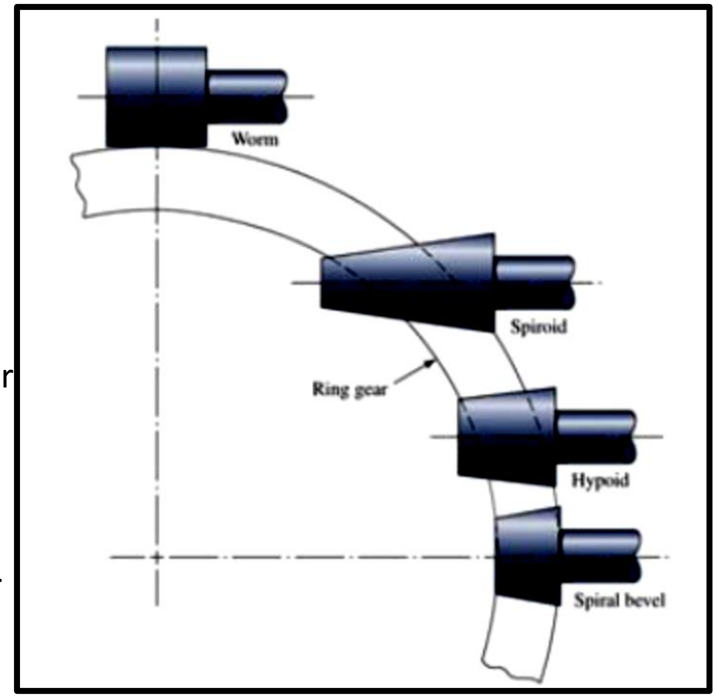
Bevel Gears - General

Bevel gears are classified as follows:

- *Straight Bevel Gears*: (*Fig. 13-35*) used for pitch line velocities up to *5m/s*, noise level is high.
- *Spiral Bevel Gears*: (*Fig. 15-1*) used for higher speeds, noise level is lower than straight gears because of gradual engagement of teeth (*similar to helical gears*). Spiral angle see Fig. 15-2.
- *Zerol Bevel Gears*: it has curved teeth but zero spiral angle. Usually used instead of straight bevel gears because of lower noise level. It has smaller thrust component than spiral gears (because of zero spiral angle).
- *Hypoid Bevel Gears*: (*Fig. 15-3*) used for offseted shafts. Teeth action is combination of rolling and sliding (*more friction*).
- *Spiroid Bevel Gears*: (*Fig 15-4*) used for shafts with large offset (the *pinion is similar to a worm*).

