2.7 Stability of Immersed and Floating Bodies:

- **Immersed Bodies**

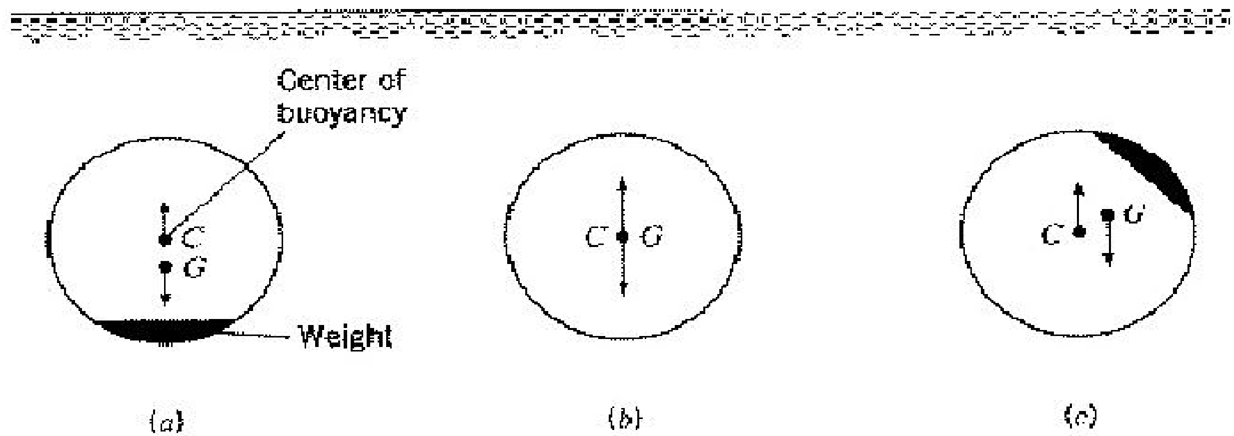

FIGURE 3.15
Conditions of stability for immersed bodies. (a) Stable. (b) Neutral. (c) Unstable.

If C is above G ⇒ body is stable

If G is above C ⇒ body is unstable

If G ≡ C ⇒ body is newtral

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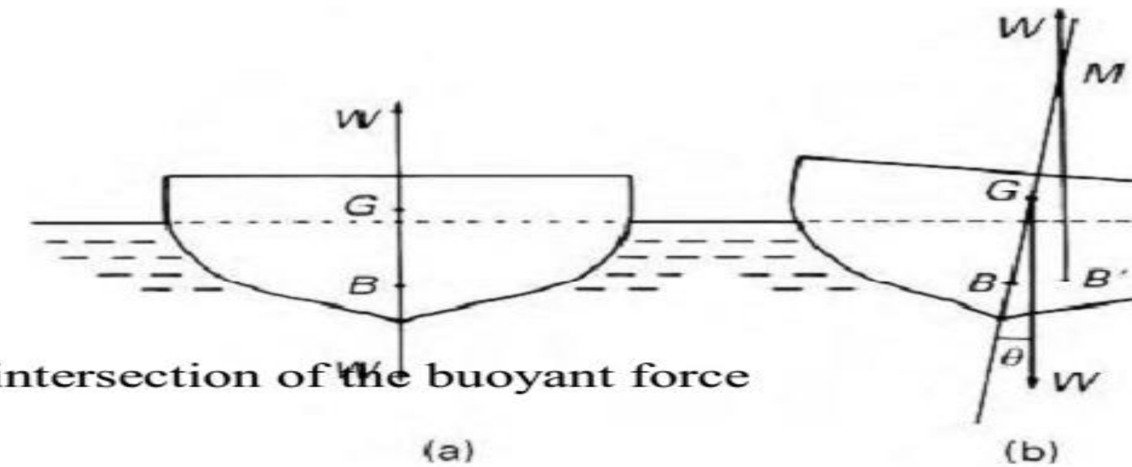
- **Floating bodies:**

1. **Linear Stability:** Any small linear displacement in any direction will set-up a restarting force tends to return the body to its original position.

