CH 15: Design of Bevel and Worm Gears

Bevel-Gears Stresses and Strengths *(Straight Bevel Gears)*

There are two major difficulties that arise for bevel gears:

- Shaft deflection: typically one of the gears has to be mounted outboard of the bearing which makes the shaft deflection more pronounced and thus have greater effect on the nature of tooth contact.
- Teeth deflection: since teeth are tapered, more deflection occurs at the smaller section and that causes non-uniform line contact (to obtain a perfect line contact the larger section needs to deflect more). To overcome this difficulty, the face width needs to be kept fairly "small".

- AGMA Stress equations:

  - Bending stress:

$$\begin{cases} S_t = \sigma = \dfrac{W^t}{F} P_d K_o K_\upsilon \dfrac{K_s K_m}{K_x J} & \text{US units} \\[2ex] \sigma_F = \dfrac{1000 W^t}{b} \dfrac{K_A K_\upsilon}{m_{et}} \dfrac{Y_X K_{H\beta}}{Y_{\beta J}} & \text{(SI units)} \end{cases}$$

  - Contact stress:

$$\begin{cases} S_C = \sigma_C = C_P \left(\dfrac{W^t}{F d_P I} K_o K_\upsilon K_m C_S C_{XC}\right)^{1/2} & \text{US units} \\[2ex] \sigma_H = Z_E \left(\dfrac{1000 W^t}{b d Z_1} K_A K_\upsilon K_{H\beta} Z_X Z_{XC}\right)^{1/2} & \text{(SI units)} \end{cases}$$

- AGMA Strength (*allowable stress*) equations:

  - Bending strength:

$$\begin{cases} S_{WT} = \sigma_{all} = \dfrac{S_{at} K_L}{S_F K_T K_R} & \text{US units} \\[2ex] \sigma_{FP} = \dfrac{\sigma_{F\lim} Y_{NT}}{S_F K_\theta Y_Z} & \text{(SI units)} \end{cases}$$

➢ Contact strength:

$$\begin{cases} S_{WC} = \sigma_{c,all} = \dfrac{S_{ac} C_L C_H}{S_H K_T C_R} & \textit{US units} \\[2ex] \sigma_{HP} = \dfrac{\sigma_{H\lim} Z_{NT} Z_W}{S_H K_\theta Z_Z} & \textit{(SI units)} \end{cases}$$

AGMA Equations Factors

- *Overload factor*, K_o (K_A): it is used to account for external loads exceeding the normal tangential load W^t.
  - ❖ See *Table 15-2* for K_o values.

Table 15–2

Overload Factors K_o (K_A)

Source: ANSI/AGMA 2003-B97.

Character of Prime Mover	Character of Load on Driven Machine: Uniform	Light Shock	Medium Shock	Heavy Shock
Uniform	1.00	1.25	1.50	1.75 or higher
Light shock	1.10	1.35	1.60	1.85 or higher
Medium shock	1.25	1.50	1.75	2.00 or higher
Heavy shock	1.50	1.75	2.00	2.25 or higher

Note: This table is for speed-decreasing drives. For speed-increasing drives, add $0.01(N/n)^2$ or $0.01(z_2/z_1)^2$ to the above factors.

- *Dynamic factor*, K_v : it is used to account for deviations from the uniform angular speed due to inaccuracies in manufacturing and meshing of gears. Transmission accuracy-level number Q_v is used to indicate the manufacturing precision.
  - ❖ The value of K_v is found from *Fig. 15-5* as a function of Q_v and pitch-line velocity v_t where:

$$v_t = \pi d_P n_P / 12 \qquad \textit{US units}$$

$$v_{et} = 5.236(10^{-5}) d_1 n_1 \qquad \textit{(SI units)}$$

✓ Also there are curve fit equations given in text.

- *Contact stress geometry factor*, I (Z_I): it is used to account for the geometry of teeth surfaces (*i.e., instantaneous radii of curvature*) and thus the contact area of the two surfaces.
  - ❖ The value of I is found from *Fig. 15-6*.

- *Bending stress geometry factor*, $J\ (Y_J)$: it is used to account for geometry of the tooth and location of the load W^t.
  - ❖ The value for J is found from *Fig. 15-7*.

Figure 15–5

Dynamic factor K_v.
(Source: ANSI/AGMA 2003-B97.)