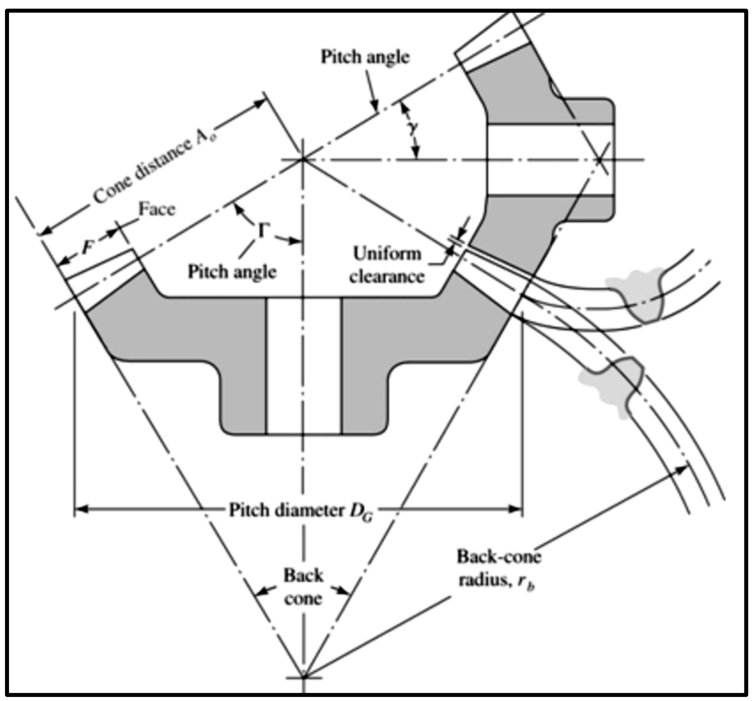


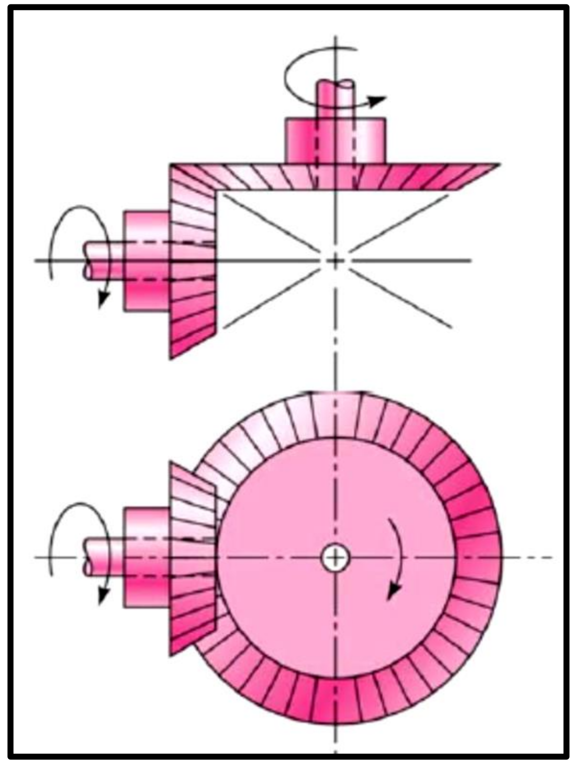
Straight Bevel Gears

Bevel gears are used to transmit motion between intersecting shafts. The shafts usually make *90* angle with each other but also other angles are possible.

The figure illustrates the terminology of bevel gears.

- The pitch diameter is measured at the large end of the tooth.
- The circular pitch and diametral pitch are calculated the same as in spur gears.
- The pitch angles are defined as shown the figure and they are related to the number of teeth as follows:





Pinion: $\boxed{\tan\gamma = \frac{N_P}{N_G}}$

Gear: $\boxed{\tan\Gamma = \frac{N_G}{N_P}}$

Standard straight-tooth bevel gears are cut using 20° pressure angle.

The shape of teeth, projected on back cone, is same as in a spur gear with radius rb

- Virtual number of teeth in this virtual spur gear is

$$N' = \frac{2\pi r_b}{p} \qquad (13\text{–}15)$$

Force Analysis-Bevel Gears

The point of action of the force is assumed to be at the midpoint of the tooth (*though it is really between the midpoint and the large end of the tooth*).

The transmitted load is found as

$$W_t = \frac{T}{r_{av}}$$

Where, T: **Torque**

r_{av}: **Pitch radius at midpoint**

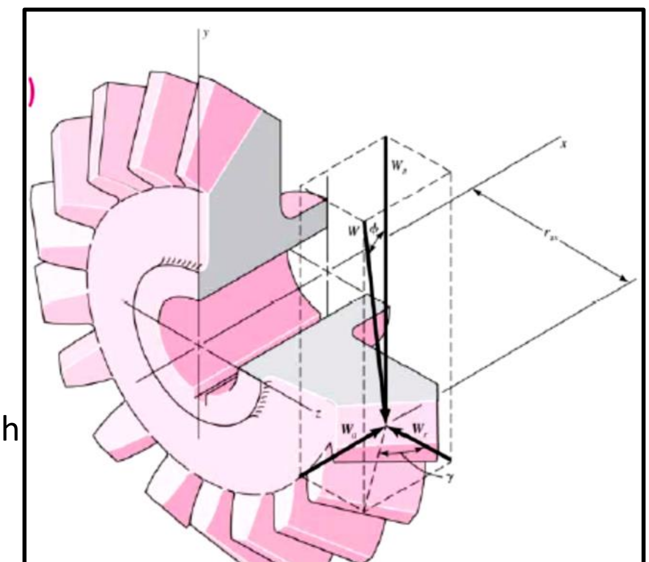


Fig. 13–35

Shigley's Mechanical Engineering Design

The forces acting on the midpoint of the tooth are shown in the figure.

From trigonometry, the radial force W_r and axial force W_a can be found as:

$$W_r = W_t \tan\phi \cos\gamma$$

$$W_a = W_t \tan\phi \sin\gamma$$

Where, ϕ: **Pressure angle**

γ: **Pitch angle**

Example 13–8

The bevel pinion in Fig. 13–36*a* rotates at 600 rev/min in the direction shown and transmits 5 hp to the gear. The mounting distances, the location of all bearings, and the average pitch radii of the pinion and gear are shown in the figure. For simplicity, the teeth have been replaced by pitch cones. Bearings *A* and *C* should take the thrust loads. Find the bearing forces on the gearshaft.

