Surface Condition Factor, $C_f(Z_R)$

This factor depends on surface finish, residual stress and work hardening.

- It is used only in the contact stress equation.
- Standard surface conditions for teeth surface are not yet defined. Thus, we will use $C_f = 1$.

Load-Distribution Factor, $K_m(K_H)$

This factor is used to account for the non-uniform load distribution along the line of contact.

- One of the causes for non-uniform load distribution is the misalignment of the gear axis resulting from the deformation of the shafts carrying the gears. Other reasons include the inaccuracy in manufacturing and assembly.
- The load-distribution factor can be found as:

$$\boxed{K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)}$$

Note that this is valid only when:
- ✓ $F/d_p \leq 2$
- ✓ $F \leq 40\ in$
- ✓ Gears mounted between two bearings
- ✓ Contact across full width of narrowest gear

where,

Crowning

$$C_{mc} = \begin{cases} 1 & \text{for non-crowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases}$$

Face width (misalignment magnification)

$$C_{pf} = \begin{cases} F/10d - 0.025 & \text{for } F \leq 1\ in \\ F/10d - 0.0375 + 0.0125F & \text{for } 1 < F \leq 17\ in \\ F/10d - 0.1109 + 0.0207F - 0.000228F^2 & \text{for } 17 < F \leq 40\ in \end{cases}$$

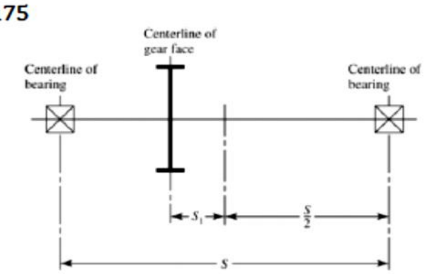
To be conservative we use d_p

➤ *Note: when $F/10d < 0.05$, use $F/10d = 0.05$ instead*

Mounting position

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \geq 0.175 \end{cases}$$

- *See the figure for S & S_1*
  S: full span
  S_1: distance from midspan



Manufacturing accuracy

$$C_{ma} = A + BF + CF^2$$

❖ Values of *A, B, & C* are found in *Table 14-9*

❖ C_{ma} can also be found from *Fig. 14-11*

Extra adjustment

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or computability improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases}$$

Table 14–9

Empirical Constants A, B, and C for Eq. (14–34), Face Width F in Inches*

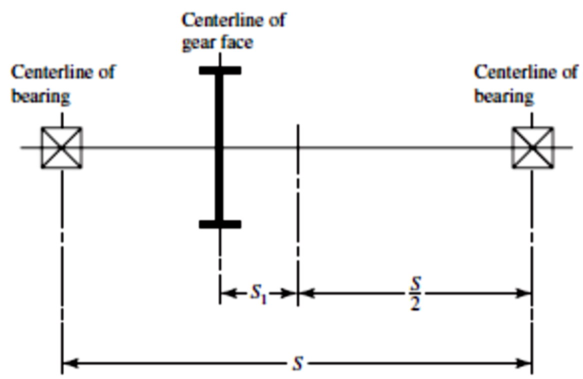
Source: ANSI/AGMA 2001-D04.

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

Figure 14–10

Definition of distances S and S_1 used in evaluating C_{pm}, Eq. (14–33). *(ANSI/AGMA 2001-D04.)*



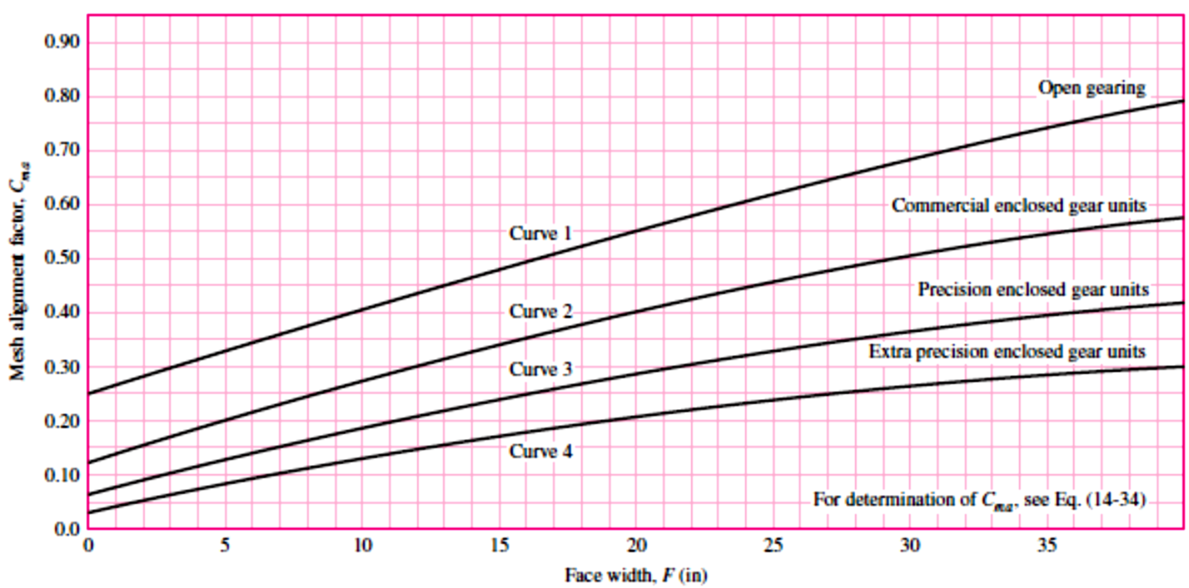


Figure 14–11

Mesh alignment factor C_{ma}. Curve-fit equations in Table 14–9. *(ANSI/AGMA 2001-D04.)*

Hardness-Ratio Factor, C_H

The pinion has less number of teeth than the gear and therefore the teeth of the pinion are subjected to more cycles of contact stress. To compensate for that, different heat treatments are used for the pinion and the gear to make the pinion harder than the gear.

