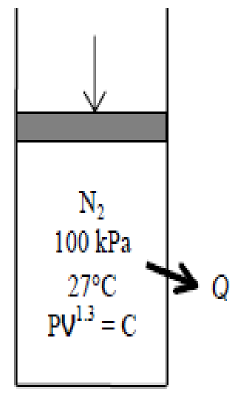
Problem:

A piston–cylinder device contains 0.8 kg of nitrogen initially at 100 kPa and 27°C. The nitrogen is now compressed slowly in a polytropic process during which $PV^{1.3}$= constant until the volume is reduced by one-half. Determine the work done and the heat transfer for this process.

Sol:

Properties the gas constant of N_2 are R = 0.2968 kPa.m3/kg.K (Table A-1). Assume the Cv value of N_2 is constant as 0.744 kJ/kg.K.



$$W_{b,in} - Q_{out} = \Delta U = m(u_2 - u_1)$$

$$W_{b,in} - Q_{out} = mc_v(T_2 - T_1)$$

The final pressure and temperature of nitrogen are

$$P_2V_2^{1.3} = P_1V_1^{1.3} \longrightarrow P_2 = \left(\frac{V_1}{V_2}\right)^{1.3} P_1 = 2^{1.3}(100\text{ kPa}) = 246.2\text{ kPa}$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \longrightarrow T_2 = \frac{P_2}{P_1}\frac{V_2}{V_1}T_1 = \frac{246.2\text{ kPa}}{100\text{ kPa}} \times 0.5 \times (300\text{ K}) = 369.3\text{ K}$$

Then the boundary work for this polytropic process can be determined from

$$W_{b,in} = -\int_1^2 P\,dV = -\frac{P_2V_2 - P_1V_1}{1-n} = -\frac{mR(T_2 - T_1)}{1-n}$$

$$= -\frac{(0.8\text{ kg})(0.2968\text{ kJ/kg}\cdot\text{K})(369.3 - 300)\text{K}}{1 - 1.3} = \mathbf{54.8\text{ kJ}}$$

Substituting into the energy balance gives

$$Q_{out} = W_{b,in} - mc_v(T_2 - T_1)$$

$$= 54.8\text{ kJ} - (0.8\text{ kg})(0.744\text{ kJ/kg.K})(369.3 - 360)\text{K}$$

$$= \mathbf{13.6\text{ kJ}}$$