

Reversible Non-Flow processes

I-Constant Volume Process (Isochoric)(Isometric)

In a constant volume process, the working substance is contained in a rigid vessel; hence the boundaries of the system are immovable and no boundary work can be done on or by the system. Other kinds of works can be applied.

In common, it will be assumed that “constant volume” implies zero work unless stated otherwise.

For the non-flow energy equation,

$$q - w = \Delta u$$

For constant volume process ($v_1 = v_2$ and $w = 0$)

$$q = \Delta u = u_2 - u_1$$

$$Q = \Delta U = m(u_2 - u_1)$$

i.e. all the heat supplied (q +ve) in a constant volume process goes to increase the internal energy. If (q -ve) it means the heat rejected from the system equals the decrease in internal energy.

For a perfect or an ideal gas,

$$q = \Delta u = u_2 - u_1 = C_v(T_2 - T_1)$$

$$Q = \Delta U = m(u_2 - u_1) = mC_v(T_2 - T_1)$$

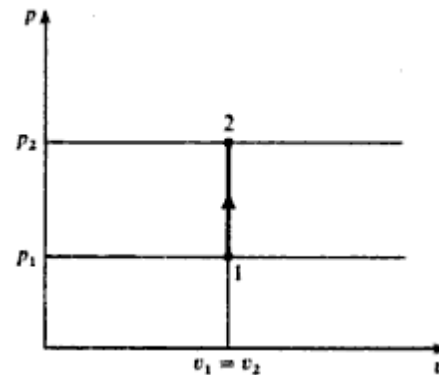
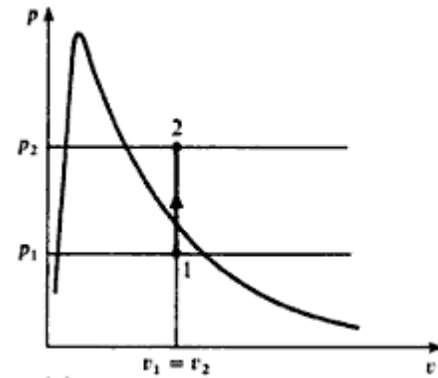
Also, from equation of state

$$P_1 v_1 = RT_1$$

$$P_2 v_2 = RT_2$$

For constant volume process, $v_1 = v_2$,

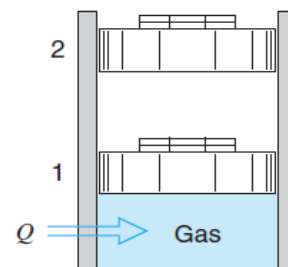
$$\frac{P_2}{P_1} = \frac{T_2}{T_1}$$



II-Constant Pressure Process (Isobaric)

For this process, the system boundary must move against external resistance as heat is supplied, for instance a fluid in a cylinder-behind a piston can be made to undergo a constant pressure process.

The boundary work done on or by the system undergoes expansion or compression process is:



$$W = \int_1^2 P dV$$

$$w = \int_1^2 P dv = P \int_1^2 dv = P(v_2 - v_1)$$

$$W = mP(v_2 - v_1) = P(V_2 - V_1)$$

For the non-flow energy equation,

$$q - w = \Delta u$$

$$q = (u_2 - u_1) + w$$

$$q = (u_2 - u_1) + P(v_2 - v_1)$$

$$q = (u_2 + P_2 v_2) - (u_1 + P_1 v_1)$$

$$\text{But, } h = u + Pv$$

$$q = h_2 - h_1$$

$$Q = mq = m(h_2 - h_1) = H_2 - H_1$$

This means that the heat supplied or rejected equals the change in enthalpy

For a perfect or an ideal gas,

$$q = \Delta h = h_2 - h_1 = C_p(T_2 - T_1)$$

$$Q = \Delta H = m(h_2 - h_1) = mC_p(T_2 - T_1)$$

Also, from equation of state

$$P_1 v_1 = RT_1$$

$$P_2 v_2 = RT_2$$

For constant pressure process, $P_1 = P_2$,

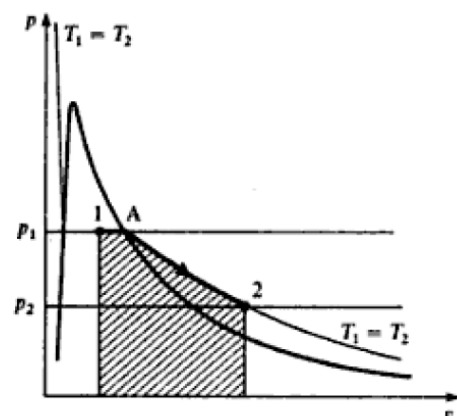
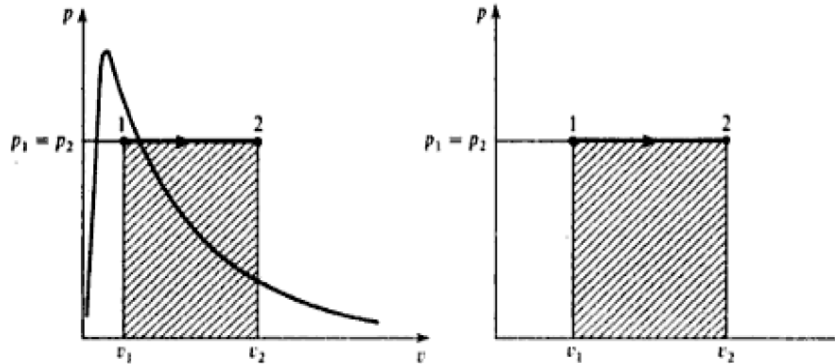
$$\frac{T_2}{T_1} = \frac{v_2}{v_1}$$

III-Constant Temperature Process (Isothermal)

In this process, the temperature remains constant.

In isothermal expansion, the pressure of the fluid decreases and there is a tendency for the temperature to fall. Therefore, heat must be added to keep the temperature at the initial value.

In an isothermal compression, the pressure of the fluid increases and there is a tendency for the temperature to rise. Therefore, heat must be rejected to maintain the temperature constant.



For a vapor, from state 1 to state A the pressure remains at P_1 , since in the wet region the pressure and temperature are the corresponding saturation values.

$$P_1 = P_A = P_2$$

$$T_1 = T_A = T_2$$

For the non-flow energy equation,

$$q - w = \Delta u$$

$$q = (u_2 - u_1) + w$$

$$Q = U_2 - U_1 + W$$

For an ideal gas, the isothermal process is more easily dealt with than that for vapor, Also,

$$P v = R T$$

$$P_1 v_1 = R T$$

$$P_2 v_2 = R T$$

$$P v = \text{constant} \quad (\text{Boyle's law})$$

$$\frac{P_2}{P_1} = \frac{v_2}{v_1}$$

$$P = c/v$$

$$c = P_1 v_1 = P_2 v_2$$

$$w = \int_1^2 P dv = \int_1^2 \frac{c}{v} dv = c \int_1^2 \frac{dv}{v} = c \ln\left(\frac{v_2}{v_1}\right) = P_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = P_2 v_2 \ln\left(\frac{v_2}{v_1}\right)$$

$$= P_1 v_1 \ln\left(\frac{P_1}{P_2}\right) = P_2 v_2 \ln\left(\frac{P_1}{P_2}\right) = R T \ln\left(\frac{P_1}{P_2}\right) = R T \ln\left(\frac{v_2}{v_1}\right) = \dots\dots\dots (*)$$

Obviously, there are a large number of equations for calculating the work done.