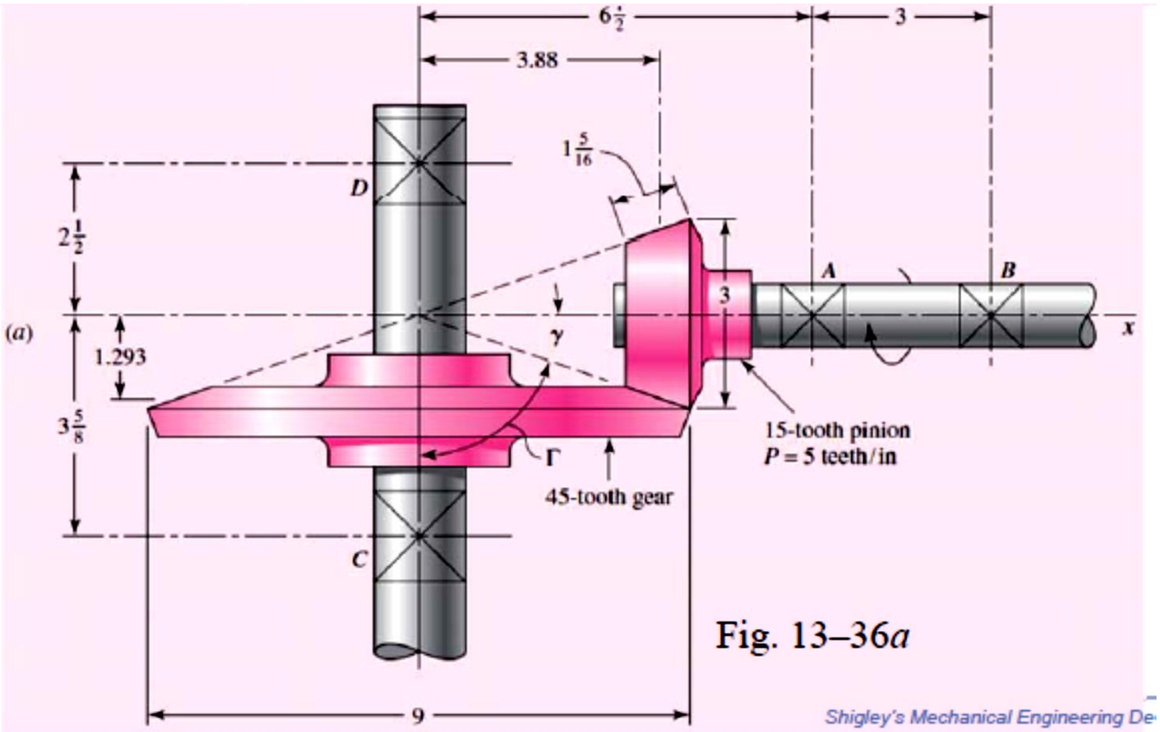


Fig. 13–36*a*

The pitch angles are

$$\gamma = \tan^{-1}\left(\frac{3}{9}\right) = 18.4^\circ \qquad \Gamma = \tan^{-1}\left(\frac{9}{3}\right) = 71.6^\circ$$

The pitch-line velocity corresponding to the average pitch radius is

$$V = \frac{2\pi r_P n}{12} = \frac{2\pi(1.293)(600)}{12} = 406\ \text{ft/min}$$

Therefore the transmitted load is

$$W_t = \frac{33\,000H}{V} = \frac{(33\,000)(5)}{406} = 406\ \text{lbf}$$

which acts in the positive z direction, as shown in Fig. 13–36*b*. We next have

$$W_r = W_t \tan\phi \cos\Gamma = 406 \tan 20^\circ \cos 71.6^\circ = 46.6 \text{ lbf}$$

$$W_a = W_t \tan\phi \sin\Gamma = 406 \tan 20^\circ \sin 71.6^\circ = 140 \text{ lbf}$$

where W_r is in the $-x$ direction and W_a is in the $-y$ direction, as illustrated in the isometric sketch of Fig. 13–36*b*.

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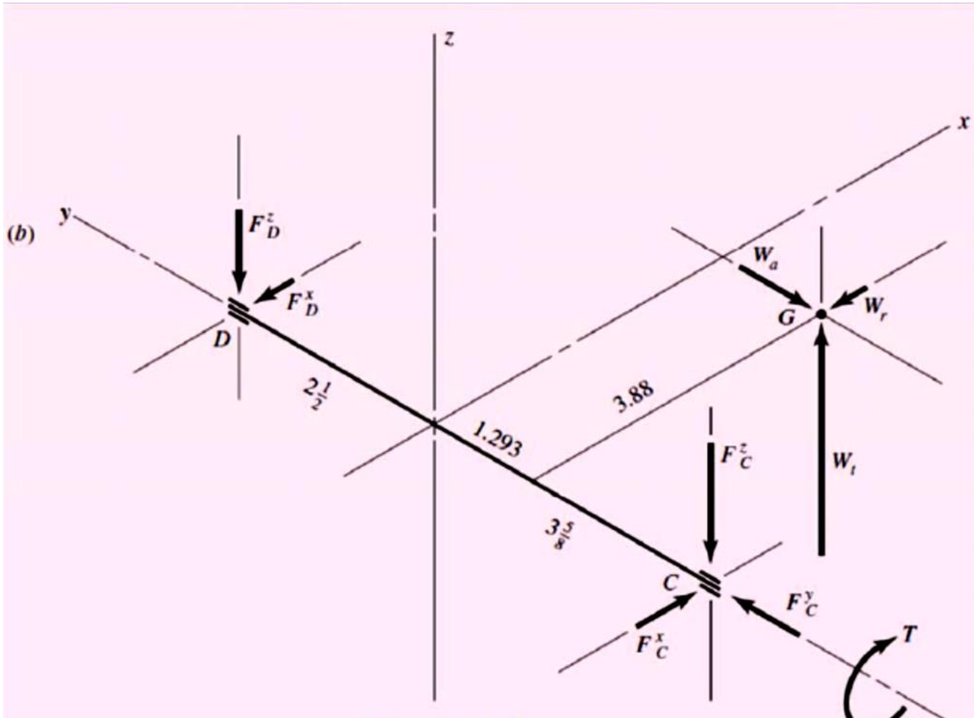

