

To find the location of this force, use the moment rule:

$$F_H \cdot y_{PH} = \int dF_H \cdot y \quad \Rightarrow \quad y_{PH} = \bar{y}_v + \frac{I_{G_v}}{A_v \bar{y}_v} \quad \text{هنا المعادلة اكتب}$$

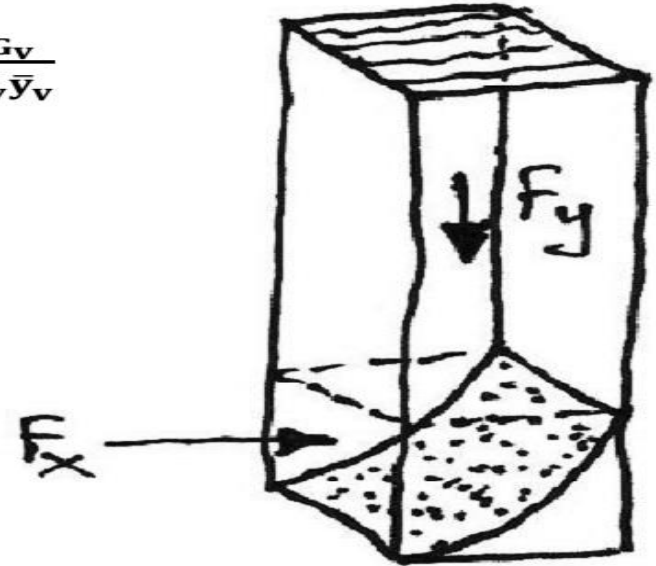
By, similar procedure:

$$\Rightarrow x_{PH} = \bar{x}_v + \frac{I_{G_v}}{A_v \bar{y}_v}$$

Now, the vertical component:

$$dF_v = dF \cdot \cos \alpha = \gamma \cdot y \cdot ds \cdot \cos \alpha = \gamma \cdot y \cdot dx$$

$$\Rightarrow F_v = \int dF_v = \gamma \int dV = \gamma \cdot V$$



Where,

V = closed volume between the curved surface and the free surface.

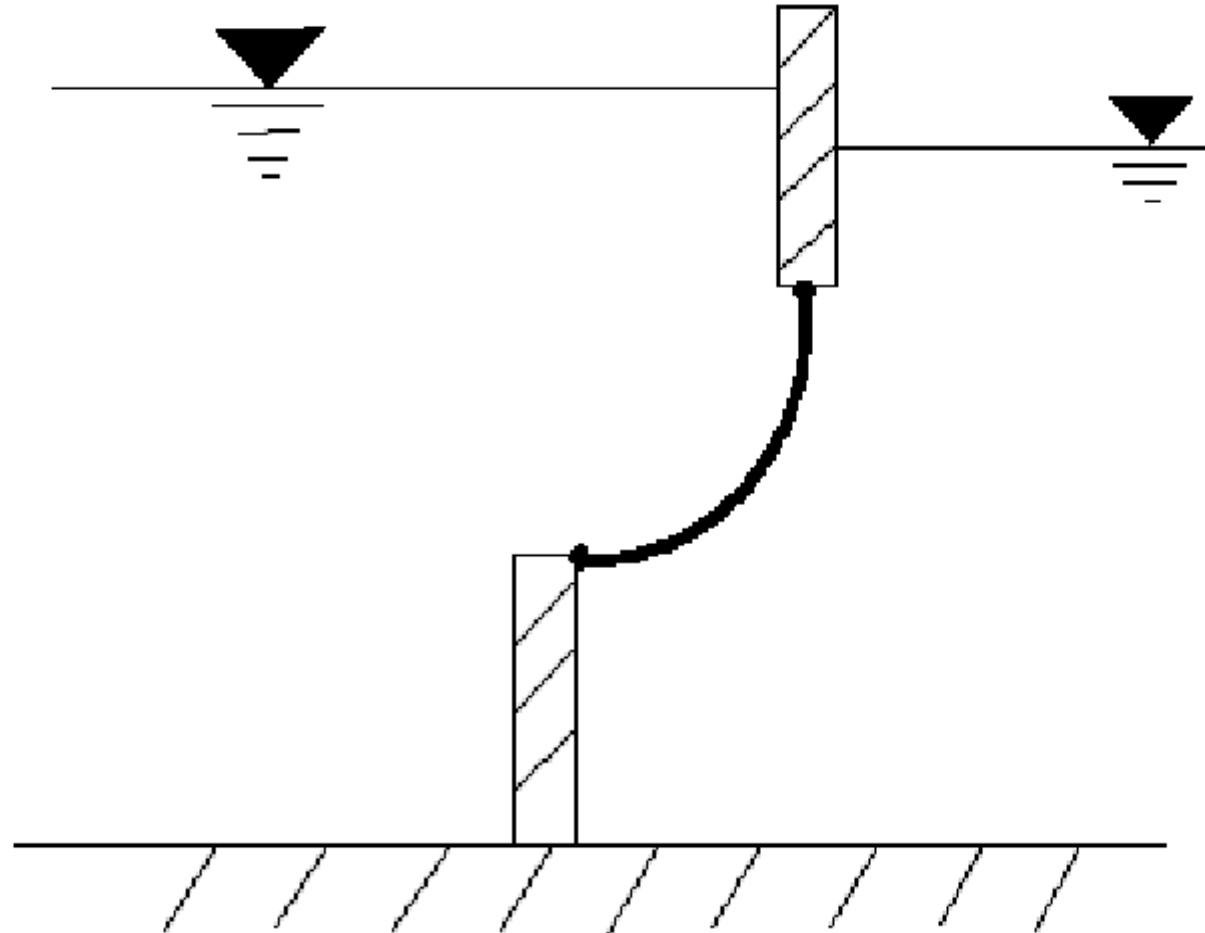
Also, by the moment rule:

$$F_v \cdot y_{pv} = \int dF_v \cdot y \quad \Rightarrow \quad y_{pv} = \bar{y}$$

$$\Rightarrow x_{pv} = \bar{x}$$

$$\Rightarrow z_{pv} = \bar{z}$$

Discuss the Example below:



Curved Surfaces – Problem

Determine the resultant force and its direction on the gate shown:

$$F_x = \gamma \cdot A \cdot \bar{y}$$

$$= 10^3 \times 9.81 \times (6 \times 1) \times \left(\frac{6}{2}\right)$$

$$= 176.6 \text{ kN}$$

And this acts at a depth $h = (2/3) \cdot 6 = 4$

$$F_y = \gamma \cdot \nabla \text{imag} = 10^3 \times 9.81 \left(\frac{\pi 6^2}{4} \times 1 \right) = 277.4 \text{ kN}$$

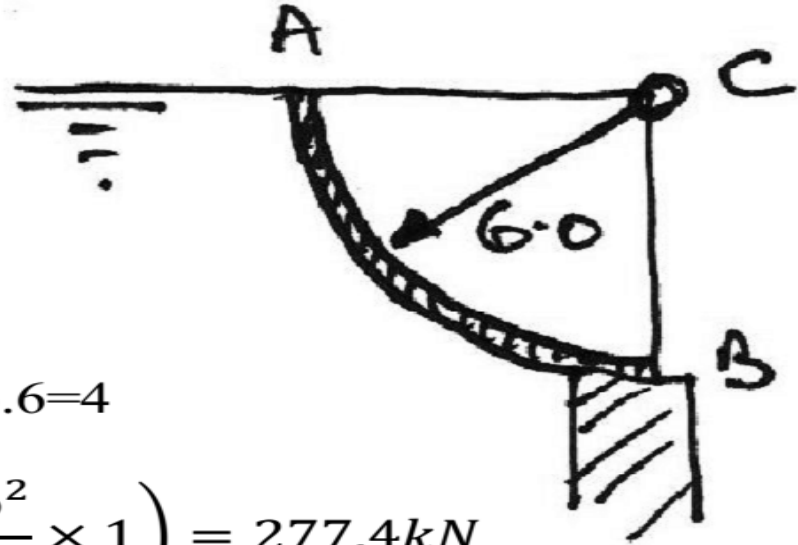
This vertical force is located at:

$$x = \frac{4R}{3\pi} = \frac{4 \times 6}{3\pi} = 2.55 \text{ m}$$

$$\Rightarrow F = \sqrt{F_x^2 + F_y^2} = \sqrt{176.6^2 + 277.4^2} = 328.8 \text{ kN}$$

And acts at an angle:

$$\theta = \tan^{-1} \frac{F_y}{F_x} = 57.5^\circ$$



H.W.