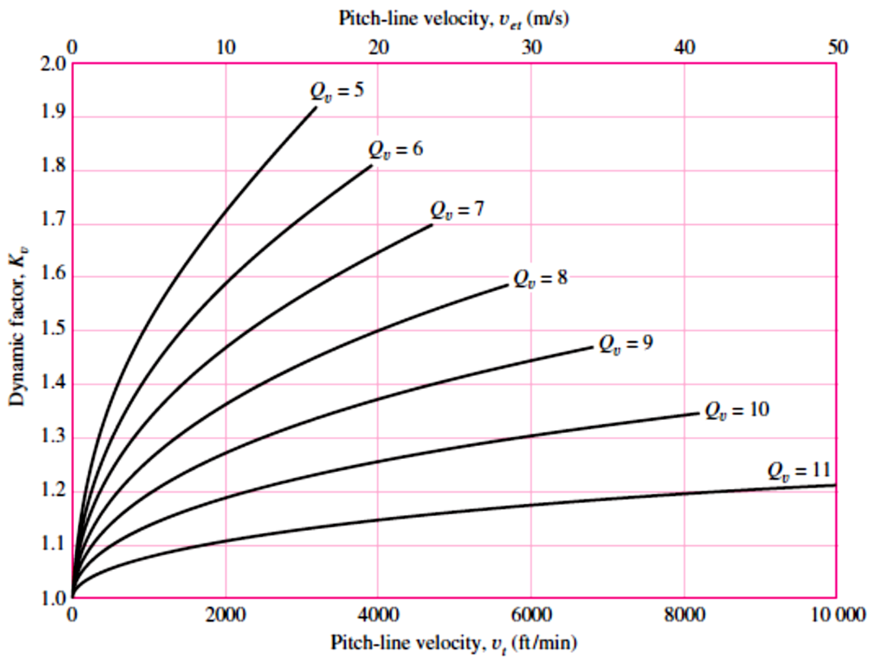


Figure 15–6

Contact geometry factor $I\ (Z_I)$ for coniflex straight-bevel gears with a 20° normal pressure angle and a 90° shaft angle.
(Source: ANSI/AGMA 2003-B97.)

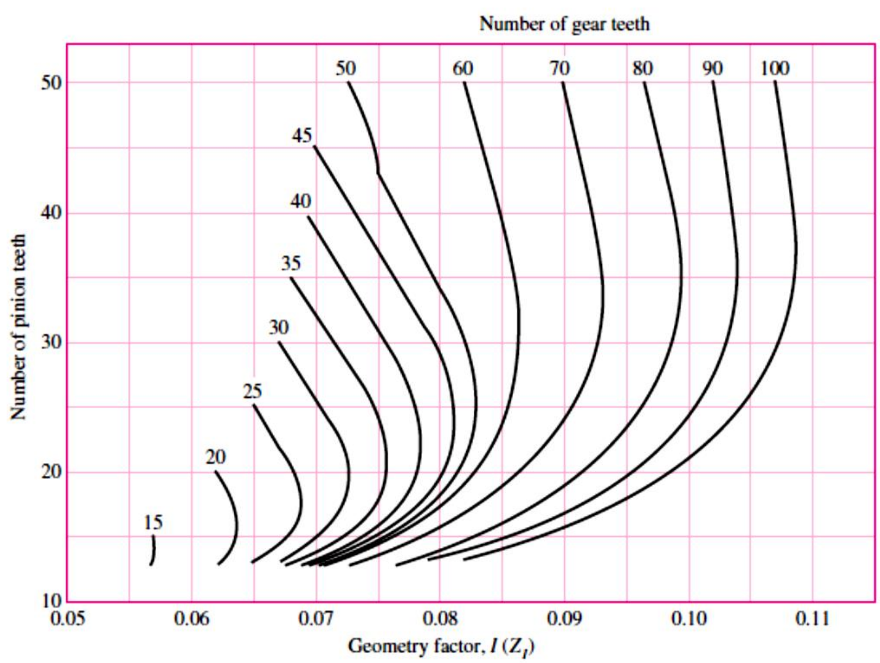
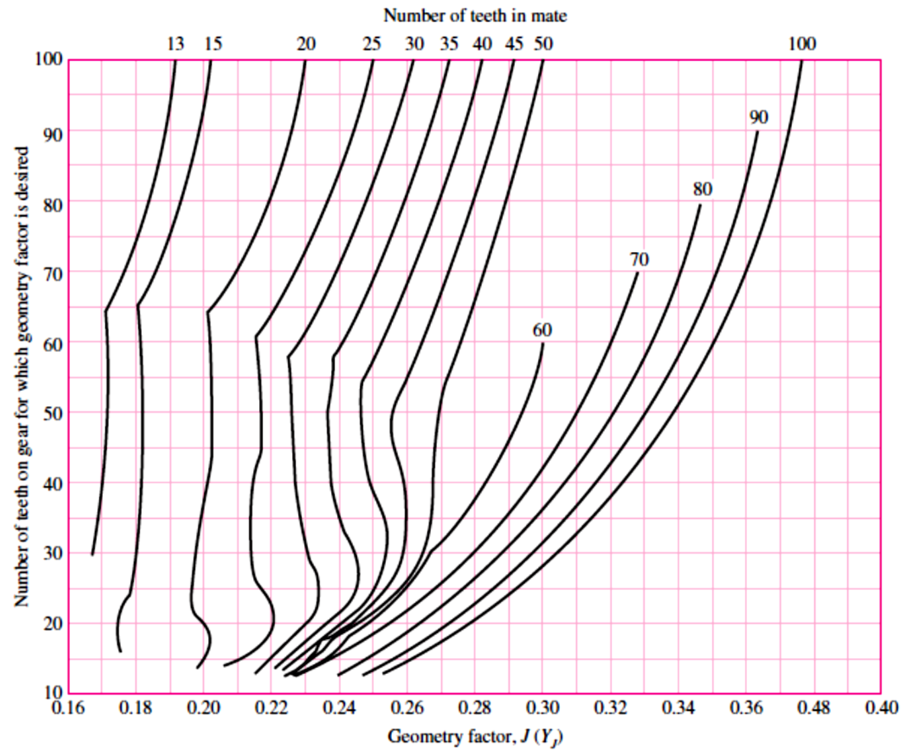