2. **Rotational stability:**



Metacenter: It is the point of intersection of the buoyant force before and after the heel.

Metacentric Height: It is the distance (**MG**)

$MG > 0$ {M above G} $\Longrightarrow$ Stable

$MG < 0$ {M below G} $\Longrightarrow$ UnStable

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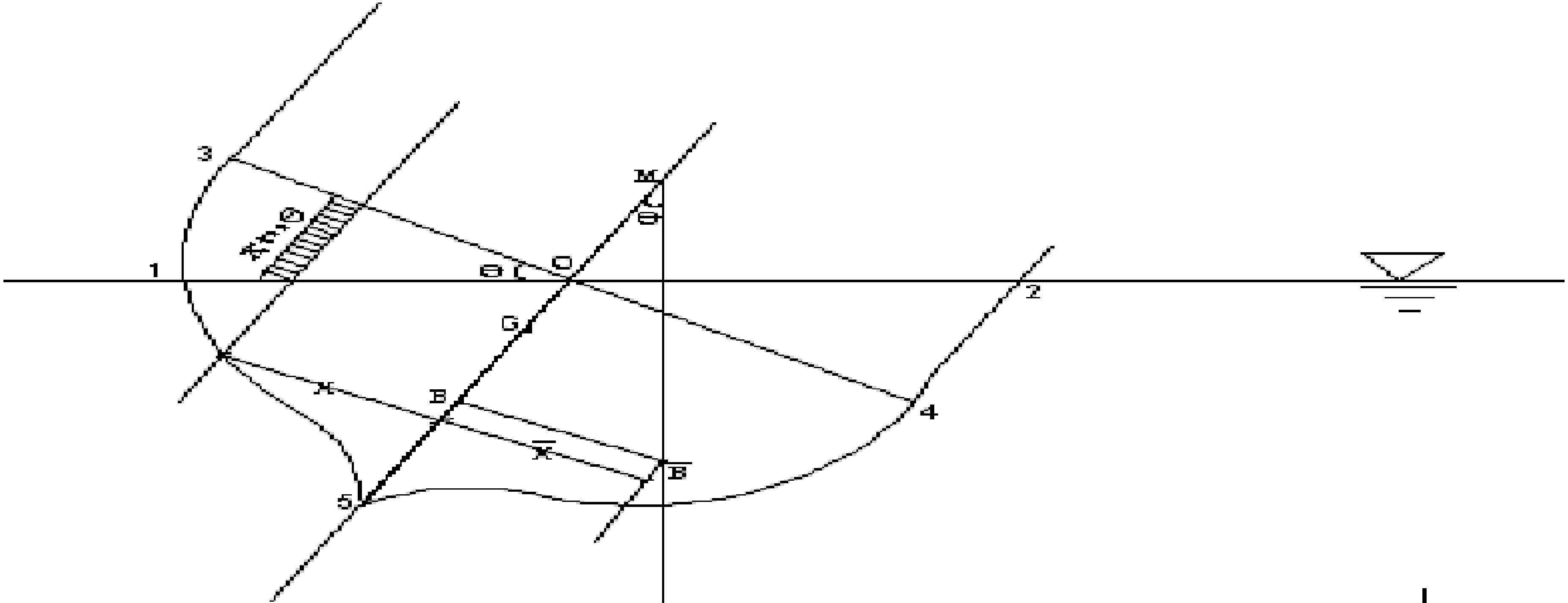