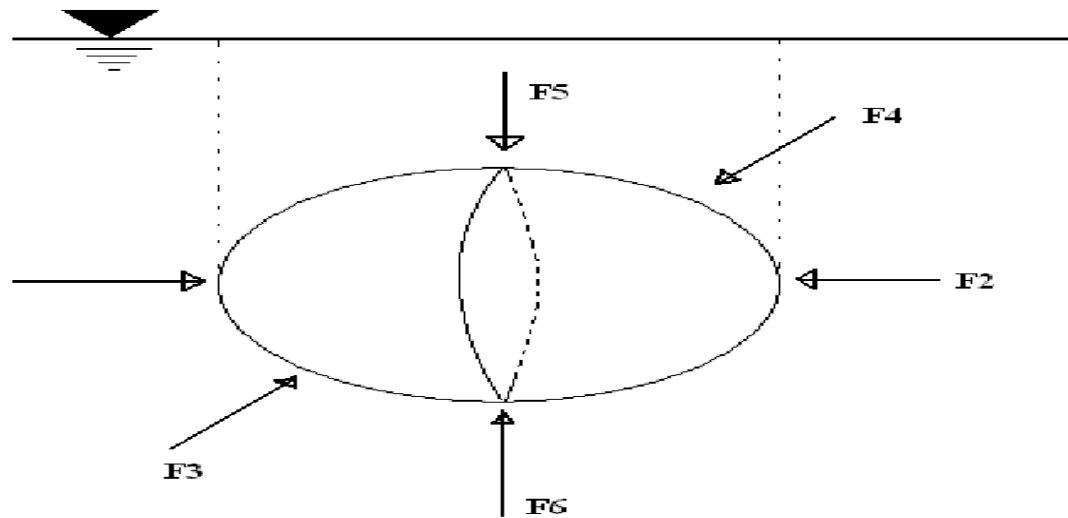
The face of a dam is curved according to the relation

$$y = \frac{x^2}{2.4}$$

where y and x are in meters, as shown in the diagram. Calculate the resultant force on each metre run of the dam. Determine the position at which the line of action of the resultant force passes through the bottom of the dam.



2.6 Buoyancy:



$$\sum_{\rightarrow}^{+} F_X = R_X = F_1 - F_2 = 0$$

$$\sum_{\swarrow}^{+} F_Z = R_Z = F_3 - F_4 = 0$$

$$\sum_{\uparrow}^{+} F_Y = R_Y = F_6 - F_5 = \gamma_f \cdot \text{[shaded circle]} - \gamma_f \cdot \text{[unshaded circle]} = \gamma_f \cdot \text{[shaded circle]}$$

$$= \gamma_f \cdot \nabla_{displaced}$$

$$\mathbf{R} = \sqrt{\mathbf{R}_X^2 + \mathbf{R}_Y^2 + \mathbf{R}_Z^2} = \mathbf{F}_B$$