- The hardness-ratio factor is used to account for the difference in hardness, and it is used *only for the gear*.

❖ For through-hardened pinion and gear, the C_H value can be found from the *Figure 14-12* (*for* $1.2 \le H_{Bp}/H_{BG} \le 1.7$), or from *Eqn. 14-36* in text.

where H_{Bp} & H_{BG} are *Brinell* hardness for pinion and gear

- If $H_{Bp}/H_{BG} < 1.2$, $C_H = 1$

- If $H_{Bp}/H_{BG} > 1.7$, $C_H = 1 + 0.00698\,(m_G - 1)$ where m_G is the speed ratio

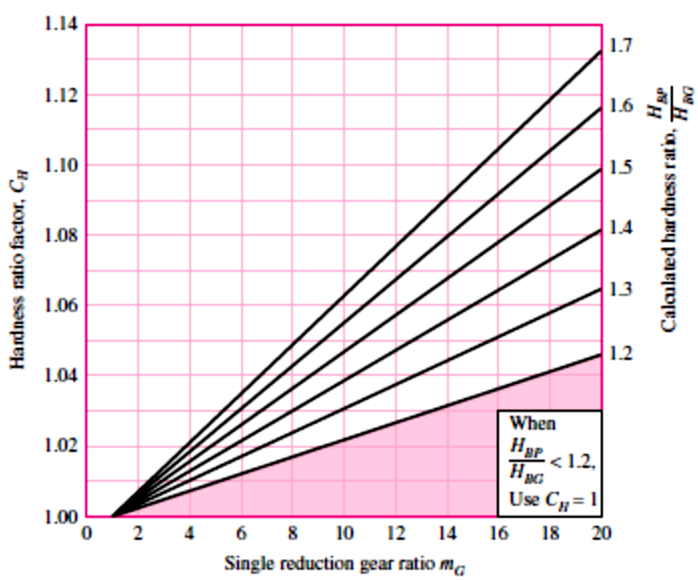
$$C_H = 1.0 + A'(m_G - 1.0) \qquad (14\text{–}36)$$

where

$$A' = 8.98(10^{-3})\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29(10^{-3})\quad 1.2 \le \frac{H_{BP}}{H_{BG}} \le 1.7$$

Figure 14–12

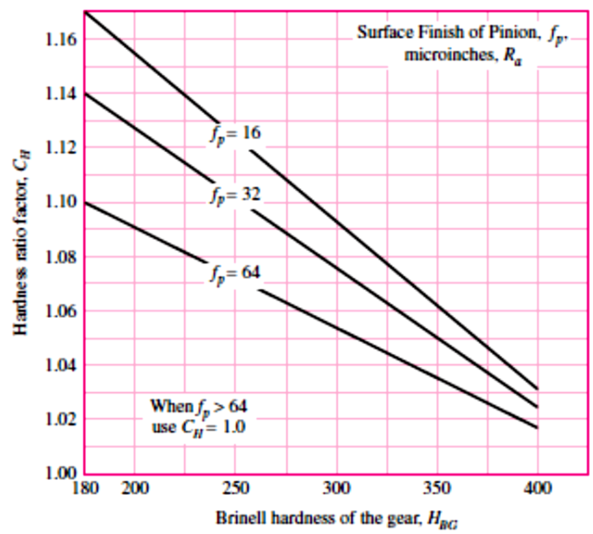
Hardness ratio factor C_H (through-hardened steel). *(ANSI/AGMA 2001-D04.)*



❖ For <u>surface-hardened pinion</u> (with hardness of *Rockwell-C: 48* or harder) run with <u>through-hardened gear</u> (*180 to 400 Brinell*), the C_H value can be found from *<u>Figure 14-13</u>* as a function of the <u>pinion surface finish</u> "f_P".

Figure 14–13

Hardness ratio factor C_H (surface-hardened steel pinion). *(ANSI/AGMA 2001-D04.)*



where $B' = 0.000\,75 \exp[-0.0112 f_P]$ and f_P is the surface finish of the pinion expressed as root-mean-square roughness R_a in μ in.

Rim-Thickness Factor, K_B

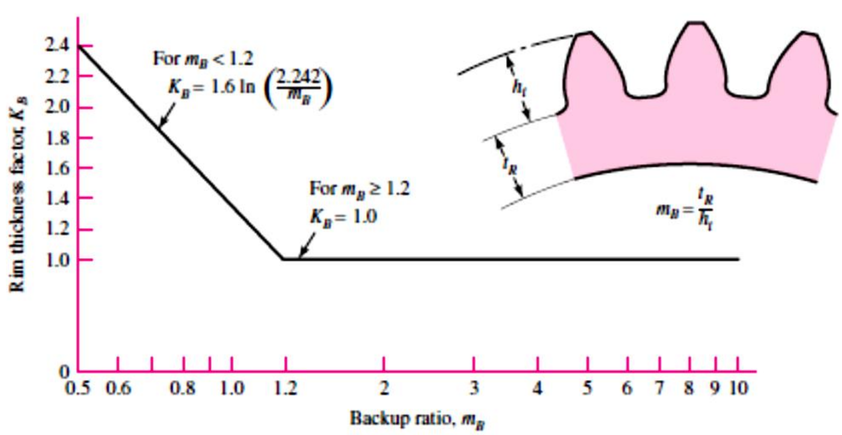
When the rim-thickness is not sufficient, it will not provide full support for the tooth causing increased stress.

- The rim-thickness factor is used to account for the <u>increase in bending stress</u> in thin-rimmed gears.

