Vibration analysis

There two approaches in vibration analysis namely:

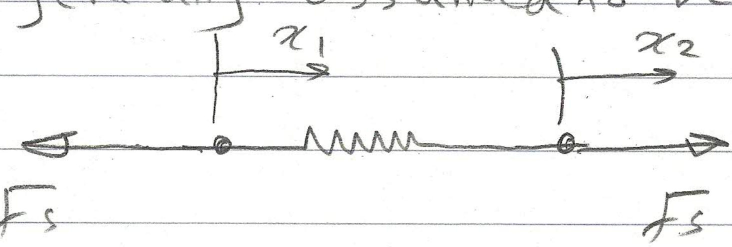
1- Discrete (lumped) System in which the number of natural frequencies = number of degrees of freedom

2. Continuous System in which infinite number of natural frequencies can be obtained.

Characteristics of discrete (lumped) System Components

There are three types of mechanical components that relate forces to displacement, velocity and acceleration:

1- Spring: It is a mechanical Component which relates force to displacement. Springs are generally assumed to be massless.



elongation $= x_2 - x_1$

K - spring constant or spring stiffness

Units: lb/in or N/m in translation and

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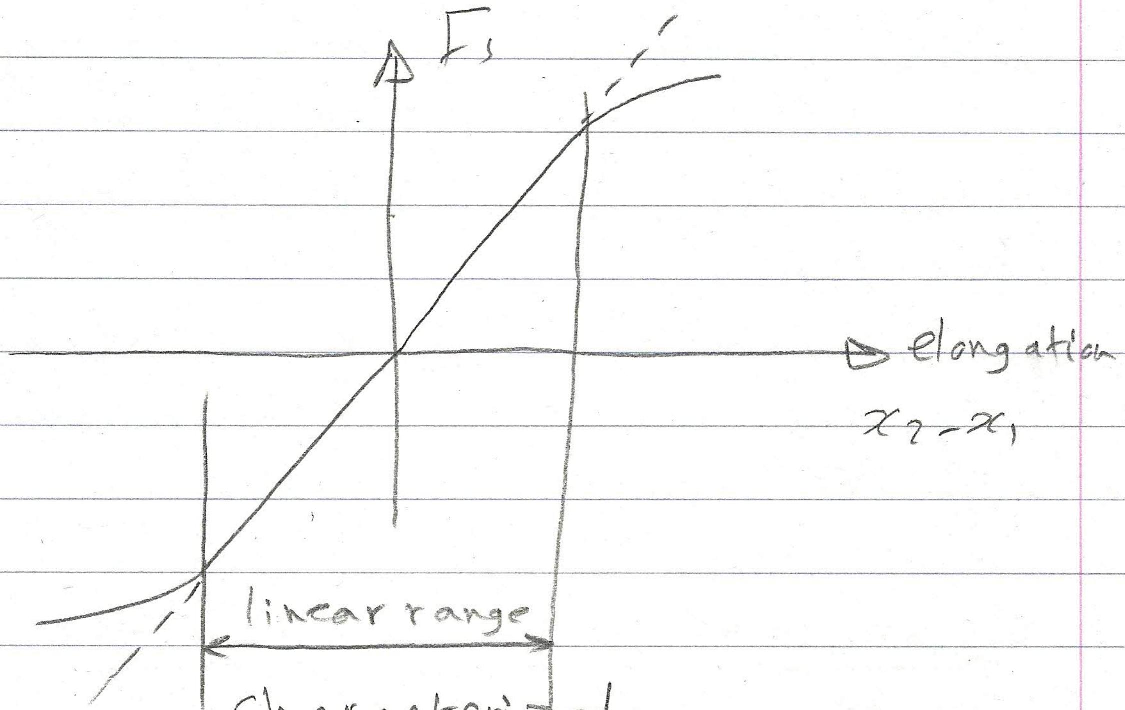
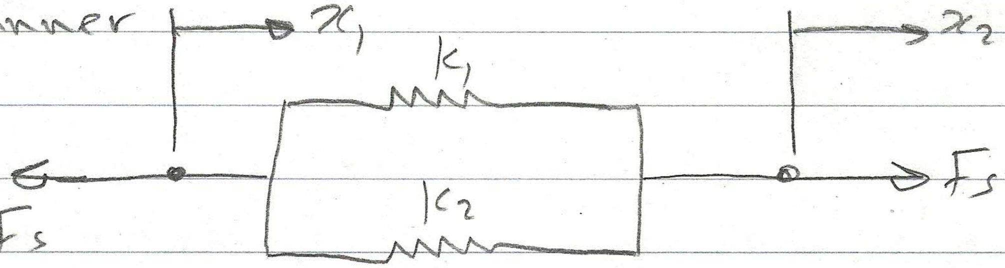
lb.in/rad or N.m/rad in rotation.
F_s - elastic or restoring force because it tends to return the spring to the unstretched state.