Figure 15–7

Bending factor J (Y_J) for coniflex straight-bevel gears with a 20° normal pressure angle and 90° shaft angle. *(Source: ANSI/AGMA 2003-B97.)*



- ***Load distribution factor***, $K_m\ (K_{H\beta})$: it is used to account for non-uniform load distribution along the line of contact.

K_m is found as :

$$K_m = K_{mb} + 0.0036F^2 \qquad \textit{US units}$$

$$K_{H\beta} = K_{mb} + 5.6(10^{-6})b^2 \qquad \textit{(SI units)}$$

where

$$K_{mb} = \begin{cases} 1.00 & \textit{both members straddle} - \textit{mounted} \\ 1.10 & \textit{one member straddle} - \textit{mounted} \\ 1.25 & \textit{neither member straddle} - \textit{mounted} \end{cases}$$

- ***Size factor for bending***, $K_s\ (Y_x)$:

$$K_s = \begin{cases} 0.4867 + \frac{0.2132}{P_d} & 0.5 \le P_d \le 16 \ \ in^{-1} \\ 0.5 & P_d > 16 \ \ in^{-1} \end{cases} \qquad \textit{US units}$$

$$Y_x = \begin{cases} 0.5 & m_{et} < 1.6 \ mm \\ 0.4867 + 0.008339\, m_{et} & 1.6 \le m_{et} \le 50 \ mm \end{cases} \qquad \textit{(SI units)}$$

- *Size factor for contact stress*, $C_s\ (Z_x)$:

$$C_s = \begin{cases} 0.5 & F < 0.5\ in \\ 0.125F + 0.4375 & 0.5 \le F \le 4.5\ in \\ 1 & F > 4.5\ in \end{cases} \qquad \textit{US units}$$

$$Z_x = \begin{cases} 0.5 & b < 12.7\ mm \\ 0.00492b + 0.4375 & 12.7 \le b \le 114.3\ mm \\ 1 & b > 114.3\ mm \end{cases} \qquad \textit{(SI units)}$$

- *Crowing factor for contact stress*, $C_{xc}\ (Z_{xc})$:

  The teeth of most bevel gears are crowned to accommodate the shaft deflections.

$$C_{xc} = Z_{xc} = \begin{cases} 1.5 & crowned\ teeth \\ 2 & non-crowned\ teeth \end{cases}$$

- *Lengthwise curvature factor for bending stress*, $K_x\ (Y_\beta)$:

$$K_x = Y_\beta = 1 \qquad \textit{for straight bevel gears}$$

- *Elastic coefficient for contact stress*, C_p : it combines the elastic properties of the gear and pinion

$$C_p = \sqrt{\frac{1}{\pi((1-v_P^2)/E_P + 1 - v_G^2)/E_G))}} \qquad \sqrt{psi}\ \ (\sqrt{MPa}\ )$$

  - Or can be found from *Table 14-8*.