❖ The value of K_B depends on the rim-thickness to tooth-height ratio, and it can be found from *<u>Figure 14-16</u>*.

Figure 14–16

Rim thickness factor K_B. *(ANSI/AGMA 2001-D04.)*



Stress Cycle Life Factors, Y_N *and* Z_N

The *AGMA* bending strength "S_t" and contact strength "S_c" are based on 10^7 load cycles.

- The load-cycle factors Y_N and Z_N, are used to modify the *AGMA* strength for lives other than 10^7 cycles.

❖ The values of Y_N and Z_N are found from *Figures 14-14 & 14-15* according to the number of load cycles for each gear.

- Note that for 10^7 Cycles $Y_N = Z_N = 1$

Figure 14–14

Repeatedly applied bending strength stress-cycle factor Y_N. (ANSI/AGMA 2001-D04.)

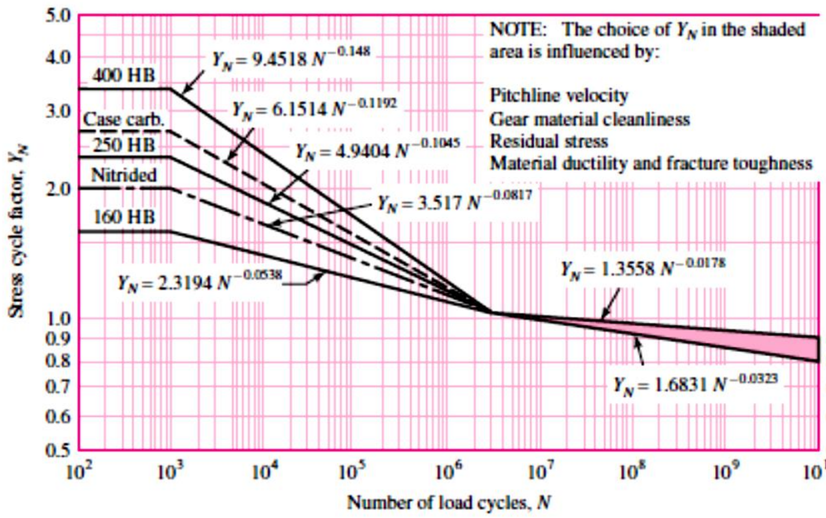
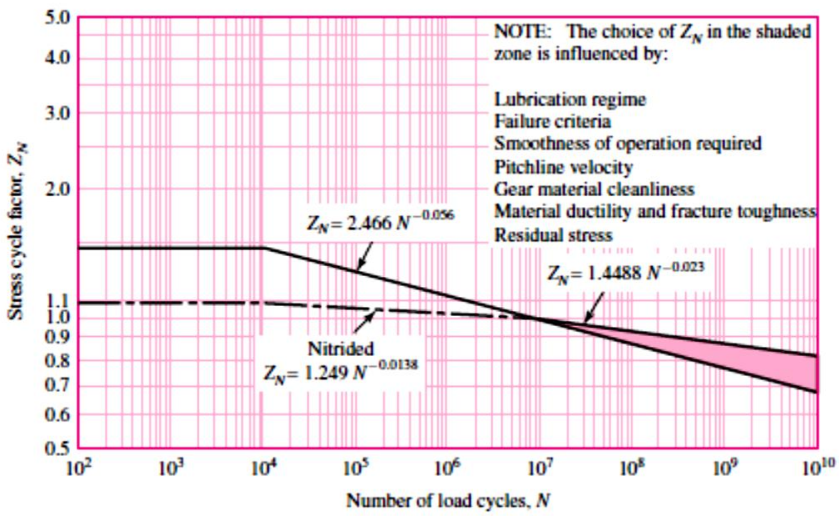


Figure 14–15

Pitting resistance stress-cycle factor Z_N. (ANSI/AGMA 2001-D04.)



Temperature Factor, $K_T(Y_\theta)$

This factor is used to modify the *AGMA* strengths for the effect of high operating temperatures.

- For lubricant (or gear-blank) temperatures up to *250°F* (*120°C*): $K_T=1$
- For temperature higher that *250°F*, K_T will be greater than one. But no data is available for such conditions.
- Heat exchangers may be used to maintain temperature below *250°F*

Reliability Factor, $K_R(Y_Z)$

The *AGMA* strengths S_t & S_c are based on *0.99* reliability.

- The reliability factor is used to modify the *AGMA* strength for reliabilities other that *0.99*.

❖ The values of K_R for some reliability values are found in *Table 14-10*.

- For reliability values other than those given in the table, the K_R value can be found as:

$$K_R = \begin{cases} 0.658 - 0.0759\ln(1-R) & \text{for } 0.5 < R < 0.99 \\ 0.5 - 0.109\ln(1-R) & \text{for } 0.99 < R < 0.9999 \end{cases}$$

Table 14–10

Reliability Factors $K_R(Y_Z)$

Source: ANSI/AGMA 2001-D04.

Reliability	$K_R\ (Y_Z)$
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

Safety Factors, S_F and S_H

A factor of safety is used to account for unquantifiable elements affecting the stress.

- When designing gear stets, a factor of safety becomes a design factor (*i.e., specified by the designer*) indicating the desired strength-to-stress ratio.
- When analyzing or doing a design assessment for a gear set, the value of safety factor is the ratio of strength to stress.
  - Bending stress factor of safety S_F is found as:

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{Fully\ corrected\ bending\ strength}{Fully\ corrected\ bending\ stress}$$

- Where S_F is linearly related to the transmitted load W^t (*since the relation between σ and* W^t *is linear*).

- Contact stress factor of safety S_H is found as:

only for the gear

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{Fully\ corrected\ contact\ strength}{Fully\ corrected\ contact\ stress}$$

- Where S_H is not linearly related to the transmitted load W^t (*since the relation between* σ_c *and* W^t *is not linear*).