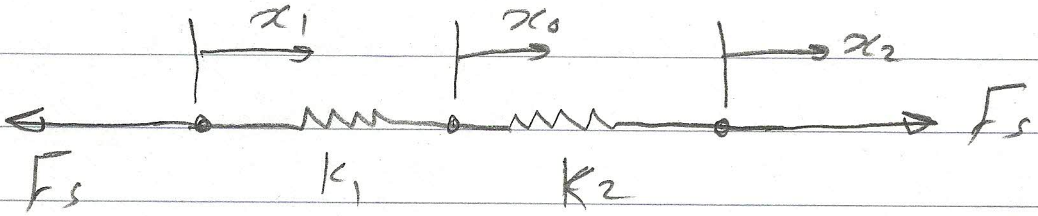
Springs may be connected in parallel manner



in which $k_{eq} = k_1 + k_2$
Generally for n springs connected in parallel manner

$$K_{eq} = \sum_{i=1}^{n} K_i$$

or in series manner

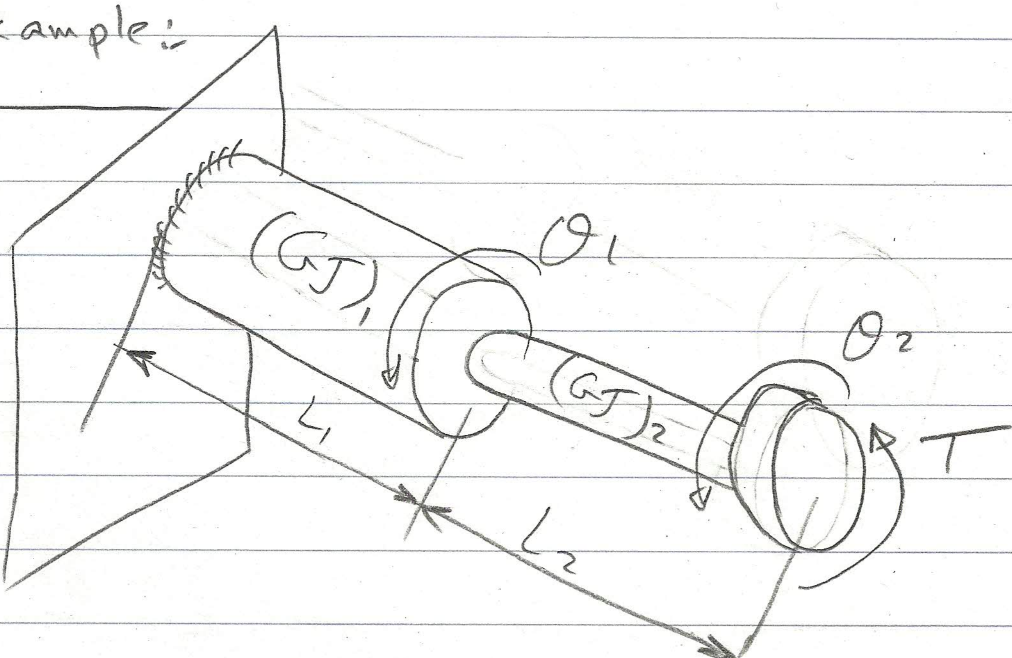


$$K_{eq} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

Generally for n springs connected in series manner

$$K_{eq} = \left[\sum_{i=1}^{n} \frac{1}{k_i}\right]^{-1}$$

Example:



For the system above
if $(GJ)_1 = 10^7 \ \mathrm{lb.in^2}$, $(GJ)_2 = 5 \times 10^6 \ \mathrm{lb.in^2}$
$L_1 = 160$ in, $L_2 = 120$ in and $T = 1000$ lb.in
Find K_{eq} and θ_2, θ_1

G_ modulus of rigidity (lb/in^2)
J_ polar moment of inertia (in^4)
From strength of material (torsion)

$$\frac{T}{J} = \frac{G\theta}{L} \longrightarrow \theta = \frac{TL}{GJ}$$

Torque acts everywhere along the shaft

$$\theta_1 = \frac{TL_1}{(GJ)_1} = \frac{1000 \times 160}{10^7} = 1.6 \times 10^{-2} \text{ rad}$$

$$\theta_2 - \theta_1 = \frac{TL_2}{(GJ)_2} = \frac{1000 \times 120}{5 \times 10^6} = 2.4 \times 10^{-2} \text{ rad}$$

$$\therefore \theta_2 = 2.4 \times 10^{-2} + 1.6 \times 10^{-2} = 4 \times 10^{-2} \text{ rad}$$

$$K_{eq} = \frac{T}{\theta_2} = \frac{1000}{4 \times 10^{-2}} = 2.5 \times 10^4 \text{ lb.in/rad}$$

Another method

$$K_1 = \frac{T}{\theta_1} = \frac{1000}{1.6 \times 10^{-2}} = \frac{10^5}{1.6}$$

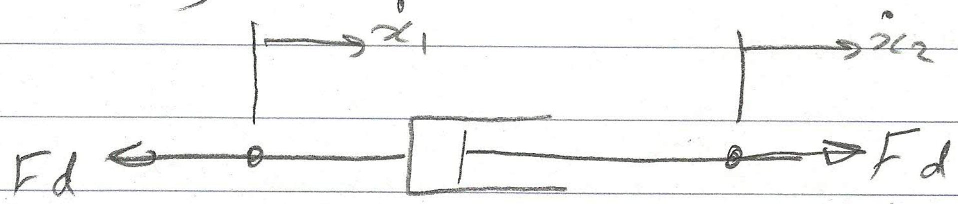
$$K_2 = \frac{T}{\theta_2 - \theta_1} = \frac{1000}{2.4 \times 10^{-2}} = \frac{10^5}{2.4}$$

Series manner

$$K_{eq} = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2}} = \frac{1}{\frac{1.6}{10^5} + \frac{2.4}{10^5}} = \frac{10^5}{4}$$

$$K_{eq} = 0.25 \times 10^5 = 2.5 \times 10^4 \text{ lb.in/rad}$$

2- Damper (dash pot) :- It is a mechanical element which relates force to velocity. It is assumed to be massless.