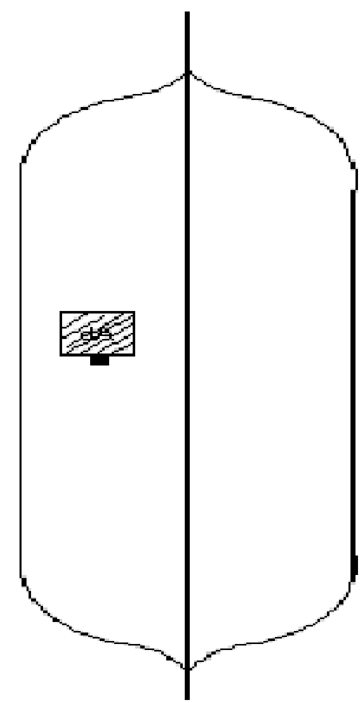

$$\overline{\mathrm{MG}} = \overline{\mathrm{MB}} - \overline{\mathrm{BG}}$$

$$\overline{\mathrm{MB}} = \frac{\bar{X}}{\tan\theta}$$

Now by moment of the areas:

$$\bar{X}\forall_{102451} = \bar{X}\forall_{024} + \bar{X}\forall_{045} - [(\bar{X}\forall_{035}) - (\bar{X}\forall_{031})]$$

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$$= \int x \cdot dv = \int x \cdot x\tan\theta \cdot dx$$

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$$\Longrightarrow \; \overline{X}\forall = \tan\theta \int x^2 . dA = \tan\theta \, I_{oo}$$

So, $\overline{X} = \tan\theta \dfrac{I_{oo}}{\forall}$

$$\Longrightarrow \; \overline{MB} = \frac{I_{oo}}{\forall}$$

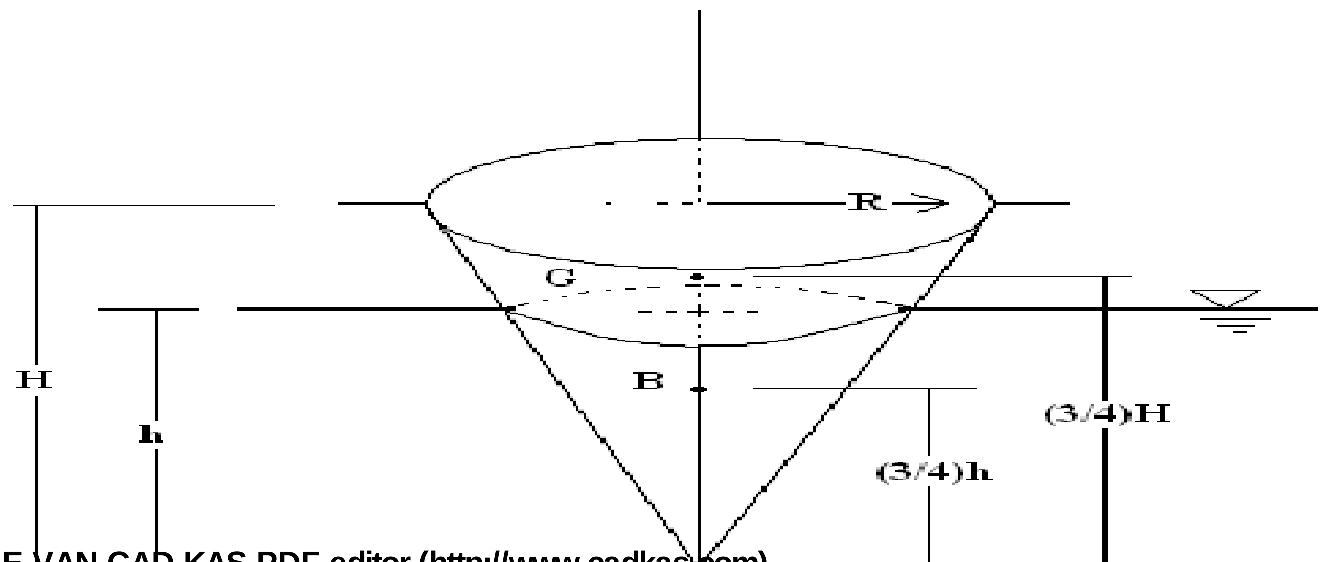
Then, $\overline{MG} = \dfrac{I_{oo}}{\forall} - \overline{BG}$

Where,

$\forall$= displaced volume

I_{oo} = Second moment of waterline area about horizontal centroidal axis oo

Discuss:



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Example : A uniform, closed cylindrical buoy, 1.5 m high, 1.0 m diameter and of mass 80 kg is to float with its axis vertical in seawater of density 1026 $kg.m^{-3}$. A body of mass 10 kg is attached to the centre of the top surface of the buoy. Show that, if the buoy floats freely, initial instability will occur.