- Because the difference in the relation of S_F and S_H with the transmitted load, if we want to compare the values of S_F and S_H in an analysis (in order to determine the nature and severity of the threat of failure), then we should:
  - Compare S_F with S_H^2 *for linear or helical contact*
  - Compare S_F with S_H^3 *for spherical contact (crowned teeth)*

Analysis

- Figures 14-17 and 14-18 give a "road map" listing the *AGMA* equations for determining the binding and contact stresses and strengths as well as the factors of safety.
- It should be clear that most of the terms in the bending and contact stress or strength equations will have the same value for the pinion and the gear.
- The factors that could have two different values for the pinion and gear are: K_S, J, K_B, S_t, S_c Y_N, Z_N.

EXAMPLE 14–4 A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 4 hp to a 52-tooth disk gear. The diametral pitch is 10 teeth/in, the face width 1.5 in, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, $J_P = 0.30$, $J_G = 0.40$, and Young's modulus is $30(10^6)$ psi. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

(*a*) Find the factor of safety of the gears in bending.

(*b*) Find the factor of safety of the gears in wear.

(*c*) By examining the factors of safety, identify the threat to each gear and to the mesh.

Solution There will be many terms to obtain so use Figs. 14–17 and 14–18 as guides to what is needed.

$$d_P = N_P/P_d = 17/10 = 1.7 \text{ in} \qquad d_G = 52/10 = 5.2 \text{ in}$$

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.7)1800}{12} = 801.1 \text{ ft/min}$$

$$W^t = \frac{33\,000\,H}{V} = \frac{33\,000(4)}{801.1} = 164.8 \text{ lbf}$$

Assuming uniform loading, $K_o = 1$. To evaluate K_v, from Eq. (14–28) with a quality number $Q_v = 6$,

$$B = 0.25(12-6)^{2/3} = 0.8255$$
$$A = 50 + 56(1 - 0.8255) = 59.77$$

Then from Eq. (14–27) the dynamic factor is

$$K_v = \left(\frac{59.77 + \sqrt{801.1}}{59.77}\right)^{0.8255} = 1.377$$

To determine the size factor, K_s, the Lewis form factor is needed. From Table 14–2, with $N_P = 17$ teeth, $Y_P = 0.303$. Interpolation for the gear with $N_G = 52$ teeth yields $Y_G = 0.412$. Thus from Eq. (*a*) of Sec. 14–10, with $F = 1.5$ in,

$$(K_s)_P = 1.192\left(\frac{1.5\sqrt{0.303}}{10}\right)^{0.0535} = 1.043$$

$$(K_s)_G = 1.192\left(\frac{1.5\sqrt{0.412}}{10}\right)^{0.0535} = 1.052$$

The load distribution factor K_m is determined from Eq. (14–30), where five terms are needed. They are, where $F = 1.5$ in when needed:

Uncrowned, Eq. (14–30): $C_{mc} = 1$,
Eq. (14–32): $C_{pf} = 1.5/[10(1.7)] - 0.0375 + 0.0125(1.5) = 0.0695$
Bearings immediately adjacent, Eq. (14–33): $C_{pm} = 1$
Commercial enclosed gear units (Fig. 14–11): $C_{ma} = 0.15$
Eq. (14–35): $C_e = 1$

Thus,

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) = 1 + (1)[0.0695(1) + 0.15(1)] = 1.22$$

Assuming constant thickness gears, the rim-thickness factor $K_B = 1$. The speed ratio is $m_G = N_G/N_P = 52/17 = 3.059$. The load cycle factors given in the problem statement, with N(pinion) $= 10^8$ cycles and N(gear) $= 10^8/m_G = 10^8/3.059$ cycles, are

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8/3.059)^{-0.0178} = 0.996$$

From Table 14.10, with a reliability of 0.9, $K_R = 0.85$. From Fig. 14–18, the temperature and surface condition factors are $K_T = 1$ and $C_f = 1$. From Eq. (14–23), with $m_N = 1$ for spur gears,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2} \frac{3.059}{3.059 + 1} = 0.121$$

From Table 14–8, $C_p = 2300\sqrt{\text{psi}}$.

Next, we need the terms for the gear endurance strength equations. From Table 14–3, for grade 1 steel with $H_{BP} = 240$ and $H_{BG} = 200$, we use Fig. 14–2, which gives

$$(S_t)_P = 77.3(240) + 12\,800 = 31\,350 \text{ psi}$$

$$(S_t)_G = 77.3(200) + 12\,800 = 28\,260 \text{ psi}$$

Similarly, from Table 14–6, we use Fig. 14–5, which gives

$$(S_c)_P = 322(240) + 29\,100 = 106\,400 \text{ psi}$$

$$(S_c)_G = 322(200) + 29\,100 = 93\,500 \text{ psi}$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$$

$$(Z_N)_G = 1.4488(10^8/3.059)^{-0.023} = 0.973$$

For the hardness ratio factor C_H, the hardness ratio is $H_{BP}/H_{BG} = 240/200 = 1.2$. Then, from Sec. 14–12,

$$A' = 8.98(10^{-3})(H_{BP}/H_{BG}) - 8.29(10^{-3})$$

$$= 8.98(10^{-3})(1.2) - 8.29(10^{-3}) = 0.002\,49$$

Thus, from Eq. (14–36),

$$C_H = 1 + 0.002\,49(3.059 - 1) = 1.005$$

(*a*) **Pinion tooth bending.** Substituting the appropriate terms for the pinion into Eq. (14–15) gives

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}\right)_P = 164.8(1)1.377(1.043)\,\frac{10}{1.5}\,\frac{1.22\,(1)}{0.30}$$

$$= 6417 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

Answer
$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P = \frac{31\,350(0.977)/[1(0.85)]}{6417} = 5.62$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 164.8(1)1.377(1.052)\frac{10}{1.5}\frac{1.22(1)}{0.40} = 4854 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