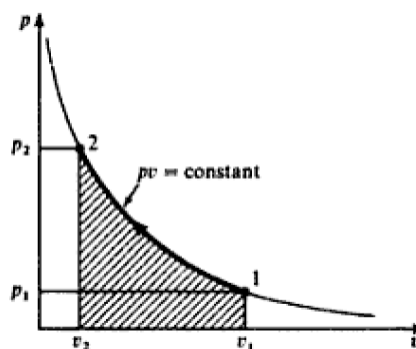
Applying Joule's law for a perfect gas:

$$u_2 - u_1 = C_v (T_2 - T_1)$$

But for isothermal process, $T_2 = T_1$, thus

$u_2 - u_1 = 0$ (internal energy remains constant in an isothermal process for a perfect gas)

Therefore, For a perfect gas only:



$$\begin{aligned}
 q &= w = P_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = P_2 v_2 \ln\left(\frac{v_2}{v_1}\right) \\
 &= P_1 v_1 \ln\left(\frac{P_1}{P_2}\right) = P_2 v_2 \ln\left(\frac{P_1}{P_2}\right) = RT \ln\left(\frac{P_1}{P_2}\right) = RT \ln\left(\frac{v_2}{v_1}\right) \\
 Q &= W = m P_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = m P_2 v_2 \ln\left(\frac{v_2}{v_1}\right) \\
 &= m P_1 v_1 \ln\left(\frac{P_1}{P_2}\right) = m P_2 v_2 \ln\left(\frac{P_1}{P_2}\right) = m R T \ln\left(\frac{P_1}{P_2}\right) = m R T \ln\left(\frac{v_2}{v_1}\right)
 \end{aligned}$$

Note

In sometimes to facilitate calculating the work done for vapor, the vapor is assumed to behave as a perfect gas in process (A-2) (referring to previous figure), while the process (1-A) has the same work that in constant pressure process.

$$w = {}_1w_A + {}_Aw_2$$

$${}_1w_A = P_1(v_A - v_1) = P_A(v_A - v_1)$$

$${}_Aw_2 = \text{The same work of Eq(*)}$$

$$\begin{aligned}
 {}_Aw_2 &= \int_1^2 P dv = \int_1^2 \frac{c}{v} dv = c \int_1^2 \frac{dv}{v} = c \ln\left(\frac{v_2}{v_1}\right) = P_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = P_2 v_2 \ln\left(\frac{v_2}{v_1}\right) \\
 &= P_1 v_1 \ln\left(\frac{P_1}{P_2}\right) = P_2 v_2 \ln\left(\frac{P_1}{P_2}\right) = RT \ln\left(\frac{P_1}{P_2}\right) = RT \ln\left(\frac{v_2}{v_1}\right)
 \end{aligned}$$

The molecular weight and gas constant of water vapor are 18.015 and 0.4615 kJ/kg.K as found in Table (A-1).

IV-Adiabatic Process

In this process, no heat is transferred to or from the system, i.e. the system is perfectly thermal insulated.

For the non-flow energy equation,

$$q - w = \Delta u$$

$$Q - W = \Delta U$$

For reversible adiabatic process ($q=0$, $Q=0$),

$$-w = \Delta u$$

$$w = u_1 - u_2$$

$$W = U_1 - U_2$$

In an adiabatic expansion process, the work done by the system causes a reduction in internal energy of the system.

For a vapor undergoing a reversible adiabatic process, the work done can be obtained by finding the internal energies u_1 and u_2 .

For perfect gas, $u = C_v T$

Then:

$$w = C_v (T_1 - T_2)$$

$$W = m C_v (T_1 - T_2)$$

Also,

$$C_v = \frac{R}{k-1}, \quad Pv = RT$$

$$w = \frac{R}{k-1} (T_1 - T_2) = \frac{(P_1 v_1 - P_2 v_2)}{k-1} \dots\dots\dots(\#)$$

$$W = \frac{mR}{k-1} (T_1 - T_2) = \frac{m(P_1 v_1 - P_2 v_2)}{k-1} = \frac{(P_1 V_1 - P_2 V_2)}{k-1}$$

For a perfect gas, a law relating P and v is derived as below:

Considering the non-flow energy equation in differential form,

$$\delta q - \delta w = du$$

For reversible adiabatic process, $\delta q = 0$

$$-\delta w = du \dots\dots\dots(1)$$

Also, from the definition of work done,

$$\delta w = P dv \dots\dots\dots(2)$$

$$-P dv = du$$

$$P dv + du = 0 \dots\dots\dots(3)$$

According to Jule's law for internal energy of perfect gas,

$$du = C_v dT \dots\dots\dots(4)$$

Applying the equation of state,

$$P = RT/v \dots\dots\dots(5)$$

Substitute equations (4)&(5) in equation (3), we get:

$$RT \frac{dv}{v} + C_v dT = 0$$

Dividing by T , we get,

$$R \frac{dv}{v} + C_v \frac{dT}{T} = 0$$

Integrating,

$$R \ln v + C_v \ln T = \text{constant}$$

Substituting $T = P v / R$

$$R \ln v + C_v \ln\left(\frac{Pv}{R}\right) = \text{constant}$$

Dividing by C_v ,

$$\frac{R}{C_v} \ln v + \ln\left(\frac{Pv}{R}\right) = \text{constant}$$

$$\text{But, } k-1 = \frac{R}{C_v}$$

Where k is the adiabatic index of the Perfect gas (Table A-2)

$$(k-1) \ln v + \ln\left(\frac{Pv}{R}\right) = \text{constant}$$

$$\ln v^{(k-1)} + \ln\left(\frac{Pv}{R}\right) = \text{constant}$$

$$\ln\left(\frac{Pv v^{k-1}}{R}\right) = \text{constant}$$

$$\ln\left(\frac{Pv^k}{R}\right) = \text{constant}$$

$$\frac{Pv^k}{R} = e^{\text{constant}}$$

$$Pv^k = c$$

For any perfect gas undergoing reversible adiabatic process from state 1 to 2,

$$P_1 v_1^k = P_2 v_2^k$$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^k = \left(\frac{V_1}{V_2}\right)^k$$

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/k}$$

Also by utilizing equation of state, we can get further relations:

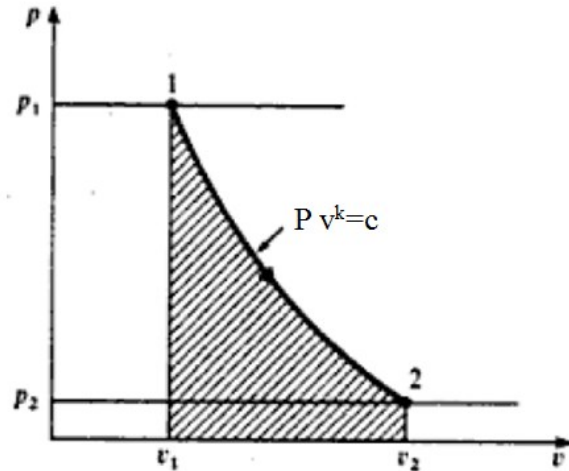
$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{1/(k-1)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

The work done (Eq (#)) can be estimated also by its definition:

$$w = \int_1^2 P dv = \frac{(P_1 v_1 - P_2 v_2)}{k-1} \quad (\text{Prove!})$$



V-Polytropic Processes

Many processes in practice approximate to reversible polytropic law of $Pv^n = \text{constant}$, where n is a constant. Both vapors and perfect gases obey this type of law.

The various previous processes are special cases of the polytropic processes for a perfect gas.

➤ When $n = \infty$

$Pv^\infty = \text{constant}$, i.e. $v = \text{constant}$

(constant volume process)

Process (1-D) constant volume heating process

Process (1-D') constant volume cooling process

➤ When $n = 0$

$Pv^0 = \text{constant}$, i.e. $P = \text{constant}$

(constant pressure process)

Process (1-A) constant pressure cooling process

Process (1-A') constant pressure heating process

➤ When $n = 1$

$Pv = \text{constant}$, i.e. $T = \text{constant}$

(constant temperature or isothermal process)

