Lectures in theory of vibration

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Weekly hours: 2 (theory) + 1 (Teturial)
+ 1 (Lab)

No. of units: 5 units

References:-

1- Theory of vibration with application
By Thomson (Text Book)
2- Element of vibration Analysis
By Meirovitch
3- The dynamical behavior of structur
By Warburton
4- Mechanical Vibration
By Den Hartog
5- Mechanical vibrations and shock Measurements
By Jens Tramp Broch
6- Fundamentals of Mechanical vibrations
By S. Graham Kelly
7- Mechanical Vibrations
By S. S. Rao
8- Mechanical Vibrations analysis
By C.S. Sharma and Anil Maheshwari
9- Schaum's Series: Mechanical vibrations
By William W. Seto.

Oscillatory Motion:

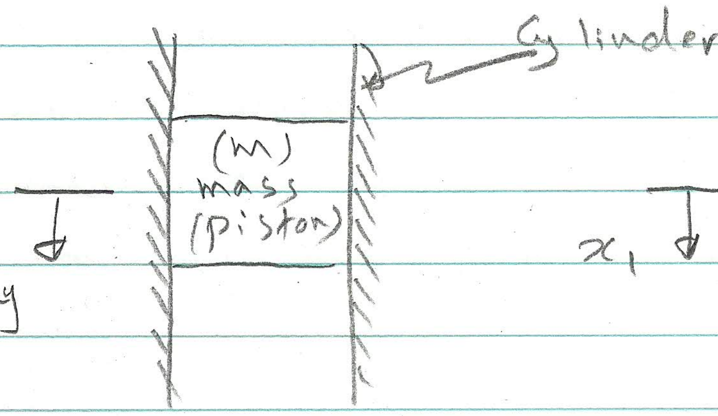
Basic Definitions:

1- Free Vibration: It occurs when a system oscillates under the action of forces inherent in the system itself and when external impressed forces are absent.

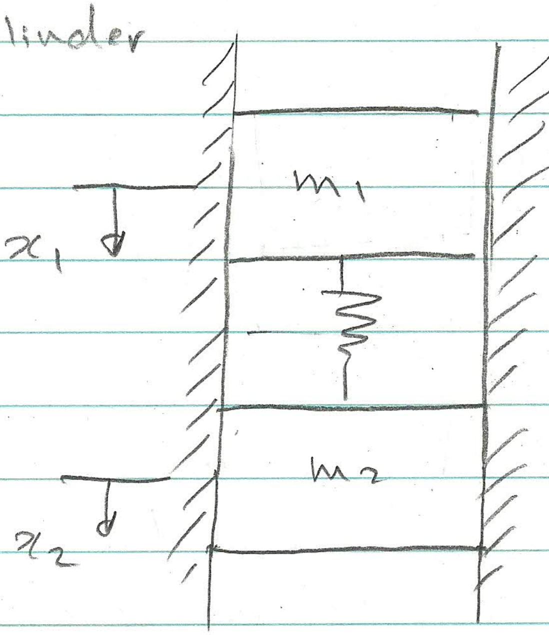
2- Forced Vibration: It takes place under the excitation of external forces. When the excitation is oscillatory, the system is forced to vibrate at the excitation frequency.

3- Degree of freedom: It represents the number of independent coordinates required to describe the motion of a system.

Examples:

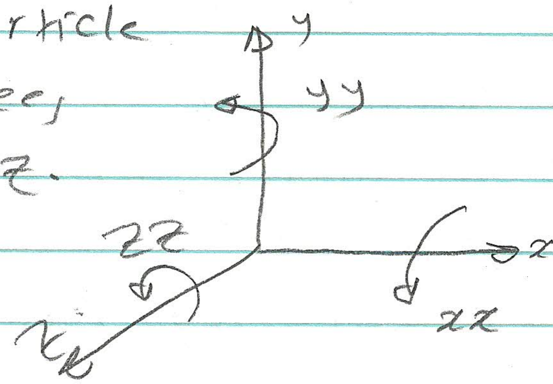


Single DOF



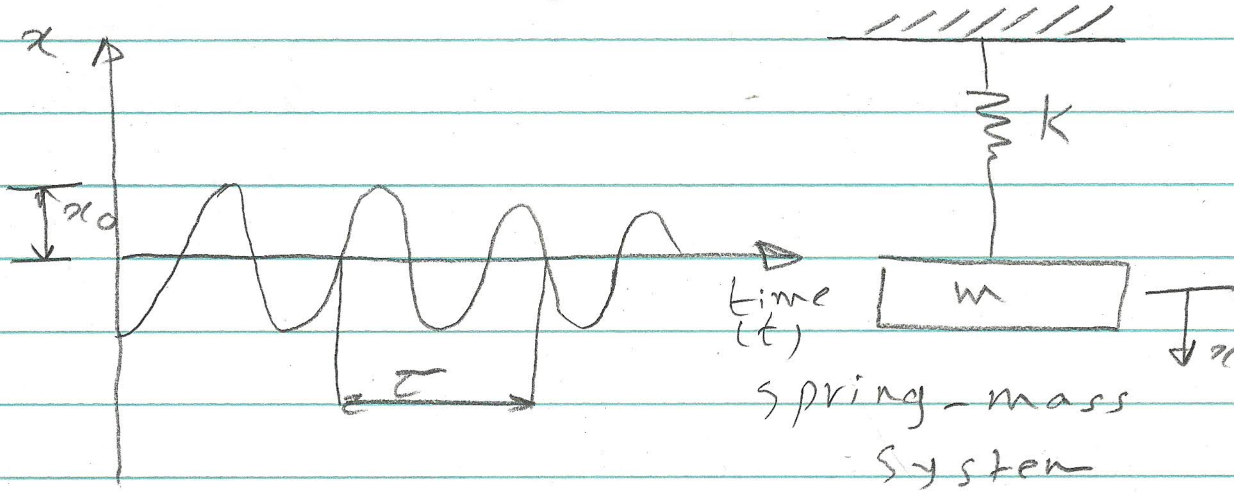
Two DOF

A free rigid body has six degrees of freedom 3 of them are translation x, y, z and the others are rotation xx, yy, zz
While a free particle has three degrees of freedom x, y, z.



12- cylinder internal combustion engine with a rigid crankshaft and rigidly mounted cylinder block has one DOF with all its moving parts (i.e. pistons, connecting rods, valves, cam shafts ---etc) because a single coordinate which represents the angle through which the crankshaft has turned determines completely the location of every moving parts in the engine.
If the cylinder block is mounted on flexible springs, so that it can freely moves in every direction, the system will have 7 DOF.

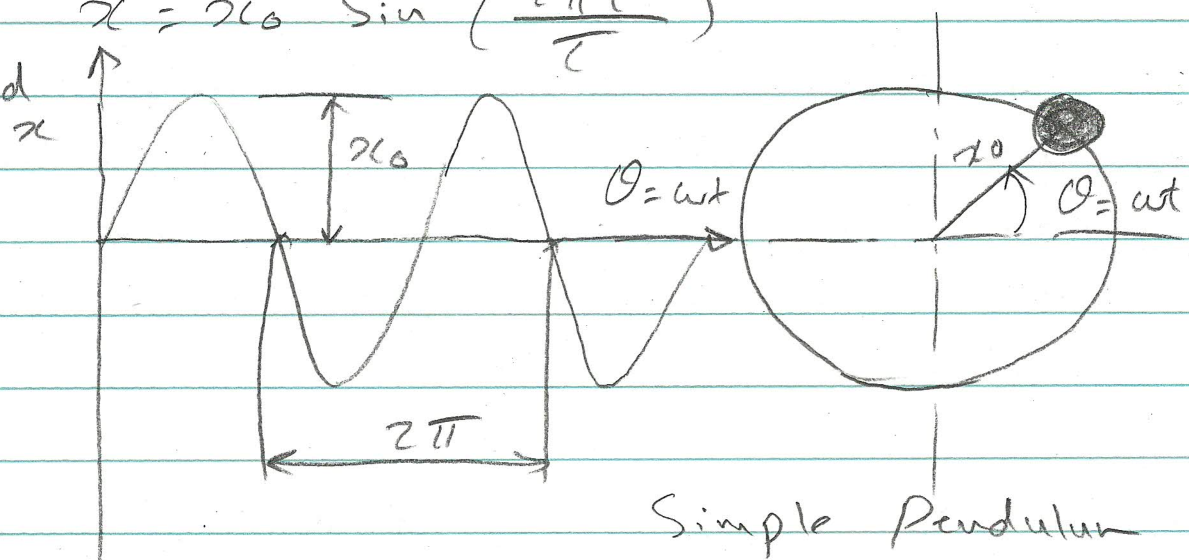
4- Harmonic motion: when the oscillation motion is repeated in equal intervals of time it is called harmonic motion.
The harmonic motion may be represented as follows:



The equation of motion is

$$x = x_0 \sin\left(\frac{2\pi t}{\tau}\right)$$

and



The equation of motion is

$$x = x_0 \sin \omega t$$

5- Amplitude:- It is the maximum value of the displacement in a vibrating system. (x_0)

6- Period (T):- It is a certain interval of time at which the vibrating system repeats its motion with all its particulars

7- Frequency (f):- It is the reciprocal of period of oscillation of a vibrating system $f = \frac{1}{T}$

Units: Either ω - circular frequency (rad/sec) $= \omega = 2\pi\,(Hz)$
or Hertz $\left(\frac{rev}{sec}\right)$ or rpm (rev/min)

$$rpm\,(cpm) = \frac{30\,\omega\,(rad/sec)}{\pi}$$

8- Natural frequency: It is a property of a dynamical system. Its value depends on the following parameters:
a- The type of material (ρ, E, ν)
b- The geometrical shape (Dimensions)
c- The boundary Condition of the system.
The system may have one or more values of natural frequencies.

9- Resonance: If the frequency excitation Coincides with one of the natural frequencies of a system, a Condition of resonance is encountered. It is resulted in a dangerous large oscillations which leads to the failure of major structure such as bridge, building or airplane wings.