- *Bending strength*, $S_{at}\ (\sigma_{F\ all})$: material property under tensile cyclic loading conditions (*tensile fatigue strength*).

  - S_{at} values are found from *Tables 15-6, 15-7* and *Fig. 15-13*.
    - S_{at} values are based on 10^7 cycles and *0.99* reliability.
    - For reversed loading such as in idler gears, *AGMA* recommends using 70% of S_{at} value.

Table 14–8

Elastic Coefficient C_p (Z_E), $\sqrt{\text{psi}}$ ($\sqrt{\text{MPa}}$) *Source:* AGMA 218.01

Pinion Material	Pinion Modulus of Elasticity E_p psi (MPa)*	Gear Material and Modulus of Elasticity E_G, lbf/in² (MPa)*: Steel 30×10^6 (2×10^5)	Malleable Iron 25×10^6 (1.7×10^5)	Nodular Iron 24×10^6 (1.7×10^5)	Cast Iron 22×10^6 (1.5×10^5)	Aluminum Bronze 17.5×10^6 (1.2×10^5)	Tin Bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.
*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

Table 15–6

Allowable Bending Stress Numbers for Steel Gears, s_{at} ($\sigma_{F\,\lim}$) *Source:* ANSI/AGMA 2003-B97.

Material Designation	Heat Treatment	Minimum Surface Hardness	Bending Stress Number (Allowable), s_{at} ($\sigma_{F\,\lim}$) lbf/in² (N/mm²): Grade 1*	Grade 2*	Grade 3*
Steel	Through-hardened	Fig. 15–13	Fig. 15–13	Fig. 15–13	
	Flame or induction hardened				
	Unhardened roots	50 HRC	15 000 (85)	13 500 (95)	
	Hardened roots		22 500 (154)		
	Carburized and case hardened†	2003-B97 Table 8	30 000 (205)	35 000 (240)	40 000 (275)
AISI 4140	Nitrided†,‡	84.5 HR15N		22 000 (150)	
Nitralloy 135M	Nitrided†,‡	90.0 HR15N		24 000 (165)	

*See ANSI/AGMA 2003-B97, Tables 8–11, for metallurgical factors for each stress grade of steel gears.
†The allowable stress numbers indicated may be used with the case depths prescribed in 21.1, ANSI/AGMA 2003-B97.
‡The overload capacity of nitrided gears is low. Since the shape of the effective S-N curve is flat, the sensitivity to shock should be investigated before proceeding with the design.

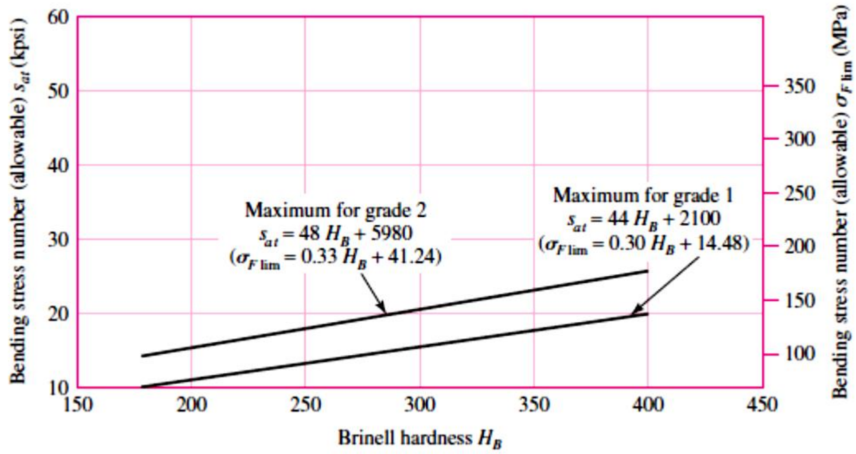
Table 15–7

Allowable Bending Stress Number for Iron Gears, s_{at} ($\sigma_{F\,\lim}$) *Source:* ANSI/AGMA 2003-B97.