Answer

$$(S_F)_G = \frac{28\,260(0.996)/[1(0.85)]}{4854} = 6.82$$

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$

$$= 2300\left[164.8(1)1.377(1.043)\frac{1.22}{1.7(1.5)}\frac{1}{0.121}\right]^{1/2} = 70\,360 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

Answer

$$(S_H)_P = \left[\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right]_P = \frac{106\,400(0.948)/[1(0.85)]}{70\,360} = 1.69$$

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1.052}{1.043}\right)^{1/2} 70\,360 = 70\,660 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

Answer

$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{70\,660} = 1.52$$

(*c*) For the pinion, we compare $(S_F)_P$ with $(S_H)_P^2$, or 5.73 with $1.69^2 = 2.86$, so the threat in the pinion is from wear. For the gear, we compare $(S_F)_G$ with $(S_H)_G^2$, or 6.96 with $1.52^2 = 2.31$, so the threat in the gear is also from wear.

- In such example, the factor of safety of the entire "gear set" is the <u>lowest</u> among all factors of safety, which is in this case the wear factor of safety for the gear $(S_H)_G^2$=*2.31*
- When the factor of safety is larger than <u>one</u>, this means that the performance of the gear set <u>exceeds</u> the requirements (*i.e., the gear set will run with the specified load for a longer life*).
- A safety factor of, for example *2.31*, for the gear set means that we can, theoretically, increase the transmitted load by *2.31* times and still get the required performance.
- Optimal design is obtained when all the different factors of safety are <u>equal</u>, however, it is <u>preferable</u> to have <u>bending factors of safety</u> that are slightly <u>higher than</u> the <u>wear factors of safety</u> because bending failure (teeth breakage) is more dangerous than surface failure (wear).
- The <u>wear resistance</u> of gears can be controlled by controlling the <u>surface hardness</u>.
- The <u>bending</u> performance of gears can be controlled by controlling the <u>core hardness</u>.

➢ Both bending and wear factors of safety are influenced by the <u>tooth size</u> (*face width & diametral pitch thus gears diameter*) but their influence on bending stress is <u>greater than</u> that on contact stress.

EXAMPLE 14–5 A 17-tooth 20° normal pitch-angle helical pinion with a right-hand helix angle of 30° rotates at 1800 rev/min when transmitting 4 hp to a 52-tooth helical gear. The normal diametral pitch is 10 teeth/in, the face width is 1.5 in, and the set has a quality number of 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion and gear are made from a through-hardened steel with surface and core hardnesses of 240 Brinell on the pinion and surface and core hardnesses of 200 Brinell on the gear. The transmission is smooth, connecting an electric motor and a centrifugal pump. Assume a pinion life of 10^8 cycles and a reliability of 0.9 and use the upper curves in Figs. 14–14 and 14–15.

(*a*) Find the factors of safety of the gears in bending.

(*b*) Find the factors of safety of the gears in wear.

(*c*) By examining the factors of safety identify the threat to each gear and to the mesh.

Solution All of the parameters in this example are the same as in Ex. 14–4 with the exception that we are using helical gears. Thus, several terms will be the same as Ex. 14–4. The reader should verify that the following terms remain unchanged: $K_o = 1$, $Y_P = 0.303$, $Y_G = 0.412$, $m_G = 3.059$, $(K_s)_P = 1.043$, $(K_s)_G = 1.052$, $(Y_N)_P = 0.977$, $(Y_N)_G = 0.996$, $K_R = 0.85$, $K_T = 1$, $C_f = 1$, $C_p = 2300\sqrt{\text{psi}}$, $(S_t)_P = 31\,350$ psi, $(S_t)_G = 28\,260$ psi, $(S_c)_P = 106\,380$ psi, $(S_c)_G = 93\,500$ psi, $(Z_N)_P = 0.948$, $(Z_N)_G = 0.973$, and $C_H = 1.005$.

For helical gears, the transverse diametral pitch, given by Eq. (13–18), is

$$P_t = P_n \cos\psi = 10\cos 30^\circ = 8.660 \text{ teeth/in}$$

Thus, the pitch diameters are $d_P = N_P/P_t = 17/8.660 = 1.963$ in and $d_G = 52/8.660 = 6.005$ in. The pitch-line velocity and transmitted force are

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(1.963)1800}{12} = 925 \text{ ft/min}$$

$$W^t = \frac{33\,000H}{V} = \frac{33\,000(4)}{925} = 142.7 \text{ lbf}$$

As in Ex. 14–4, for the dynamic factor, $B = 0.8255$ and $A = 59.77$. Thus, Eq. (14–27) gives

$$K_v = \left(\frac{59.77 + \sqrt{925}}{59.77}\right)^{0.8255} = 1.404$$

The geometry factor I for helical gears requires a little work. First, the transverse pressure angle is given by Eq. (13–19)

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ$$

The radii of the pinion and gear are $r_P = 1.963/2 = 0.9815$ in and $r_G = 6.004/2 = 3.002$ in, respectively. The addendum is $a = 1/P_n = 1/10 = 0.1$, and the base-circle radii of the pinion and gear are given by Eq. (13–6) with $\phi = \phi_t$:

$$(r_b)_P = r_P \cos\phi_t = 0.9815 \cos 22.80^\circ = 0.9048 \text{ in}$$

$$(r_b)_G = 3.002 \cos 22.80^\circ = 2.767 \text{ in}$$

From Eq. (14–25), the surface strength geometry factor

$$Z = \sqrt{(0.9815 + 0.1)^2 - 0.9048^2} + \sqrt{(3.004 + 0.1)^2 - 2.769^2}$$
$$- (0.9815 + 3.004) \sin 22.80^\circ$$
$$= 0.5924 + 1.4027 - 1.5444 = 0.4507 \text{ in}$$