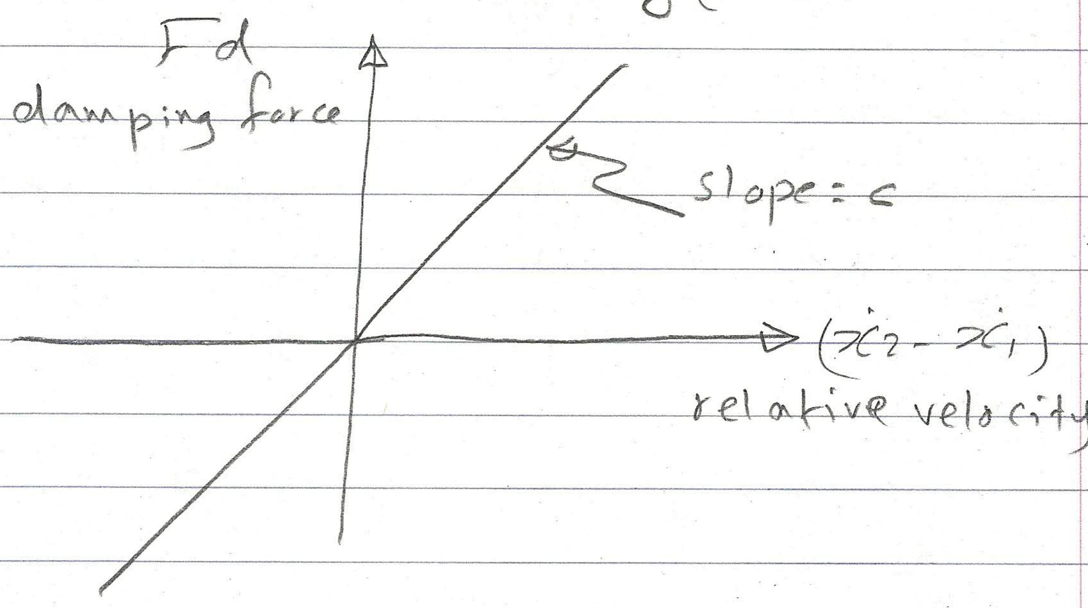


$$Fd = c(\dot{x}_2 - \dot{x}_1)$$

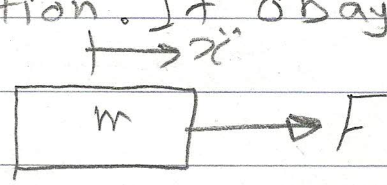
c- coefficient of viscous damping

<u>unit</u>: in translation lb.sec/in or N.sec/m and in rotation lb.in.sec/rad or N.m.sec/rad

Fd - damping force because it resists an increase in relative velocity. $(\dot{x}_2 - \dot{x}_1)$



3- mass :- It is a mechanical element which relates force to acceleration. It obeys Newton's second law



$$F = m\ddot{x}$$

unit: lb. sec^2/in in B.S unit or
kg, gm in SI unit.

Springs and masses stored and release energy. Springs have potential energy while masses have kinetic energy: In translation

$$PE = \frac{1}{2} k x^2 \quad , \quad KE = \frac{1}{2} m v^2$$

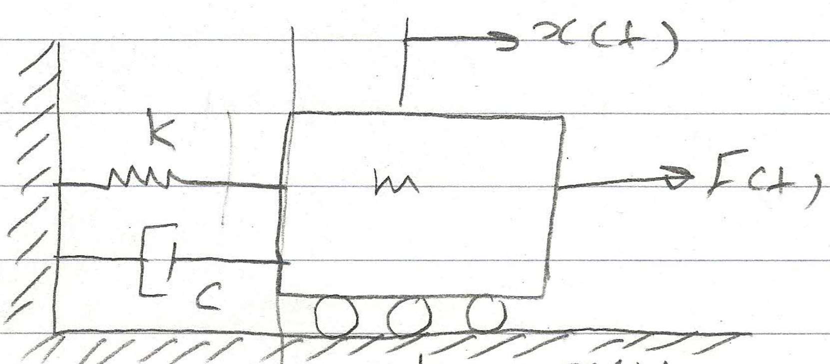
while in rotation

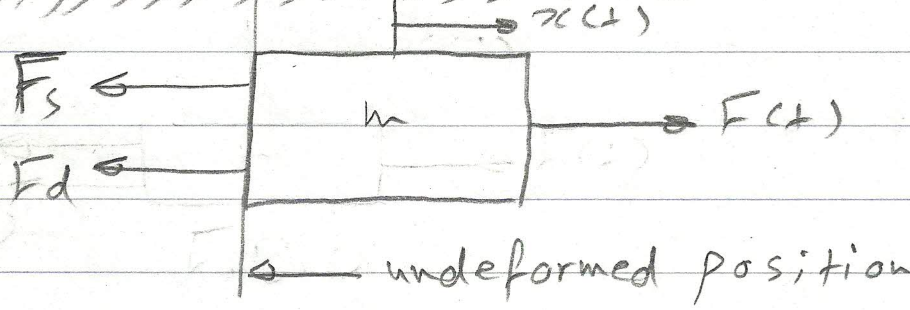
$$PE = \frac{1}{2} k \theta^2 \quad , \quad KE = \frac{1}{2} J \omega^2$$