Fig. 13–36*b*

In preparing to take a sum of the moments about bearing D, define the position vector from D to G as

$$\mathbf{R}_G = 3.88\mathbf{i} - (2.5 + 1.293)\mathbf{j} = 3.88\mathbf{i} - 3.793\mathbf{j}$$

We shall also require a vector from D to C:

$$\mathbf{R}_C = -(2.5 + 3.625)\mathbf{j} = -6.125\mathbf{j}$$

Then, summing moments about D gives

$$\mathbf{R}_G \times \mathbf{W} + \mathbf{R}_C \times \mathbf{F}_C + \mathbf{T} = \mathbf{0} \qquad (1)$$

When we place the details in Eq. (1), we get

$$(3.88\mathbf{i} - 3.793\mathbf{j}) \times (-46.6\mathbf{i} - 140\mathbf{j} + 406\mathbf{k}) + (-6.125\mathbf{j}) \times (F_C^x\mathbf{i} + F_C^y\mathbf{j} + F_C^z\mathbf{k}) + T\mathbf{j} = \mathbf{0} \qquad (2)$$

After the two cross products are taken, the equation becomes

$$(-1540\mathbf{i} - 1575\mathbf{j} - 720\mathbf{k}) + (-6.125F_C^z\mathbf{i} + 6.125F_C^x\mathbf{k}) + T\mathbf{j} = \mathbf{0}$$

from which

$$\mathbf{T} = 1575\mathbf{j}\ \text{lbf} \cdot \text{in} \qquad F_C^x = 118\ \text{lbf} \qquad F_C^z = -251\ \text{lbf} \qquad (3)$$

Now sum the forces to zero. Thus

$$\mathbf{F}_D + \mathbf{F}_C + \mathbf{W} = \mathbf{0} \qquad (4)$$

When the details are inserted, Eq. (4) becomes

$$(F_D^x\mathbf{i} + F_D^z\mathbf{k}) + (118\mathbf{i} + F_C^y\mathbf{j} - 251\mathbf{k}) + (-46.6\mathbf{i} - 140\mathbf{j} + 406\mathbf{k}) = \mathbf{0} \qquad (5)$$

First we see that $F_C^y = 140$ lbf, and so

$$\mathbf{F}_C = 118\mathbf{i} + 140\mathbf{j} - 251\mathbf{k}\ \text{lbf}$$

Then, from Eq. (5),

$$\mathbf{F}_D = -71.4\mathbf{i} - 155\mathbf{k}\ \text{lbf}$$

Worm Gears

- The worm and worm gear have the same hand of helix, but the helix angles are different.
- Helix angle for the worm, ψ_W, is large (see fig. 13-24) and usually the lead angle, λ , is specified instead.
- Lead angle for the worm, λ , is equal to the gear helix angle, ψ_G, (for shafts at 90°).
- Typically, the axial pitch "P_x" for the worm and the transverse pitch " P_t " for the gear are specified.
- The lead " l "and the axial pitch " P_x " of the worm are related as, (where : number of teeth of the worm).

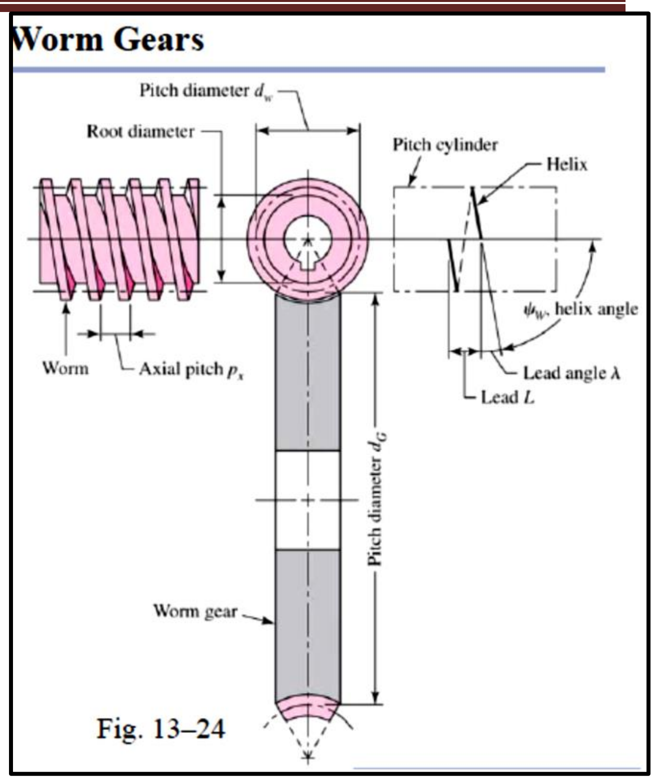

Fig. 13–24

- The lead " l " and the lead angle " λ " of the worm are related as ,

$$L = p_x N_W \qquad (13\text{–}27)$$

$$\tan\lambda = \frac{l}{\pi d_w}$$

- Pitch diameter of gear is measured on plane containing worm axis

$$d_G = \frac{N_G p_t}{\pi} \qquad (13\text{–}25)$$

Worm may have any pitch diameter.