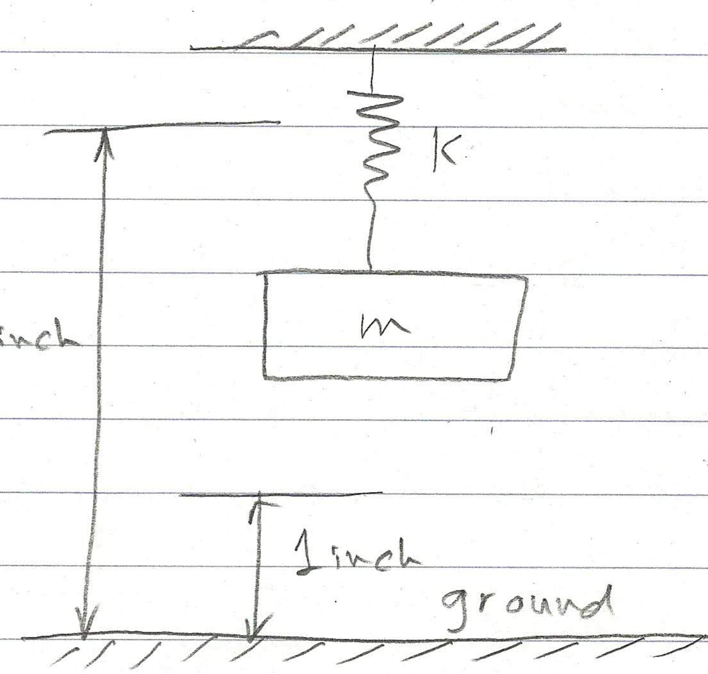
10- Damping: It is resulted by the energy dissipated by friction and other resistances. It is of great importance in limiting the amplitude of oscillation at resonance.

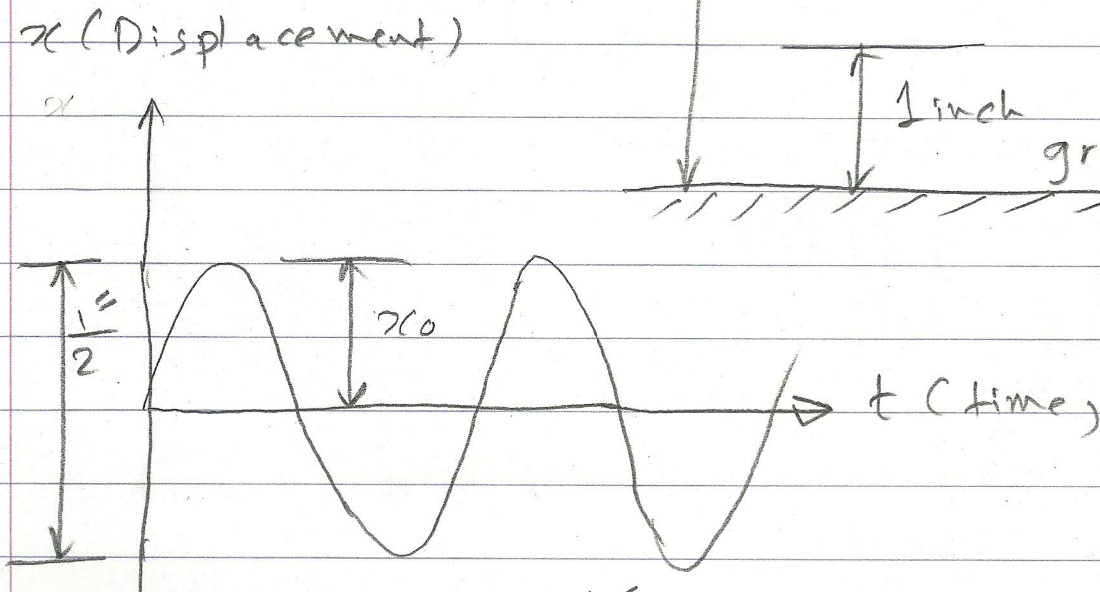
There are three types of damping:

a. Viscous damping
b. Coulomb (Dry friction) damping
c. Structural damping

Example:

A body is suspended from a spring vibrates vertically up and down between two positions 1 and 1.5 inches above the ground. During each second it reaches the top position (1.5 inch) twenty times. What are T, f, ω, and x_0?





$$x_0 = \frac{1}{2} \times \frac{1''}{2} = \frac{1}{4} \text{ inch}$$

$T = \frac{1}{20}$ sec $f = 20$ cycle/sec

$f = 20$ Hz

$\omega = 2\pi f = 2\pi \times 20 = 40\pi$ rad/sec

Review to Dynamics:

Newton's second law in translation motion

$$\sum F = ma$$

where m - mass and a - acceleration

Any force in the direction of motion is considered to be (+ve) and vise versa.

Newton's second law in rotation motion

$$\sum T_o = J_o \ddot{\theta}$$

where J_o - mass moment of inertia about axis o and $\ddot{\theta}$ - angular acceleration

Any torque about axis o in the direction of rotation is considered to be (+ve) and vise versa.

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \ddot{x}$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = \alpha = \frac{d\omega}{dt}$$

mass in translation is replaced by J (mass moment of inertia)