Solution

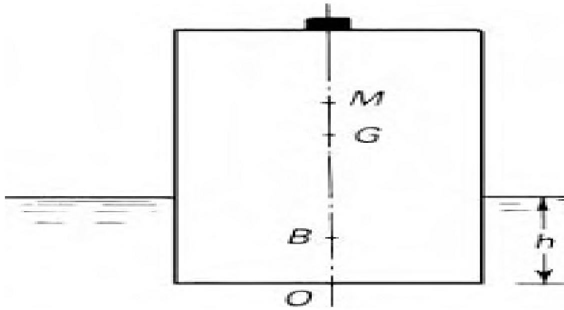
Moments of mass about horizontal axis through O:

$$(10\text{kg}).(1.5\text{m}) + (80\text{kg}).\left(\frac{1.5}{2}\text{m}\right) = \{(80+10)\text{kg}\}(\text{OG})$$

$\Rightarrow$ OG = 0.8333 m

For vertical equilibrium,

buoyancy = weight.



$$\therefore \quad \frac{\pi}{4}(1m)^2.h \times 1026kg.m^{-3} = (80+10)kg$$

$$\Longrightarrow h = 0.1117m$$

$$\therefore \quad BM = \frac{AK^2}{\forall} = \frac{\pi}{64}d^4 / \frac{\pi}{4}d^2h = \frac{d^2}{16h} = \frac{1^2}{16\times 0.1117}$$

$$= 0.560m$$

And:

$$GM = OB + BM - OG = \left(\frac{0.1117}{2} + 0.560 - 0.8333\right)m$$
$$= -0.2175m$$

Since this is negative (i.e. M is below G) buoy is unstable.

Discuss some other solved problems:

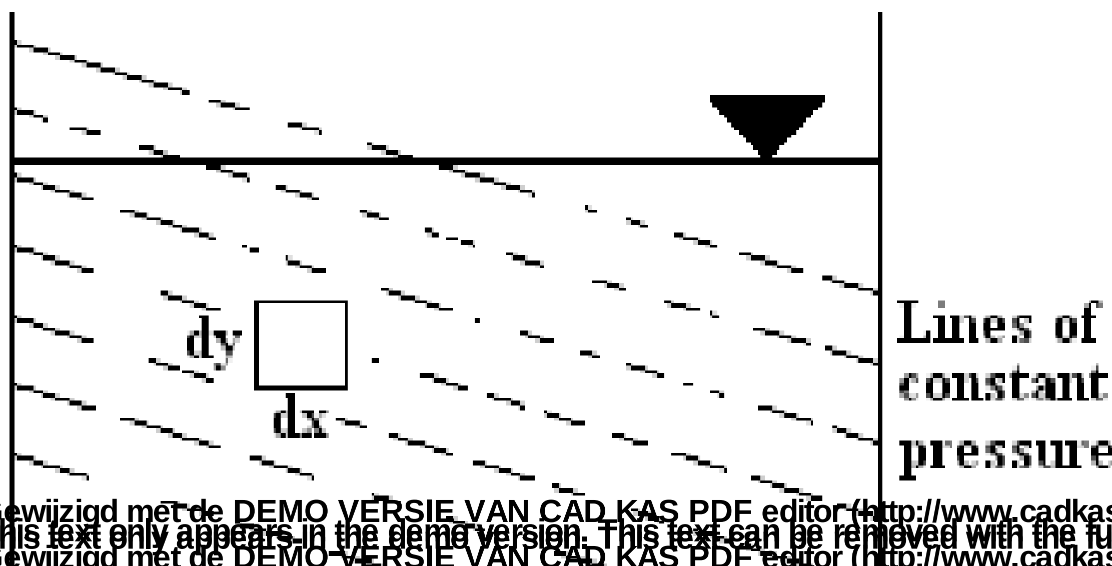
2.8 Relative Equilibrium:

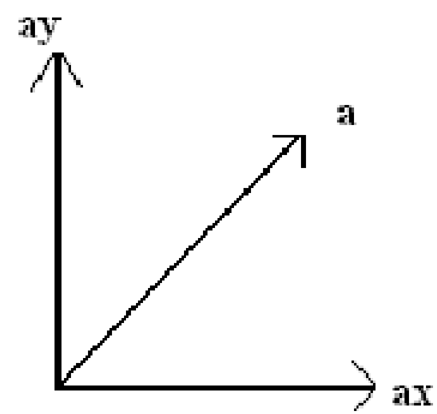
Fluid in motion in such a way that no layer move relative to an adjacent layer.

Examples:-

1. Motion with uniform linear velocity.
2. Motion with uniform linear acceleration.
3. Uniform rotation.

Uniform acceleration





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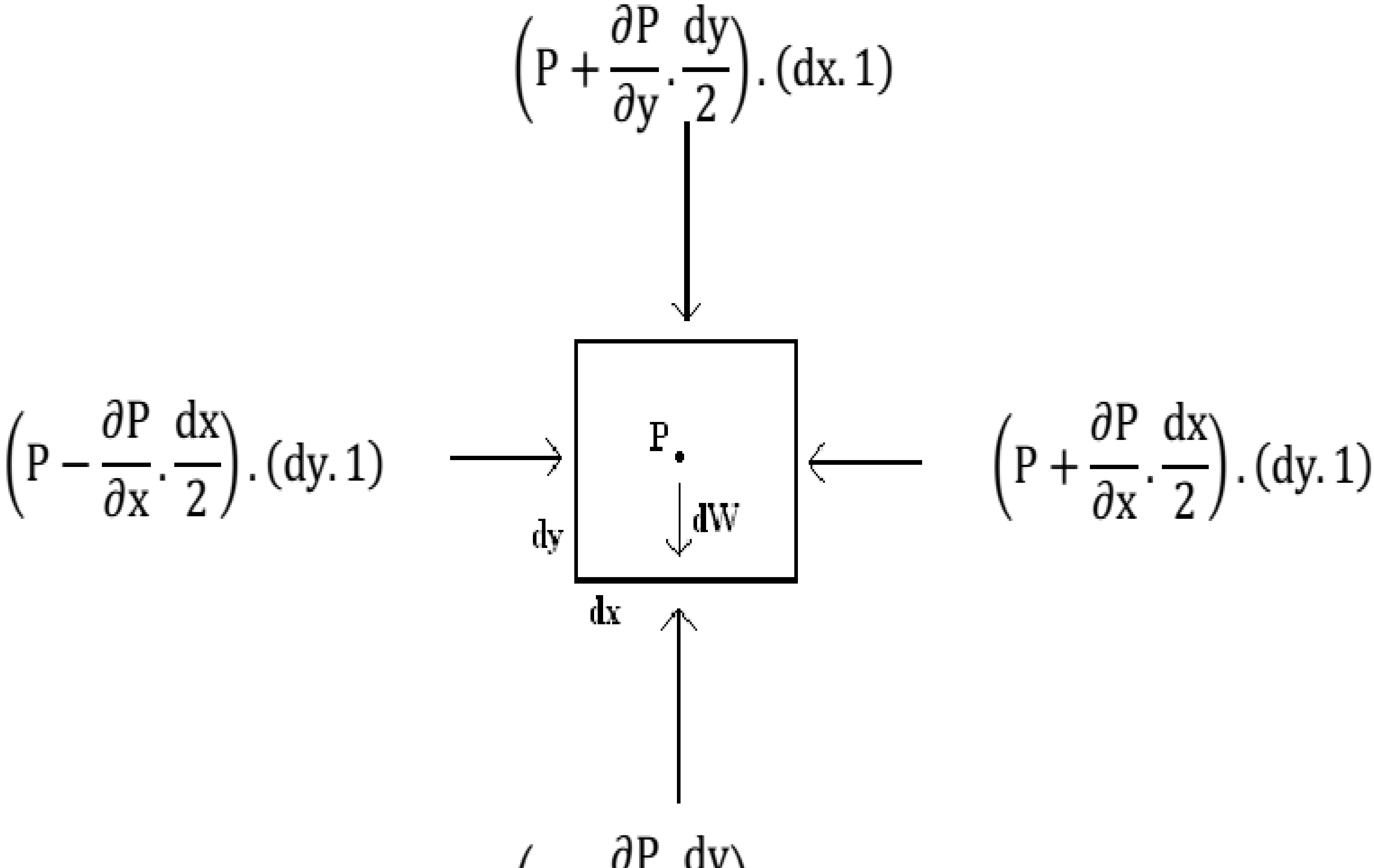