- Should be same as hob used to cut the gear teeth
- Recommended range for worm pitch diameter as a function of center distance C,

$$\frac{C^{0.875}}{3.0} \le d_W \le \frac{C^{0.875}}{1.7} \qquad (13\text{–}26)$$

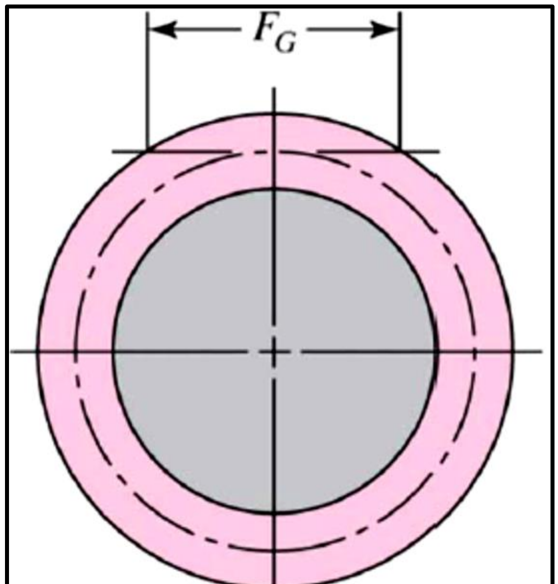

Fig. 13–25

Face Width of Worm Gear

- Face width F_G of a worm gear should

be equal to the length of a tangent to the worm pitch circle between its points of intersection with the addendum circle

Force Analysis - Worm Gears

With friction neglected, the only force acting (x, y and z) is the normal force W. The components of the force W^x, W^y & W^z are shown in Fig. 13-40. Using subscripts W and G to indicate forces acting on the Worm and Gear

We note that

$$\begin{aligned} W^x &= W_{wt} = -W_{Ga} \\ W^y &= W_{wr} = -W_{Gr} \\ W^z &= W_{wa} = -W_{Gt} \end{aligned} \qquad (13\text{–}41)$$

For gear axis in the x direction and worm axis in the z direction

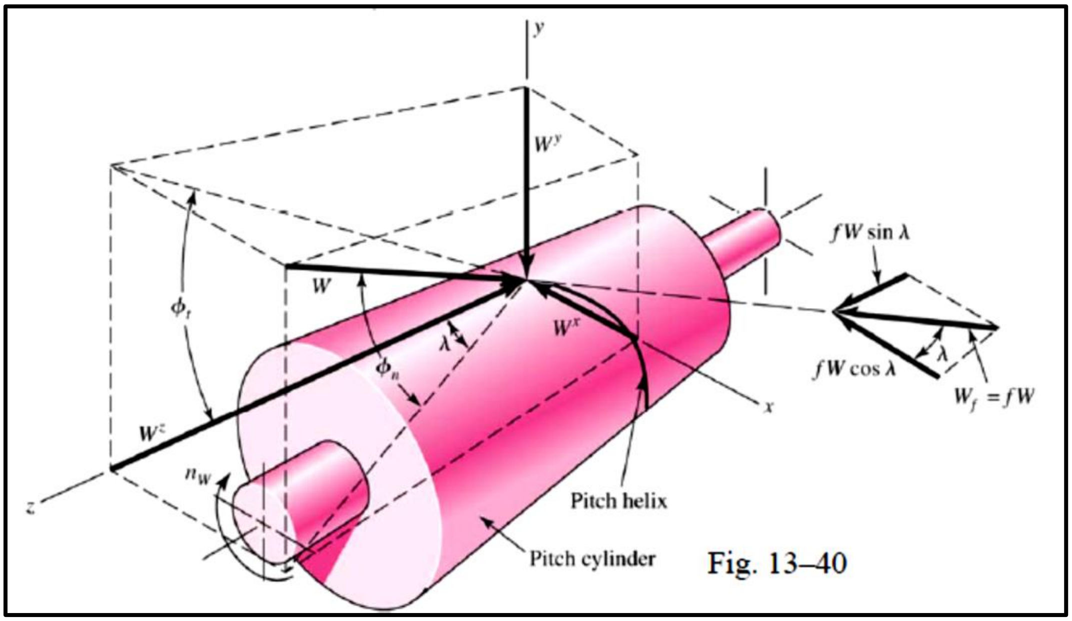

Fig. 13–40

$$
\begin{aligned}
W_{Wt} &= -W_{Ga} = W^x \\
W_{Wr} &= -W_{Gr} = W^y \\
W_{Wa} &= -W_{Gt} = W^z
\end{aligned}
\qquad (13\text{–}42)
$$

In spur gears, the relative motion of mating teeth is primarily rolling thus friction is negligible. Where as in worm gears, the relative motion between the worm and worm gear teeth is pure sliding therefore friction is significant in worm gears performance.