Material	Material Designation: ASTM	ISO	Heat Treatment	Typical Minimum Surface Hardness	Bending Stress Number (Allowable), s_{at} ($\sigma_{F\,\lim}$) lbf/in² (N/mm²)
Cast iron	ASTM A48	ISO/DR 185			
	Class 30	Grade 200	As cast	175 HB	4500 (30)
	Class 40	Grade 300	As cast	200 HB	6500 (45)
Ductile (nodular) iron	ASTM A536	ISO/DIS 1083			
	Grade 80-55-06	Grade 600-370-03	Quenched and tempered	180 HB	10 000 (70)
	Grade 120-90-02	Grade 800-480-02		300 HB	13 500 (95)

Figure 15–13

Allowable bending stress number for through-hardened steel gears, $s_{at}(\sigma_{F\,lim})$. *(Source: ANSI/AGMA 2003-B97.)*



- ***Contact strength***, S_{ac} $(\sigma_{H\,all})$: material property under compressive cyclic loading conditions (*compressive fatigue strength*).

  - S_{ac} values are found from *Tables 15-4, 15-5* and *Fig. 15-12*.
    - ***Note:*** S_{ac} values are based on *10^9 cycles* and *0.99* reliability.

Table 15–4

Allowable Contact Stress Number for Steel Gears, s_{ac} $(\sigma_{H\,lim})$ *Source:* ANSI/AGMA 2003-B97.

Material Designation	Heat Treatment	Minimum Surface* Hardness	Allowable Contact Stress Number, s_{ac} $(\sigma_{H\,lim})$ lbf/in² (N/mm²) Grade 1†	Grade 2†	Grade 3†
Steel	Through-hardened‡	Fig. 15–12	Fig. 15–12	Fig. 15–12	
	Flame or induction hardened§	50 HRC	175 000 (1210)	190 000 (1310)	
	Carburized and case hardened§	2003-B97 Table 8	200 000 (1380)	225 000 (1550)	250 000 (1720)
AISI 4140	Nitrided§	84.5 HR15N		145 000 (1000)	
Nitralloy 135M	Nitrided§	90.0 HR15N		160 000 (1100)	

*Hardness to be equivalent to that at the tooth middepth in the center of the face width.

†See ANSI/AGMA 2003-B97, Tables 8 through 11, for metallurgical factors for each stress grade of steel gears.

‡These materials must be annealed or normalized as a minumum.

§The allowable stress numbers indicated may be used with the case depths prescribed in 21.1, ANSI/AGMA 2003-B97.

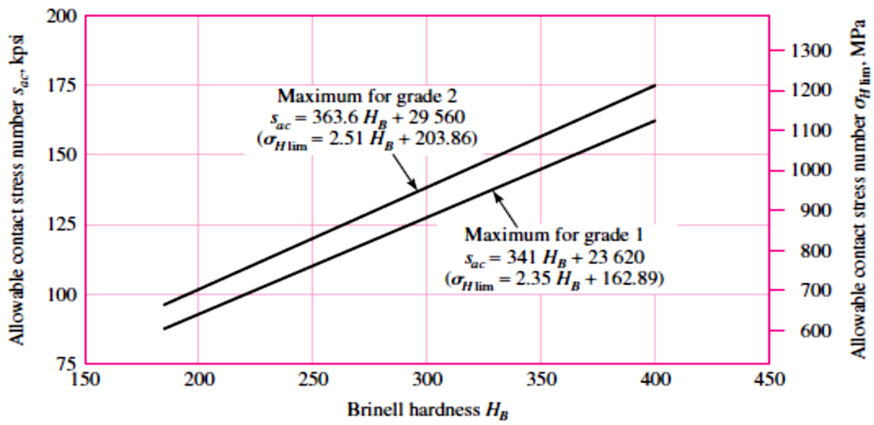
Table 15–5

Allowable Contact Stress Number for Iron Gears, s_{ac} ($\sigma_{H\,lim}$) *Source:* ANSI/AGMA 2003-B97.