Since the first two terms are less than 1.5444, the equation for Z stands. From Eq. (14–24) the normal circular pitch p_N is

$$p_N = p_n \cos\phi_n = \frac{\pi}{P_n} \cos 20^\circ = \frac{\pi}{10} \cos 20^\circ = 0.2952 \text{ in}$$

From Eq. (14–21), the load sharing ratio

$$m_N = \frac{p_N}{0.95Z} = \frac{0.2952}{0.95(0.4507)} = 0.6895$$

Substituting in Eq. (14–23), the geometry factor I is

$$I = \frac{\sin 22.80^\circ \cos 22.80^\circ}{2(0.6895)} \frac{3.06}{3.06 + 1} = 0.195$$

From Fig. 14–7, geometry factors $J'_P = 0.45$ and $J'_G = 0.54$. Also from Fig. 14–8 the J-factor multipliers are 0.94 and 0.98, correcting J'_P and J'_G to

$$J_P = 0.45(0.94) = 0.423$$

$$J_G = 0.54(0.98) = 0.529$$

The load-distribution factor K_m is estimated from Eq. (14–32):

$$C_{pf} = \frac{1.5}{10(1.963)} - 0.0375 + 0.0125(1.5) = 0.0577$$

with $C_{mc} = 1$, $C_{pm} = 1$, $C_{ma} = 0.15$ from Fig. 14–11, and $C_e = 1$. Therefore, from Eq. (14–30),

$$K_m = 1 + (1)[0.0577(1) + 0.15(1)] = 1.208$$

(*a*) **Pinion tooth bending.** Substituting the appropriate terms into Eq. (14–15) using P_t gives

$$(\sigma)_P = \left(W^t K_o K_v K_s \frac{P_t}{F} \frac{K_m K_B}{J} \right)_P = 142.7(1)1.404(1.043) \frac{8.66}{1.5} \frac{1.208(1)}{0.423}$$
$$= 3445 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–41) gives

Answer
$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma} \right)_P = \frac{31\,350(0.977)/[1(0.85)]}{3445} = 10.5$$

Gear tooth bending. Substituting the appropriate terms for the gear into Eq. (14–15) gives

$$(\sigma)_G = 142.7(1)1.404(1.052)\frac{8.66}{1.5}\frac{1.208(1)}{0.529} = 2779 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–41) gives

Answer
$$(S_F)_G = \frac{28\,260(0.996)/[1(0.85)]}{2779} = 11.9$$

(*b*) **Pinion tooth wear.** Substituting the appropriate terms for the pinion into Eq. (14–16) gives

$$(\sigma_c)_P = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$

$$= 2300\left[142.7(1)1.404(1.043)\frac{1.208}{1.963(1.5)}\frac{1}{0.195}\right]^{1/2} = 48\,230 \text{ psi}$$

Substituting the appropriate terms for the pinion into Eq. (14–42) gives

Answer
$$(S_H)_P = \left(\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right)_P = \frac{106\,400(0.948)/[1(0.85)]}{48\,230} = 2.46$$

Gear tooth wear. The only term in Eq. (14–16) that changes for the gear is K_s. Thus,

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1.052}{1.043}\right)^{1/2} 48\,230 = 48\,440 \text{ psi}$$

Substituting the appropriate terms for the gear into Eq. (14–42) with $C_H = 1.005$ gives

Answer
$$(S_H)_G = \frac{93\,500(0.973)1.005/[1(0.85)]}{48\,440} = 2.22$$

(*c*) For the pinion we compare S_F with S_H^2, or 10.5 with $2.46^2 = 6.05$, so the threat in the pinion is from wear. For the gear we compare S_F with S_H^2, or 11.9 with $2.22^2 = 4.93$, so the threat is also from wear in the gear. For the meshing gearset wear controls.

- It is desirable to make the <u>bending factors of safety</u> for the pinion and gear, <u>equal</u>. This can be done by controlling the *"core"* hardness (*and thus bending strength*) of the pinion and gear.

  - The bending factors of safety of the pinion and gear are,

$$(S_F)_P = \left(\frac{\sigma_{all}}{\sigma}\right)_P = \left(\frac{S_t Y_N/(K_T K_R)}{W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}}\right)_P, \quad (S_F)_G = \left(\frac{\sigma_{all}}{\sigma}\right)_G = \left(\frac{S_t Y_N/(K_T K_R)}{W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}}\right)_G$$

  - Equating both factors of safety and canceling identical terms (*including K_s which is almost equal*) and solving for $(S_t)_G$

$$(S_t)_G = (S_t)_P \frac{(Y_N)_P}{(Y_N)_G}\frac{J_P}{J_G}$$

- Knowing that $Y_N = \alpha N^{\beta}$, we can write $(Y_N)_P = \alpha N_P^{\beta}$
  and $(Y_N)_G = \alpha (N_P/m_G)^{\beta}$
- Substituting and simplifying we get,

$$\boxed{(S_t)_G = (S_t)_P m_G^{\beta} \frac{J_P}{J_G}}$$

The gear can be less strong than the pinion for the same factor of safety

- Similarly, the contact-stress factors of safety for the pinion and gear can be made equal by controlling the "surface" hardness (*and thus contact strength*).
  The relation between contact-strengths for pinion and gear can be found to be,

$$\boxed{(S_c)_G = (S_c)_P m_G^{\beta}}$$

EXAMPLE 14–6 In a set of spur gears, a 300-Brinell 18-tooth 16-pitch 20° full-depth pinion meshes with a 64-tooth gear. Both gear and pinion are of grade 1 through-hardened steel. Using $\beta = -0.023$, what hardness can the gear have for the same factor of safety?