Dampers dissipate energy

Differential Equation of motion of Second order linear system

Consider spring-damper-mass system in horizontal motion





F.B.D

Applying Newton's second law $\sum F = m\ddot{x}$

$$F(t) - F_s(t) - F_d(t) = m\ddot{x}(t)$$

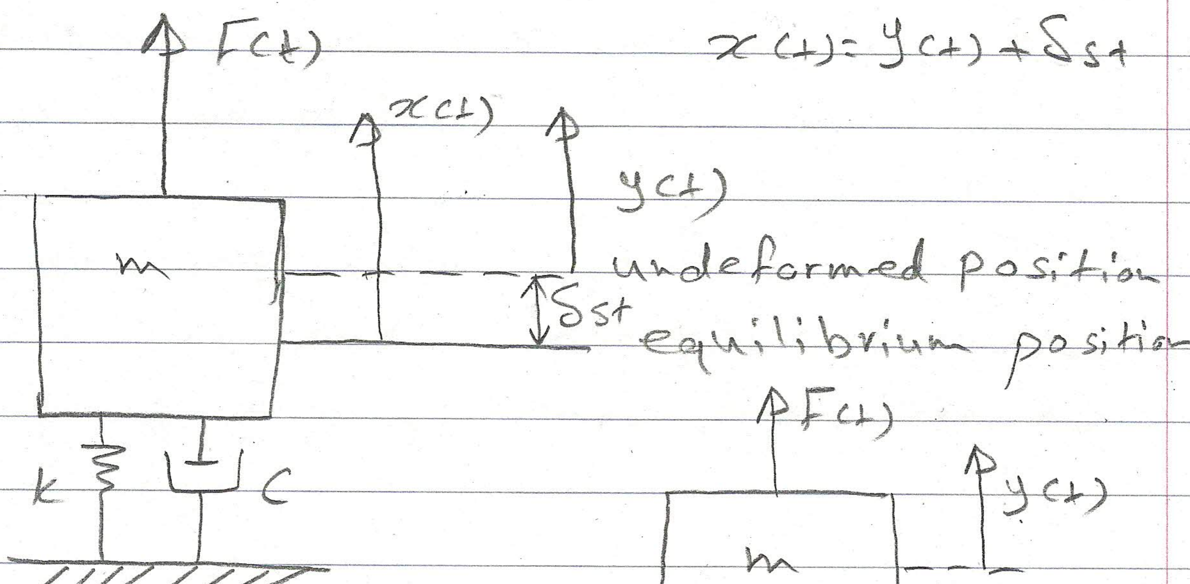
But $F_s(t) = kx(t)$, $F_d = c\dot{x}(t)$

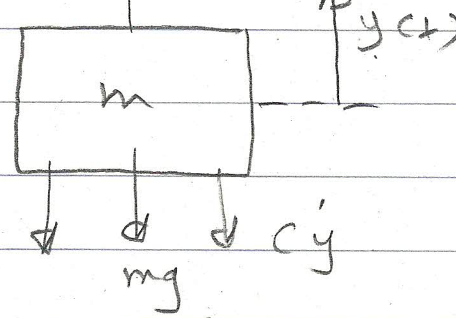
$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t) \quad \text{--- (1)}$$

It is a second order linear ordinary differential equation with constant coefficients m, c, k which are called System parameter

Now consider the same spring-damper-mass system but in vertical motion

$$x(t) = y(t) + \delta_{st}$$





Applying Newton's second law $\sum F = m\ddot{y}$

$$F(t) - mg - c\dot{y} - ky = m\ddot{y}(t)$$

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) + mg = F(t)$$

$$\ddot{y}(t) = \ddot{x}(t) \qquad \dot{y}(t) = \dot{x}(t) \qquad y(t) = x(t) - \delta_{st}$$

$$m\ddot{x}(t) + c\dot{x}(t) + k(x(t) - \delta_{st}) + mg = F(t)$$

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) - k\delta_{st} + mg = F(t)$$

For static equilibrium $k\delta_{st} = mg$ — (2)

So the final equation of motion will be of form

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t) \quad \text{--- (3)}$$

Equation (3) is the same of Eq. (1)

Conclusions.

1- In measuring displacements of linear system from the static equilibrium position, the gravity force mg can be ignored because it is balanced at all times by an additional force $k\delta_{st}$ in the spring.

2- The static equilibrium position is more convenient reference than the unstretched state of the spring