Material	Material Designation ASTM	ISO	Heat Treatment	Typical Minimum Surface Hardness	Allowable Contact Stress Number, s_{ac} ($\sigma_{H\,lim}$) lbf/in² (N/mm²)
Cast iron	ASTM A48	ISO/DR 185			
	Class 30	Grade 200	As cast	175 HB	50 000 (345)
	Class 40	Grade 300	As cast	200 HB	65 000 (450)
Ductile (nodular) iron	ASTM A536	ISO/DIS 1083			
	Grade 80-55-06	Grade 600-370-03	Quenched	180 HB	94 000 (650)
	Grade 120-90-02	Grade 800-480-02	and tempered	300 HB	135 000 (930)

Figure 15–12

Allowable contact stress number for through-hardened steel gears, s_{ac}($\sigma_{H\,lim}$). *(Source: ANSI/AGMA 2003-B97.)*



- <u>*Stress-cycle factor for bending strength,*</u> $K_L(Y_{NT})$: it accounts for lives other than 10^7 cycles.
  - The value of $K_L(Y_{NT})$ is found from <u>*Fig. 15-9*</u>.

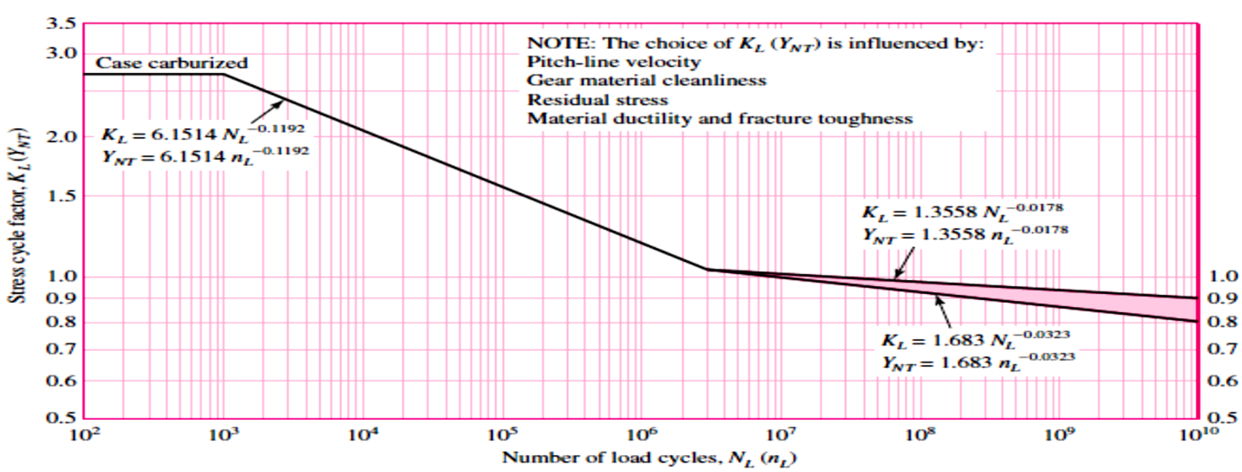


Figure 15–9

Stress cycle factor for bending strength K_L (Y_{NT}) for carburized case-hardened steel bevel gears. *(Source: ANSI/AGMA 2003-B97.)*

- *Stress-cycle factor for contact strength*, C_L (Z_{NT}): it accounts for lives other than 10^9 cycles.

  ❖ The value of C_L (Z_{NT}) is found from *Fig 15-8*.