Solution For through-hardened grade 1 steel the pinion strength $(S_t)_P$ is given in Fig. 14–2:

$$(S_t)_P = 77.3(300) + 12\,800 = 35\,990 \text{ psi}$$

From Fig. 14–6 the form factors are $J_P = 0.32$ and $J_G = 0.41$. Equation (14–44) gives

$$(S_t)_G = 35\,990 \left(\frac{64}{18}\right)^{-0.023} \frac{0.32}{0.41} = 27\,280 \text{ psi}$$

Use the equation in Fig. 14–2 again.

Answer

$$(H_B)_G = \frac{27\,280 - 12\,800}{77.3} = 187 \text{ Brinell}$$

EXAMPLE 14–7 For $\beta = -0.056$ for a through-hardened steel, grade 1, continue Ex. 14–6 for wear.

Solution From Fig. 14–5,

$$(S_c)_P = 322(300) + 29\,100 = 125\,700 \text{ psi}$$

From Eq. (14–45),

$$(S_c)_G = (S_c)_P \left(\frac{64}{18}\right)^{-0.056} = 125\,700 \left(\frac{64}{18}\right)^{-0.056} = 117\,100 \text{ psi}$$

Answer

$$(H_B)_G = \frac{117\,100 - 29\,200}{322} = 273 \text{ Brinell}$$

which is slightly less than the pinion hardness of 300 Brinell.

Design of Gear Mesh

The decisions needed when designing a gear set are divided in two categories:

Priori decisions
- Function: load, speed, reliability, life, K_o
- Unquantifiable risk: design factor n_d ← Design factor of safet
- Tooth system: ϕ, ψ, a, b, root fillet radius
- Gear ratio m_G: N_P, N_G
- Quality number: Q_V

Design decisions
(Total of eight decisions)
- Diametral pitch, P_d
- Face width, F

Gear Size

- Pinion maternal, core hardness, case hardness
- Gear material, core hardness, case hardness

Gear Strength

- When designing, some iterations will be required until a satisfactory design is reached. Thus, it is important to place the design decisions in order of importance (*i.e., impact on the amount of work to be redone in iterations*).

The suggested design strategy is as follows:

1. Select a trial Diametral pitch, P
2. Take the face width to be $F = 4\pi/P$ (*face width should be within* $3\pi/P \leq F \leq 5\pi/P$)
3. Start with bending analysis

   3.1 Pinion: find the bending stress σ (take W^t as n_d W^t)

   - 3.1.1 Choose a material and core hardness
   - 3.1.2 Solve for, F , such that $\sigma = \sigma_{all}$(if F is not within range return to 3.1.1 or to 1)
   - 3.1.3 Choose a value for, F, slightly larger than the calculated value & check the factor of safety S_F

   3.2 Gear: Find necessary core hardness such that $(S_F)_G = (S_F)_P$

   - 3.2.1 Choose a material and core hardness
   - 3.2.2 Find stress σ, then check factor of safety

4. Start wear analysis

   4.1 Pinion: find contact stress σ_c (take W^t as n_d W^t)

4.1.1 Find S_c such that $\sigma_c = \sigma_{c,all}$
4.1.2 Find attendant case hardness & choose larger harness
4.1.3 Check factor of safety S_H^2

4.2 Gear: find necessary case hardness such that $(S_H)_G = (S_H)_p$
4.2.1 Choose larger case hardness
4.2.2 Check factor of safety S_H^2

EXAMPLE 14–8 Design a 4:1 spur-gear reduction for a 100-hp, three-phase squirrel-cage induction motor running at 1120 rev/min. The load is smooth, providing a reliability of 0.95 at 10^9 revolutions of the pinion. Gearing space is meager. Use Nitralloy 135M, grade 1 material to keep the gear size small. The gears are heat-treated first then nitrided.

Solution Make the a priori decisions:

- Function: 100 hp, 1120 rev/min, $R = 0.95$, $N = 10^9$ cycles, $K_o = 1$
- Design factor for unquantifiable exingencies: $n_d = 2$
- Tooth system: $\phi_n = 20°$
- Tooth count: $N_P = 18$ teeth, $N_G = 72$ teeth (no interference)
- Quality number: $Q_v = 6$, use grade 1 material
- Assume $m_B \geq 1.2$ in Eq. (14–40), $K_B = 1$

Pitch: Select a trial diametral pitch of $P_d = 4$ teeth/in. Thus, $d_P = 18/4 = 4.5$ in and $d_G = 72/4 = 18$ in. From Table 14–2, $Y_P = 0.309$, $Y_G = 0.4324$ (interpolated). From Fig. 14–6, $J_P = 0.32$, $J_G = 0.415$.

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi(4.5)1120}{12} = 1319 \text{ ft/min}$$

$$W^t = \frac{33\,000H}{V} = \frac{33\,000(100)}{1319} = 2502 \text{ lbf}$$

From Eqs. (14–28) and (14–27),

$$B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

$$K_v = \left(\frac{59.77 + \sqrt{1319}}{59.77}\right)^{0.8255} = 1.480$$

From Eq. (14–38), $K_R = 0.658 - 0.0759 \ln(1 - 0.95) = 0.885$. From Fig. 14–14,

$$(Y_N)_P = 1.3558(10^9)^{-0.0178} = 0.938$$

$$(Y_N)_G = 1.3558(10^9/4)^{-0.0178} = 0.961$$

From Fig. 14–15,

$$(Z_N)_P = 1.4488(10^9)^{-0.023} = 0.900$$

$$(Z_N)_G = 1.4488(10^9/4)^{-0.023} = 0.929$$

From the recommendation after Eq. (14–8), $3p \le F \le 5p$. Try $F = 4p = 4\pi/P = 4\pi/4 = 3.14$ in. From Eq. (*a*), Sec. 14–10,

