrightarrow Y1V1 = Y2V2 \qquad \ldots\ldots(2)$$

$$M.E. \Longrightarrow \ F1 - F2 = \dot{m}(V2 - V1)$$

$$\text{Or: } \gamma.\frac{Y1}{2}.Y1 - \gamma.\frac{Y2}{2}.Y2 = \rho.Y2.V2^2 - \rho.Y1.V1^2 \ \ldots\ldots\ldots.(3)$$

Then by 1,2, and 3 get: $\mathbf{h_J} = \dfrac{(\mathbf{Y2} - \mathbf{Y1})^3}{\mathbf{4Y1Y2}}$

$$\text{And, } Y2 = -\frac{Y1}{2} + \sqrt{\left(\frac{Y1}{2}\right)^2 + \frac{2V1^2Y1}{g}}$$

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Example (1):

3.97 Air flows from a hole of diameter 0.03 m in a flat plate as shown in Fig. P3.97. A circular disk of diameter D is placed a distance h from the lower plate. The pressure in the tank is maintained at 1 kPa. Determine the flowrate as a function of h if viscous effects and elevation changes are assumed negligible and the flow exits radially from the circumference of the circular disk with uniform velocity.

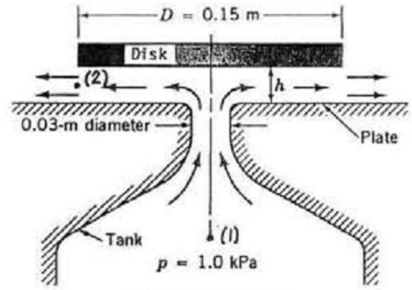

FIGURE P3.97

$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } p_0 = 1\,\frac{kN}{m^2},\ p_2 = 0,\ z_0 \approx z_2, \text{ and } V_0 = 0$$

Thus,

$$V_2 = \sqrt{\frac{2p_0}{\rho}} = \sqrt{\frac{2\left(1\times10^3\,\frac{N}{m^2}\right)}{1.23\,\frac{kg}{m^3}}} = 40.3\,\frac{m}{s}$$

so that

$$Q = A_2 V_2 = \pi D_2 h V_2 = \pi(0.15m)h\left(40.3\,\frac{m}{s}\right)$$

or

$$Q = \underline{\underline{19.0\,h\,\frac{m^3}{s}}} \quad \text{where } h \sim m$$

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Example (2):

3.101 Water flows down the sloping ramp shown in Fig. P3.101 with negligible viscous effects. The flow is uniform at sections (1) and (2). For the conditions given show that three solutions for the downstream depth, h_2, are obtained by use of the Bernoulli and continuity equations. However, show that only two of these solutions are realistic. Determine these values.

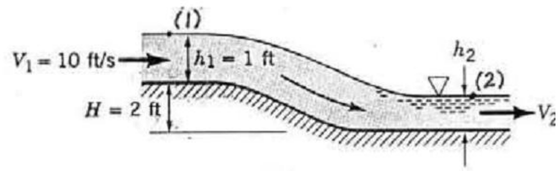

FIGURE P3.101

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $p_1 = 0$, $p_2 = 0$, $z_1 = 3\,ft$, and $z_2 = h_2$

Also, $A_1 V_1 = A_2 V_2$

or

$$V_2 = \frac{h_1}{h_2} V_1 = \frac{(1\,ft)(10\,\frac{ft}{s})}{h_2} = \frac{10}{h_2}$$

Thus, Eq. (1) becomes

$$\frac{(10\,\frac{ft}{s})^2}{2\,(32.2\,\frac{ft}{s^2})} + 3\,ft = \frac{\left(\frac{10}{h_2}\right)^2}{2(32.2\,\frac{ft}{s^2})} + h_2$$

or

$$64.4\,h_2^3 - 293\,h_2^2 + 100 = 0$$

By using a root finding program the three roots to this cubic equation are found to be:

$h_2 = 0.630\,ft$

$h_2 = 4.48\,ft$

or

$h_2 =$ a negative root

Clearly it is not possible (physically) to have $h_2 < 0$ Thus, $h_2 = \underline{\underline{0.630\,ft}}$ or $h_2 = \underline{\underline{4.48\,ft}}$

Example (3):

5.41 Water enters the horizontal, circular cross-sectional, sudden contraction nozzle sketched in Fig. P5.41 at section (1) with a uniformly distributed velocity of 25 ft/s and a pressure of 75 psi. The water exits from the nozzle into the atmosphere at section (2) where the uniformly distributed velocity is 100 ft/s. Determine the axial component of the anchoring force required to hold the contraction in place.

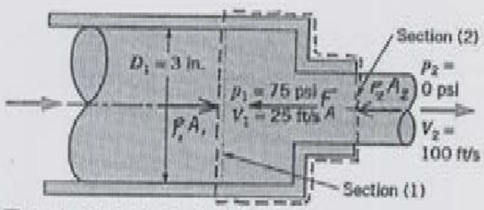

FIGURE P5.41

For this problem we include in the control volume the nozzle as well as the water at an instant between sections (1) and (2) as indicated in the sketch above. The horizontal forces acting on the contents of the control volume are shown in the sketch. Note that the atmospheric forces cancel out and are not shown. Application of the horizontal or x-direction component of the linear momentum equation (Eq. 5.22) to the flow through this control volume yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = P_1 A_1 - F_A - P_2 A_2 \qquad (1)$$

From the conservation of mass equation (Eq. 5.12) we obtain

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2$$

Thus Eq. (1) may be expessed as

$$\dot{m}(u_2 - u_1) = P_1 A_1 - F_A - P_2 A_2$$

or

$$F_A = P_1 A_1 - P_2 A_2 + \dot{m}(u_2 - u_1) = P_1 \frac{\pi}{4} D_1^2 - P_2 \frac{\pi D_2^2}{4} - \rho u_1 \frac{\pi}{4} D_1^2 (u_2 - u_1)$$

and

$$F_A = \left(75 \frac{lb}{in^2}\right) \frac{\pi (3\,in.)^2}{4} - 0\,lb - \left(1.94 \frac{slugs}{ft^3}\right)\left(25 \frac{ft}{s}\right) \frac{\pi (3\,in.)^2}{4} \frac{\left(100 \frac{ft}{s} - 25 \frac{ft}{s}\right)}{\left(144 \frac{in.^2}{ft^2}\right)} \left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right)$$

$$F_A = \underline{\underline{352}}\ lb$$

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