Introducing a coefficient of friction "f" the frictional force (acting tangential to the surface of the tooth) " $W_f = fW$ "contributes in the force components:
Relative motion in worm gearing is sliding action

- Friction is much more significant than in other types of gears
- Including friction components, Eq. (13-41) can be expanded to

$$
\begin{aligned}
W^x &= W(\cos\phi_n \sin\lambda + f\cos\lambda) \\
W^y &= W\sin\phi_n \\
W^z &= W(\cos\phi_n \cos\lambda - f\sin\lambda
\end{aligned}
\qquad (13\text{–}43)
$$

Where,
ϕ_n:normal pressure angle
λ: lead angle

- Combining with Eqs. (13-42) and (13-43), the frictional force can be found as

$$W_f = fW = \frac{fW_{Gt}}{f\sin\lambda - \cos\phi_n \cos\lambda} \qquad (13\text{–}44)$$

The tangential forces of the worm " W_{wt} " and the gear " W_{Gt} " can be related as:

$$W_{wt} = W_{Gt}\frac{\cos\phi_n \sin\lambda + f\cos\lambda}{f\sin\lambda - \cos\phi_n\cos\lambda} \qquad (13\text{–}45)$$

The efficiency of force transmission by the worm set can be defined as: · From Eq. (13–45) with f = 0 in the numerator,

$$\eta = \frac{W_{wt}(no\ friction)}{W_{wt}(with\ friction)} = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} \qquad (13\text{–}45)$$

- Using this equation it can be seen that the efficiency increases with increasing helix angle " " (i.e., decreasing λ), see Table 13-6.
- Experiments have shown that the coefficient of friction depends on the sliding velocity, "V_s".
- The relative sliding velocity " V_s " can be found as, (see Fig 13-41)

$$V_s = \frac{V_W}{\cos\lambda} \qquad V_w\text{: Pitch-Line velocity of the worm}$$

- The coefficient of friction " f " can be found as function of V_s from Fig 13-42.

Worm Gearing Efficiency

- With typical value of f = 0.05, and ϕ_n = 20º, efficiency as a function of helix angle is given in the table.

Helix Angle ψ, deg	Efficiency η, %
1.0	25.2
2.5	45.7
5.0	62.0
7.5	71.3
10.0	76.6
15.0	82.7
20.0	85.9
30.0	89.1

Table 13–6 Shigley

Worm Gearing Efficiency

- Coefficient of friction is dependent on relative or sliding velocity V_S
- V_G is pitch line velocity of gear
- V_W is pitch line velocity of worm

$$\mathbf{V}_W = \mathbf{V}_G + \mathbf{V}_S$$

$$V_S = \frac{V_W}{\cos\lambda} \qquad (13\text{–}47)$$

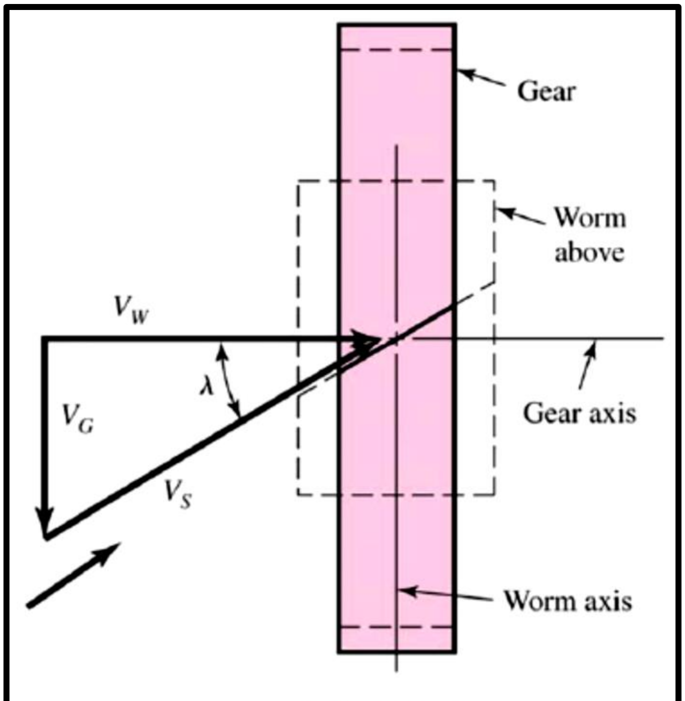

Fig. 13–41

Coefficient of Friction for Worm Gearing

- Graph shows representative values
- Curve A is for when more friction is expected, such as when gearsare cast iron
- Curve B is for high-quality materials