Process (1-B) isothermal compression process

Process (1-B') isothermal expansion process

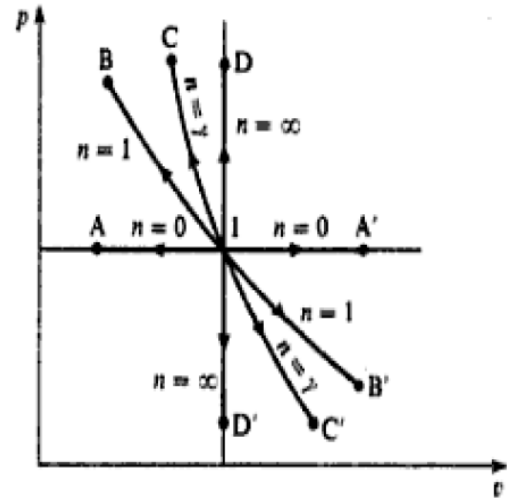
➤ When $n = k$

$Pv^k = \text{constant}$, i.e. $q = 0$

(adiabatic process)

Process (1-C) adiabatic compression process

Process (1-C') adiabatic expansion process



The boundary work of reversible polytropic process can be computed as:

$$w = \int_1^2 P dv$$

$$P = \frac{c}{v^n} = cv^{-n}$$

$$w = \int_1^2 cv^{-n} dv = c \left[\frac{v^{1-n}}{1-n} \right]_{v_1}^{v_2} = \left[\frac{cv_2^{1-n} - cv_1^{1-n}}{1-n} \right]$$

$$c = P_1 v_1^n = P_2 v_2^n$$

$$w = \left[\frac{P_2 v_2 v_2^{1-n} - P_1 v_1 v_1^{1-n}}{1-n} \right] = \frac{P_2 v_2 - P_1 v_1}{1-n} = \frac{P_1 v_1 - P_2 v_2}{n-1} \quad (*)$$

$$W = \frac{m(P_2 v_2 - P_1 v_1)}{1-n} = \frac{m(P_1 v_1 - P_2 v_2)}{n-1} = \frac{P_1 V_1 - P_2 V_2}{n-1}$$

The last equation is applied for any working substance (*vapors and perfect gases*) undergoing a reversible polytropic process. Also,

$$Pv^n = c$$

$$P_1 v_1^n = P_2 v_2^n = c$$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^n = \left(\frac{V_1}{V_2}\right)^n$$

$$\frac{V_2}{V_1} = \frac{v_2}{v_1} = \left(\frac{P_1}{P_2}\right)^{1/n}$$

For **perfect gas** only:

$$Pv = RT$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

$$\frac{V_2}{V_1} = \frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{1/(n-1)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

Furthermore, the work done can be calculated as:

$$w = \frac{P_1 v_1 - P_2 v_2}{n-1} = \frac{R(T_1 - T_2)}{n-1}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$$

The amount of heat supplied or rejected in reversible polytropic process can be evaluated by non-flow energy equation

$$q - w = \Delta u$$

$$q = \Delta u + w$$

For **vapor** the work done is calculated from equation (*), while the change in internal energy ($u_2 - u_1$) can be found by vapor table after determining the states of vapor.

For **perfect gas**:

$$q = \Delta u + w$$

$$q = C_v(T_2 - T_1) + \frac{R(T_1 - T_2)}{n - 1}$$

$$\text{But } C_v = \frac{R}{k - 1}$$

$$q = \frac{R}{k - 1}(T_2 - T_1) + \frac{R(T_1 - T_2)}{n - 1}$$

$$q = \frac{R(T_1 - T_2)}{1 - k} + \frac{R(T_1 - T_2)}{n - 1}$$

$$q = R(T_1 - T_2)\left(\frac{1}{1 - k} + \frac{1}{n - 1}\right)$$

$$q = R(T_1 - T_2)\left(\frac{1 - k + n - 1}{(1 - k)(n - 1)}\right)$$

$$q = \frac{R(T_1 - T_2)}{n - 1}\left(\frac{n - k}{1 - k}\right)$$

$$q = \left(\frac{n - k}{1 - k}\right)w$$

$$Q = \left(\frac{n - k}{1 - k}\right)W \quad (\text{This equation is applied for ideal gas only})$$