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by: $\sum F_x = m.a_x$

$$\left(P - \frac{\partial P}{\partial x}.\frac{dx}{2}\right).dy - \left(P + \frac{\partial P}{\partial x}.\frac{dx}{2}\right).dy = (\rho.dx.dy.1)\,a_x$$

$$\Rightarrow \frac{\partial \mathbf{P}}{\partial \mathbf{x}} = -\boldsymbol{\rho}.\mathbf{a}_{\mathbf{x}}$$

$$\sum F_y = m.a_y$$

$$\left(P - \frac{\partial P}{\partial y}.\frac{dy}{2}\right).(dx.1) - \left(P + \frac{\partial P}{\partial y}.\frac{dy}{2}\right).(dx.1)$$
$$= \left(\frac{\gamma}{g}.dx.dy.1\right)(a_y + g)$$

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$$\Rightarrow \; -\frac{\partial P}{\partial y}\, dy.\, dx = \; \rho.\, dx\, .\, dy.\left(a_y + g\right)$$

$$\Rightarrow \; \boldsymbol{\frac{\partial P}{\partial y} = \; -\rho\left(a_y + g\right)}$$

$$\Rightarrow \; d_P = \frac{\partial P}{\partial x}\, dx + \; \frac{\partial P}{\partial y}\, dy$$

$$\int d_P = \; \int -\rho.\, a_x\, .\, dx - \; \int \rho.\left(a_y + g\right)dy$$

$$\Rightarrow \; P = -\rho.\, a_x.\, x - \rho\left(a_y + g\right).\, y + C$$

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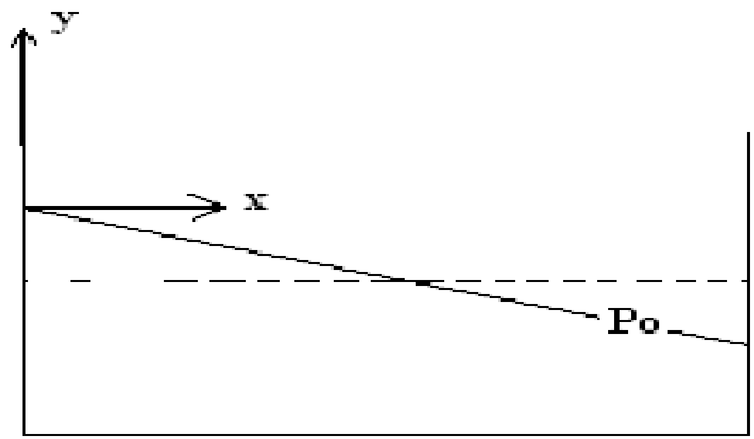
To find C apply a boundary condition
At, $x = y = 0$ then, $p = po$ sub. Above to get:
$Po = C$



$$\Longrightarrow \; \mathbf{P = P_o - \rho . a_x . x - \rho(a_y + g)y}$$

For points on a line of constant pressure (dp=0):
Thus:

$$\frac{dy}{dx} = \tan\theta = \frac{-a_x}{a_y + g}$$

Discuss some example for open and closed tanks.