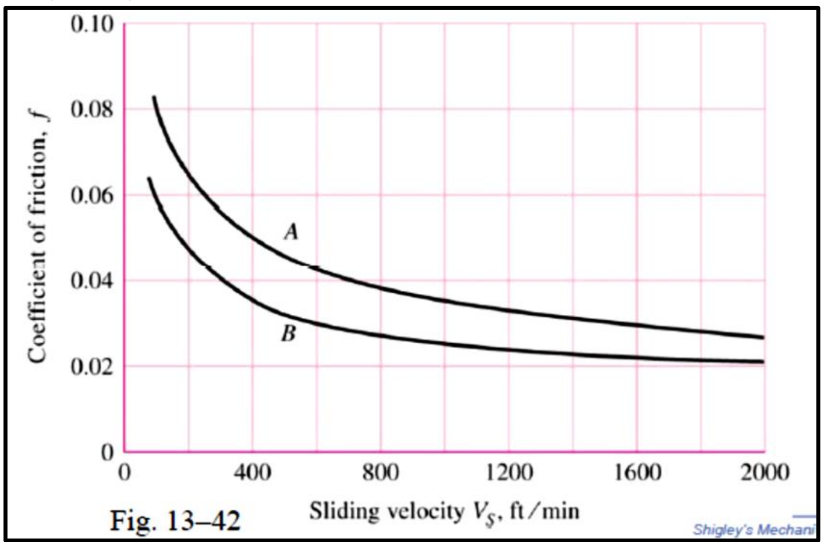

Fig. 13–42

Example 13–10

A 2-tooth right-hand worm transmits 1 hp at 1200 rev/min to a 30-tooth worm gear. The gear has a transverse diametral pitch of 6 teeth/in and a face width of 1 in. The worm has a pitch diameter of 2 in and a face width of $2\frac{1}{2}$ in. The normal pressure angle is $14\frac{1}{2}^{\circ}$. The materials and quality of work needed are such that curve *B* of Fig. 13–42 should be used to obtain the coefficient of friction.

(*a*) Find the axial pitch, the center distance, the lead, and the lead angle.

(*b*) Figure 13–43 is a drawing of the worm gear oriented with respect to the coordinate system described earlier in this section; the gear is supported by bearings *A* and *B*. Find the forces exerted by the bearings against the worm-gear shaft, and the output torque.

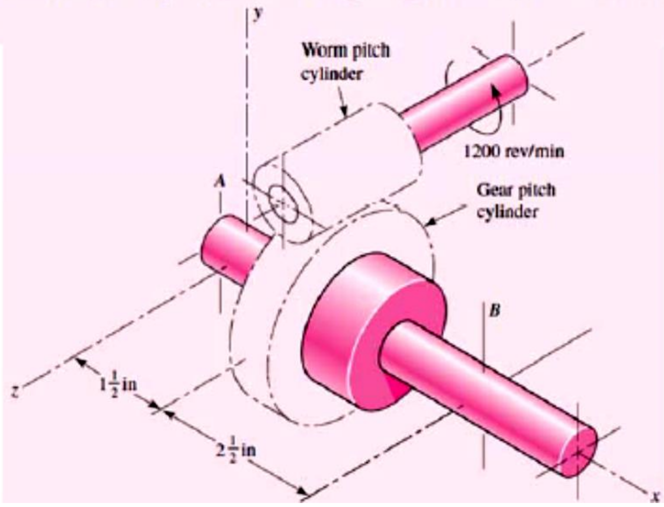

Fig. 13–43

Shigley's Mechanical Engineering Design

(*a*) The axial pitch is the same as the transverse circular pitch of the gear, which is

$$p_x = p_t = \frac{\pi}{P} = \frac{\pi}{6} = 0.5236 \text{ in}$$

The pitch diameter of the gear is $d_G = N_G/P = 30/6 = 5$ in. Therefore, the center distance is

$$C = \frac{d_W + d_G}{2} = \frac{2+5}{2} = 3.5 \text{ in}$$

From Eq. (13–27), the lead is

$$L = p_x N_W = (0.5236)(2) = 1.0472 \text{ in}$$

Also using Eq. (13–28), find

$$\lambda = \tan^{-1}\frac{L}{\pi d_W} = \tan^{-1}\frac{1.0472}{\pi(2)} = 9.46^\circ$$

(*b*) Using the right-hand rule for the rotation of the worm, you will see that your thumb points in the positive z direction. Now use the bolt-and-nut analogy (the worm is right-handed, as is the screw thread of a bolt), and turn the bolt clockwise with the right hand while preventing nut rotation with the left. The nut will move axially along the bolt toward your right hand. Therefore the surface of the gear (Fig. 13–43) in contact with the worm will move in the negative z direction. Thus, the gear rotates clockwise about x, with your right thumb pointing in the negative x direction.

The pitch-line velocity of the worm is

$$V_W = \frac{\pi d_W n_W}{12} = \frac{\pi(2)(1200)}{12} = 628 \text{ ft/min}$$

The speed of the gear is $n_G = (\frac{2}{30})(1200) = 80$ rev/min. Therefore the pitch-line velocity of the gear is

$$V_G = \frac{\pi d_G n_G}{12} = \frac{\pi(5)(80)}{12} = 105 \text{ ft/min}$$

Then, from Eq. (13–47), the sliding velocity V_S is found to be

$$V_S = \frac{V_W}{\cos\lambda} = \frac{628}{\cos 9.46^\circ} = 637 \text{ ft/min}$$

Getting to the forces now, we begin with the horsepower formula

$$W_{Wt} = \frac{33\,000H}{V_W} = \frac{(33\,000)(1)}{628} = 52.5 \text{ lbf}$$

This force acts in the negative x direction, the same as in Fig. 13–40. Using Fig. 13–42, we find $f = 0.03$. Then, the first equation of group (13–42) and (13–43) gives

$$W = \frac{W^x}{\cos\phi_n \sin\lambda + f\cos\lambda}$$

$$= \frac{52.5}{\cos 14.5^\circ \sin 9.46^\circ + 0.03\cos 9.46^\circ} = 278 \text{ lbf}$$