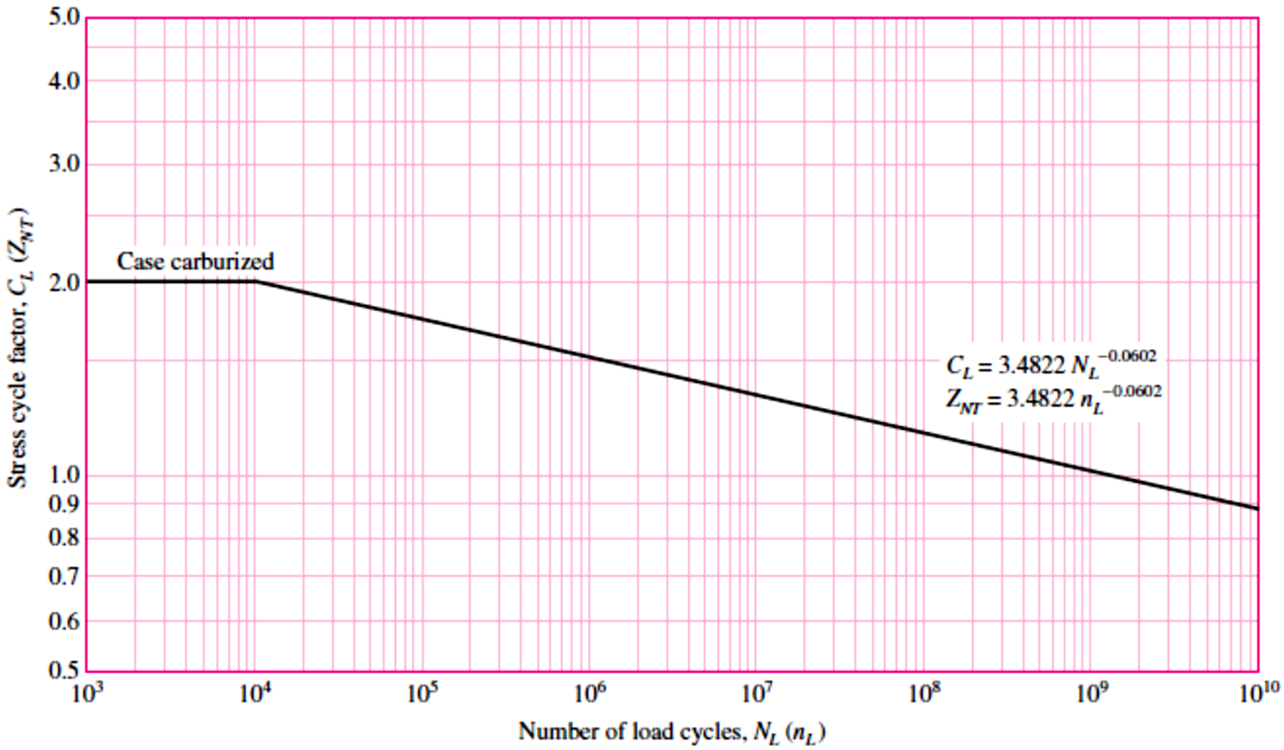


Figure 15–8

Contact stress cycle factor for pitting resistance C_L (Z_{NT}) for carburized case-hardened steel bevel gears. *(Source: ANSI/AGMA 2003-B97.)*

- *Hardness ratio factor*, C_H (Z_W) : it is used to account for the difference of the hardness between gear and pinion.

  ❖ For through-hardened gear and pinion , use *Fig. 15-10*.
  ❖ For surface-hardened pinion mating with through-hardened gear, use *Fig. 15-11*.

Figure 15–10

Hardness-ratio factor C_H (Z_W) for through-hardened pinion and gear. *(Source: ANSI/AGMA 2003-B97.)*