$$\Rightarrow \mathbf{F}_B = \gamma_f \cdot \nabla_{disp.}$$

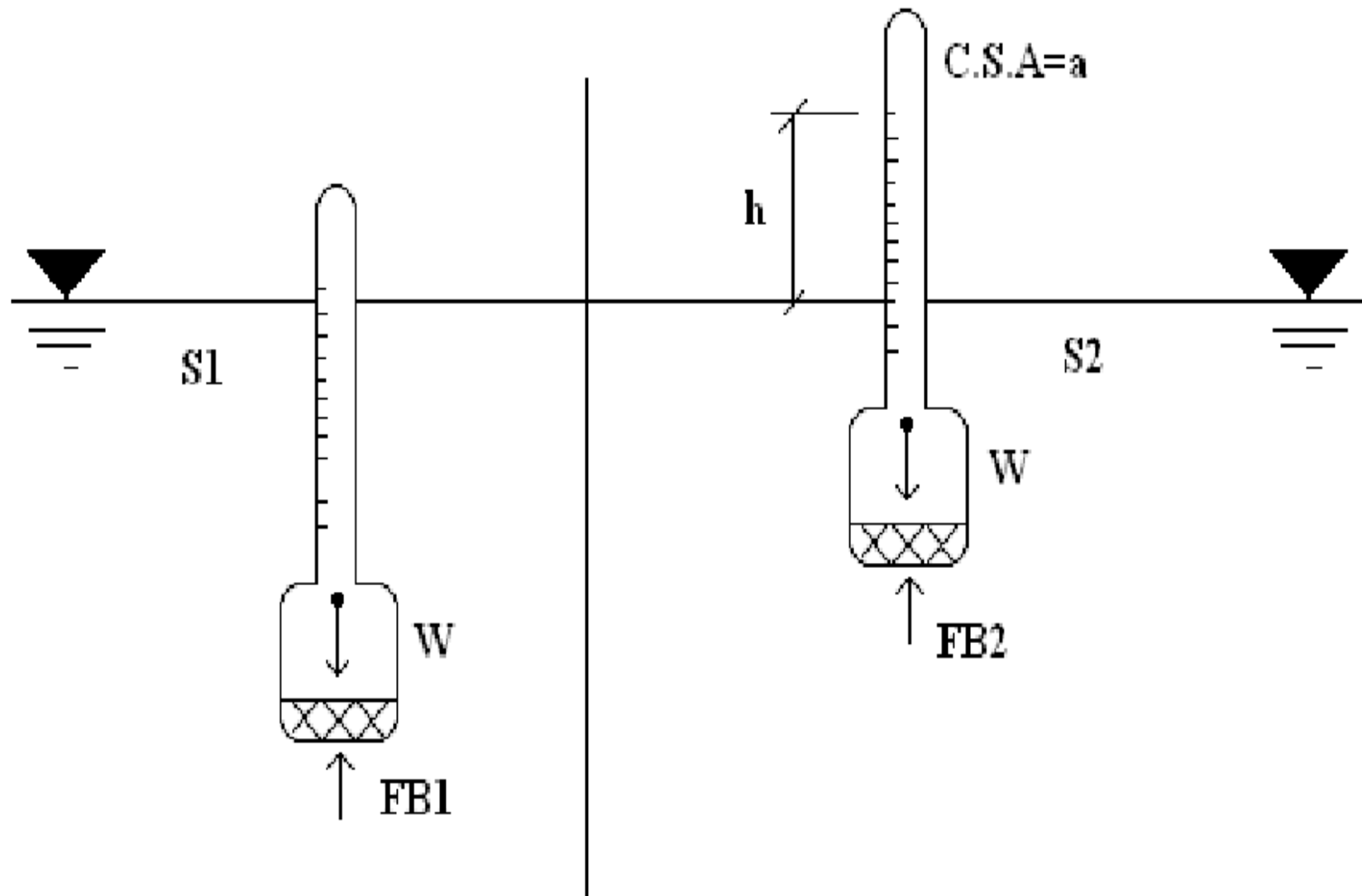
Where,

\mathbf{F}_B = Buoyancy force

γ_f = Specific weight of fluid

$\nabla_{disp.}$ = Displaced volume

- Hydrometer:



$$W = F_{B1} = S1 \cdot \gamma_w \cdot V_o$$

Where V_o = displaced volume

$$\Rightarrow S1 \cdot \gamma_w \cdot V_o = S2 \cdot \gamma_w \cdot (V_o - a \cdot h)$$

$$\text{Or: } h = \frac{V_o}{a} = \frac{S2 - S1}{S2}$$

EXAMPLE: A block of concrete weighs 100 kg in air and weighs 60 kg when immersed in water. What is the average specific weight of the block?

SOLUTION: The buoyant force is,

$$\text{By, } \sum F_y = 0$$

$$60 + F_B - 100 = 0 \quad \therefore F_B = 40\text{kg} = \gamma_{\text{water}} \times \text{Volume of block}$$

$$\Rightarrow V = \frac{40}{1000} = 0.04\text{m}^3$$

$$\text{Therefore, } \gamma = \frac{100}{0.04} = 2500\text{kg/m}^3 = 2.5\text{ ton/m}^3$$

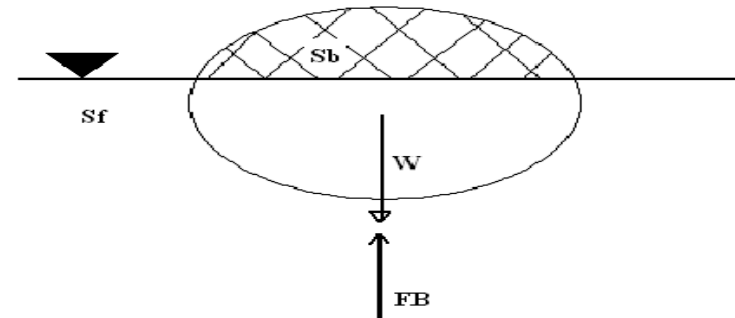
EXAMPLE 2: Find \bar{V}/V when $V = \text{total volume}$, $\bar{V} = \text{displaced volume}$.

$$\text{By, } \sum F_y = 0$$

$$\Rightarrow W = F_B$$

$$\therefore S_b \cdot \gamma_w \cdot V = S_f \cdot \gamma_w \cdot \bar{V}$$

$$\Rightarrow \frac{V}{\bar{V}} = \frac{S_b}{S_f}$$



EXAMPLE: A block of steel ($S = 7.85$) floats at a mercury – water interface as shown. Find the ratio (a/b) for this condition.

SOLUTION:

$$\text{by } \sum F_y = 0$$

$$W = F_{B \text{ total}} = F_{B(w)} + F_{B(Hg)}$$

$$\Rightarrow 7.85\gamma_w \cdot A \cdot (a + b) = \gamma_w \cdot A \cdot a + 13.6\gamma_w \cdot A \cdot b$$

$$\Rightarrow 7.85(a + b) = a + 13.6 \cdot b$$

$$\Rightarrow \frac{a}{b} = 0.839$$