Also, from Eq. (13–43),

$$W^y = W \sin \phi_n = 278 \sin 14.5° = 69.6 \text{ lbf}$$

$$W^z = W(\cos \phi_n \cos \lambda - f \sin \lambda)$$

$$= 278(\cos 14.5° \cos 9.46° - 0.03 \sin 9.46°) = 264 \text{ lbf}$$

We now identify the components acting on the gear as

$$W_{Ga} = -W^x = 52.5 \text{ lbf}$$

$$W_{Gr} = -W^y = -69.6 \text{ lbf}$$

$$W_{Gt} = -W^z = -264 \text{ lbf}$$

At this point a three-dimensional line drawing should be made in order to simplify the work to follow. An isometric sketch, such as the one of Fig. 13–44, is easy to make and will help you to avoid errors.

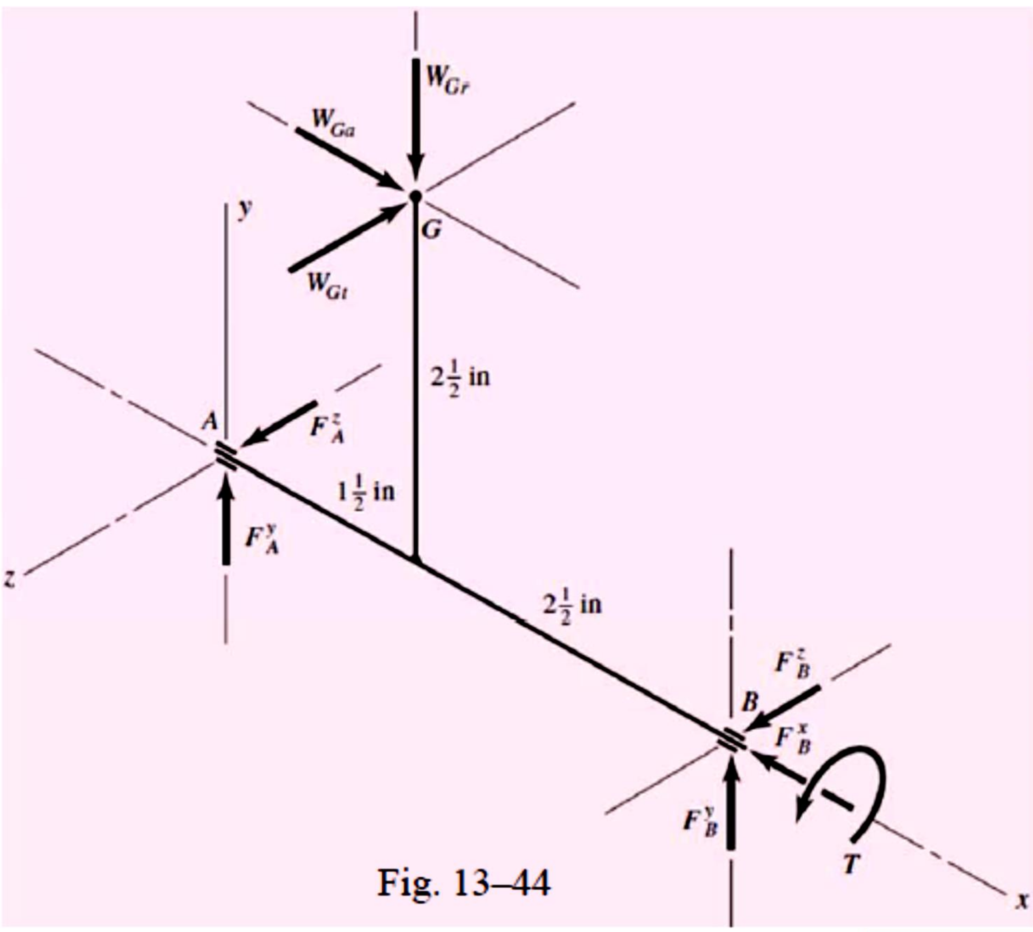

Fig. 13–44

We shall make B a thrust bearing in order to place the gearshaft in compression. Thus, summing forces in the x direction gives

$$F_B^x = -52.5 \text{ lbf}$$

Taking moments about the z axis, we have

$$-(52.5)(2.5) - (69.6)(1.5) + 4F_B^y = 0 \qquad F_B^y = 58.9 \text{ lbf}$$

Taking moments about the y axis,

$$(264)(1.5) - 4F_B^z = 0 \qquad F_B^z = 99 \text{ lbf}$$

These three components are now inserted on the sketch as shown at B in Fig. 13–44.

Summing forces in the y direction,

$$-69.6 + 58.9 + F_A^y = 0 \qquad F_A^y = 10.7 \text{ lbf}$$

Similarly, summing forces in the z direction,

$$-264 + 99 + F_A^z = 0 \qquad F_A^z = 165 \text{ lbf}$$

These two components can now be placed at A on the sketch. We still have one more equation to write. Summing moments about x,

$$-(264)(2.5) + T = 0 \qquad T = 660 \text{ lbf} \cdot \text{in}$$

It is because of the frictional loss that this output torque is less than the product of the gear ratio and the input torque.