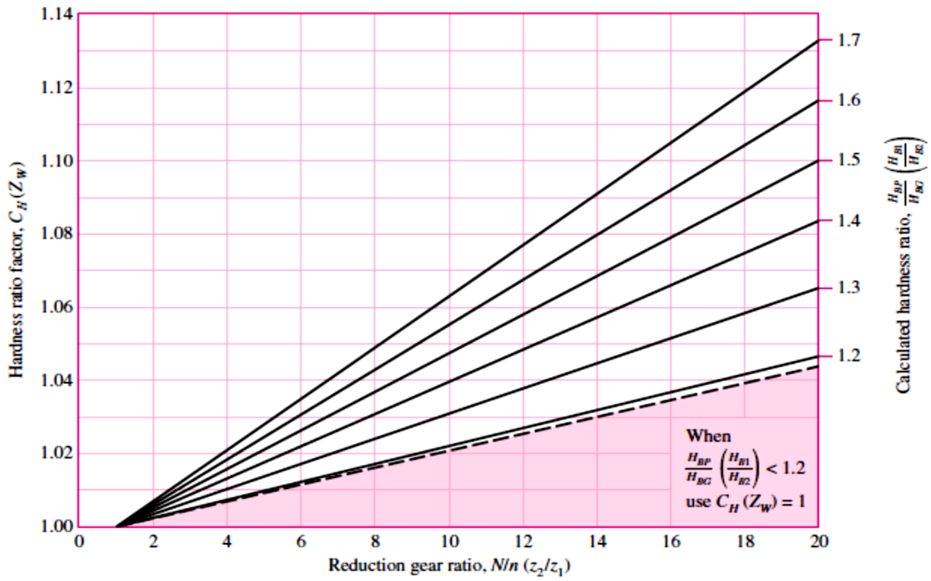
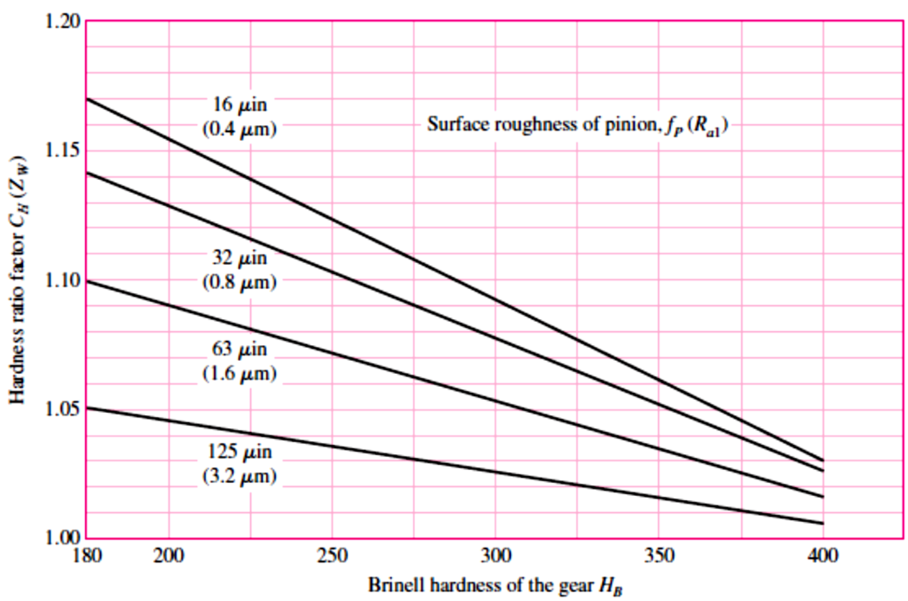


Figure 15–11

Hardness-ratio factor C_H (Z_W) for surface-hardened pinions. *(Source: ANSI/AGMA 2003-B97.)*



- ***Temperature factor***, K_T (K_θ): it accounts for the change in material strength at increased temperature.

$$K_T = \begin{cases} 1 & 32°\text{F} \le T \le 250\ °\text{F} \\ (460 + T)/710 & T > 250\ °\text{F} \end{cases}$$

$$K_\theta = \begin{cases} 1 & 0\ °\text{C} \le T \le 120\ °\text{C} \\ (273 + T)/393 & T > 120\ °\text{C} \end{cases}$$

- *Reliability factors*, K_R and C_R : used to account for reliabilities other than *0.99*

  K_R: reliability factor for bending strength
  C_R: reliability factor for contact strength
  where $C_R = \sqrt{K_R}$
  - Values of K_R and C_R are found from *Table 15-3*.

Table 15–3

Reliability Factors

Source: ANSI/AGMA 2003-B97.

Requirements of Application	Reliability Factors for Steel* C_R (Z_Z)	K_R (Y_Z)†
Fewer than one failure in 10 000	1.22	1.50
Fewer than one failure in 1000	1.12	1.25
Fewer than one failure in 100	1.00	1.00
Fewer than one failure in 10	0.92	0.85‡
Fewer than one failure in 2	0.84	0.70§

*At the present time there are insufficient data concerning the reliability of bevel gears made from other materials.

†Tooth breakage is sometimes considered a greater hazard than pitting. In such cases a greater value of K_R (Y_Z) is selected for bending.

‡At this value plastic flow might occur rather than pitting.

§From test data extrapolation.

- *Safety factors* S_F and S_H: used to account for unquantifiable elements affecting the stresses.
  - When designing the safety factor become a design factor.
  - When analyzing, the safety factor is the ratio of strength-to-stress.
  - when comparing bending and contact factors of safety, we compare S_F with $(S_H)^2$

Summary: Figures 15-14 & 15-15 give "roadmaps" for bending and contact equations for straight bevel gears.

Straight Bevel Gear Analysis