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} = 1.192\left(\frac{3.14\sqrt{0.309}}{4}\right)^{0.0535} = 1.140$$

From Eqs. (14–31), (14–33), (14–35), $C_{mc} = C_{pm} = C_e = 1$. From Fig. 14–11, $C_{ma} = 0.175$ for commercial enclosed gear units. From Eq. (14–32), $F/(10d_P) = 3.14/[10(4.5)] = 0.0698$. Thus,

$$C_{pf} = 0.0698 - 0.0375 + 0.0125(3.14) = 0.0715$$

From Eq. (14–30),

$$K_m = 1 + (1)[0.0715(1) + 0.175(1)] = 1.247$$

From Table 14–8, for steel gears, $C_p = 2300\sqrt{\text{psi}}$. From Eq. (14–23), with $m_G = 4$ and $m_N = 1$,

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2}\frac{4}{4+1} = 0.1286$$

Pinion tooth bending. With the above estimates of K_s and K_m from the trial diametral pitch, we check to see if the mesh width F is controlled by bending or wear considerations. Equating Eqs. (14–15) and (14–17), substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{\text{bend}}$ necessary to resist bending fatigue, we obtain

$$(F)_{\text{bend}} = n_d W^t K_o K_v K_s P_d \frac{K_m K_B}{J_P}\frac{K_T K_R}{S_t Y_N} \tag{1}$$

Equating Eqs. (14–16) and (14–18), substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{\text{wear}}$ necessary to resist wear fatigue, we obtain

$$(F)_{\text{wear}} = \left(\frac{C_p Z_N}{S_c K_T K_R}\right)^2 n_d W^t K_o K_v K_s \frac{K_m C_f}{d_P I} \tag{2}$$

From Table 14–5 the hardness range of Nitralloy 135M is Rockwell C32–36 (302–335 Brinell). Choosing a midrange hardness as attainable, using 320 Brinell. From Fig. 14–4,

$$S_t = 86.2(320) + 12\,730 = 40\,310 \text{ psi}$$

Inserting the numerical value of S_t in Eq. (1) to estimate the face width gives

$$(F)_{\text{bend}} = 2(2502)(1)1.48(1.14)4\frac{1.247(1)(1)0.885}{0.32(40\,310)0.938} = 3.08 \text{ in}$$

From Table 14–6 for Nitralloy 135M, $S_c = 170\,000$ psi. Inserting this in Eq. (2), we find

$$(F)_{\text{wear}} = \left(\frac{2300(0.900)}{170\,000(1)0.885}\right)^2 2(2502)1(1.48)1.14\frac{1.247(1)}{4.5(0.1286)} = 3.44 \text{ in}$$

Decision Make face width 3.50 in. Correct K_s and K_m:

$$K_s = 1.192\left(\frac{3.50\sqrt{0.309}}{4}\right)^{0.0535} = 1.147$$

$$\frac{F}{10d_P} = \frac{3.50}{10(4.5)} = 0.0778$$

$$C_{pf} = 0.0778 - 0.0375 + 0.0125(3.50) = 0.0841$$

$$K_m = 1 + (1)[0.0841(1) + 0.175(1)] = 1.259$$

The bending stress induced by W^t in bending, from Eq. (14–15), is

$$(\sigma)_P = 2502(1)1.48(1.147)\frac{4}{3.50}\frac{1.259(1)}{0.32} = 19\,100 \text{ psi}$$

The AGMA factor of safety in bending of the pinion, from Eq. (14–41), is

$$(S_F)_P = \frac{40\,310(0.938)/[1(0.885)]}{19\,100} = 2.24$$

Decision **Gear tooth bending.** Use cast gear blank because of the 18-in pitch diameter. Use the same material, heat treatment, and nitriding. The load-induced bending stress is in the ratio of J_P/J_G. Then

$$(\sigma)_G = 19\,100\frac{0.32}{0.415} = 14\,730 \text{ psi}$$

The factor of safety of the gear in bending is

$$(S_F)_G = \frac{40\,310(0.961)/[1(0.885)]}{14\,730} = 2.97$$

Pinion tooth wear. The contact stress, given by Eq. (14–16), is

$$(\sigma_c)_P = 2300\left[2502(1)1.48(1.147)\frac{1.259}{4.5(3.5)}\frac{1}{0.129}\right]^{1/2} = 118\,000 \text{ psi}$$

The factor of safety from Eq. (14–42), is

$$(S_H)_P = \frac{170\,000(0.900)/[1(0.885)]}{118\,000} = 1.465$$

By our definition of factor of safety, pinion bending is $(S_F)_P = 2.24$, and wear is $(S_H)_P^2 = (1.465)^2 = 2.15$.

Gear tooth wear. The hardness of the gear and pinion are the same. Thus, from Fig. 14–12, $C_H = 1$, the contact stress on the gear is the same as the pinion, $(\sigma_c)_G = 118\,000$ psi. The wear strength is also the same, $S_c = 170\,000$ psi. The factor of safety of the gear in wear is

$$(S_H)_G = \frac{170\,000(0.929)/[1(0.885)]}{118\,000} = 1.51$$

So, for the gear in bending, $(S_F)_G = 2.97$, and wear $(S_H)_G^2 = (1.51)^2 = 2.29$.

Rim. Keep $m_B \geq 1.2$. The whole depth is h_t = addendum + dedendum = $1/P_d + 1.25/P_d = 2.25/P_d = 2.25/4 = 0.5625$ in. The rim thickness t_R is

$$t_R \geq m_B h_t = 1.2(0.5625) = 0.675 \text{ in}$$

In the design of the gear blank, be sure the rim thickness exceeds 0.675 in; if it does not, review and modify this mesh design.