Spur and Helical Gears

This chapter is devoted to analysis and design of spur and helical gears such that they will resist bending failure of teeth and pitting failure of tooth surfaces.

The Lewis Bending Equation

The Lewis equation is used to estimate the bending stress in gear teeth (*max. bending stress at the root of a gear tooth*).

To derive the basic Lewis equation, refer to Fig. 14–1a, which shows a cantilever of cross-sectional dimensions ***F*** and ***t***, having a length ***l*** and a load W^t, uniformly distributed across the face width ***F***. The section modulus ***I/c*** is $Ft^2/6$, and therefore the bending stress is

$$\sigma = \frac{M}{I/c} = \frac{6W^t l}{Ft^2} \qquad (a)$$

Table 14–1

Symbols, Their Names, and Locations*

Symbol	Name	Where Found
b	Net width of face of narrowest member	Eq. (14–16)
C_e	Mesh alignment correction factor	Eq. (14–35)
C_f	Surface condition factor	Eq. (14–16)
C_H	Hardness-ratio factor	Eq. (14–18)
C_{ma}	Mesh alignment factor	Eq. (14–34)
C_{mc}	Load correction factor	Eq. (14–31)
C_{mf}	Face load-distribution factor	Eq. (14–30)
C_p	Elastic coefficient	Eq. (14–13)
C_{pf}	Pinion proportion factor	Eq. (14–32)
C_{pm}	Pinion proportion modifier	Eq. (14–33)
d	Operating pitch diameter of pinion	Ex. (14–1)
d_P	Pitch diameter, pinion	Eq. (14–22)
d_G	Pitch diameter, gear	Eq. (14–22)
E	Modulus of elasticity	Eq. (14–10)
F	Net face width of narrowest member	Eq. (14–15)
f_P	Pinion surface finish	Fig. 14–13
H	Power	Fig. 14–17
H_B	Brinell hardness	Ex. 14–3
H_{BG}	Brinell hardness of gear	Sec. 14–12
H_{BP}	Brinell hardness of pinion	Sec. 14–12
hp	Horsepower	Ex. 14–1
h_t	Gear-tooth whole depth	Sec. 14–16
I	Geometry factor of pitting resistance	Eq. (14–16)
J	Geometry factor for bending strength	Eq. (14–15)

K	Contact load factor for pitting resistance	Eq. (6–65)
K_B	Rim-thickness factor	Eq. (14–40)
K_f	Fatigue stress-concentration factor	Eq. (14–9)
K_m	Load-distribution factor	Eq. (14–30)
K_o	Overload factor	Eq. (14–15)
K_R	Reliability factor	Eq. (14–17)
K_s	Size factor	Sec. 14–10
K_T	Temperature factor	Eq. (14–17)
K_v	Dynamic factor	Eq. (14–27)
m	Metric module	Eq. (14–15)
m_B	Backup ratio	Eq. (14–39)
m_G	Gear ratio (never less than 1)	Eq. (14–22)
m_N	Load-sharing ratio	Eq. (14–21)
N	Number of stress cycles	Fig. 14–14
N_G	Number of teeth on gear	Eq. (14–22)
N_P	Number of teeth on pinion	Eq. (14–22)
n	Speed	Ex. 14–1

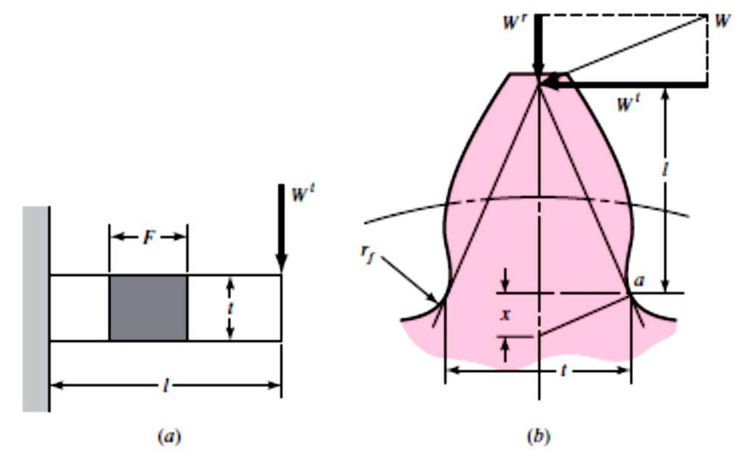
n_P	Pinion speed	Ex. 14–4
P	Diametral pitch	Eq. (14–2)
P_d	Diametral pitch of pinion	Eq. (14–15)
p_N	Normal base pitch	Eq. (14–24)
p_n	Normal circular pitch	Eq. (14–24)
p_x	Axial pitch	Eq. (14–19)
Q_v	Transmission accuracy level number	Eq. (14–29)
R	Reliability	Eq. (14–38)
R_a	Root-mean-squared roughness	Fig. 14–13
r_f	Tooth fillet radius	Fig. 14–1
r_G	Pitch-circle radius, gear	In standard
r_P	Pitch-circle radius, pinion	In standard
r_{bP}	Pinion base-circle radius	Eq. (14–25)
r_{bG}	Gear base-circle radius	Eq. (14–25)
S_C	Buckingham surface endurance strength	Ex. 14–3
S_c	AGMA surface endurance strength	Eq. (14–18)
S_t	AGMA bending strength	Eq. (14–17)
S	Bearing span	Fig. 14–10
S_1	Pinion offset from center span	Fig. 14–10
S_F	Safety factor—bending	Eq. (14–41)
S_H	Safety factor—pitting	Eq. (14–42)
W^t or W_t^t	Transmitted load	Fig. 14–1
Y_N	Stress cycle factor for bending strength	Fig. 14–14
Z_N	Stress cycle factor for pitting resistance	Fig. 14–15
β	Exponent	Eq. (14–44)
σ	Bending stress	Eq. (14–2)

σ_C	Contact stress from Hertzian relationships	Eq. (14–14)
σ_c	Contact stress from AGMA relationships	Eq. (14–16)
σ_{all}	Allowable bending stress	Eq. (14–17)
$\sigma_{c,all}$	Allowable contact stress, AGMA	Eq. (14–18)
ϕ	Pressure angle	Eq. (14–12)
ϕ_t	Transverse pressure angle	Eq. (14–23)
ψ	Helix angle at standard pitch diameter	Ex. 14–5

*Because ANSI/AGMA 2001-C95 introduced a significant amount of new nomenclature, and continued in ANSI/AGMA 2001-D04, this summary and references are provided for use until the reader's vocabulary has grown.

†See preference rationale following Eq. (a), Sec. 14–1.

Figure 14–1



- ❖ It treats the gear tooth using a factor called "*Lewis from factor, Y*" (*Table 14-2*).
- ❖ It also includes a correction for dynamic effects "K_v" (*due to rotation of the gear*).

Table 14–2

Values of the Lewis Form Factor Y (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

- Lewis equation forms the basis of the *AGMA* bending stress equation used nowadays.

Surface Durability

This section is concerned with the failure of teeth surfaces *(wear)*.

- The most common type of surface failure is pitting which is caused by the repeating high contact stress.
- An expression for the max contact stress " σ_c " between mating gear teeth can be derived from the *Hertz* equation for two cylinders in contact.
- By adapting the notation used in gearing and including a velocity factor " K_v ", the contact-stress can be found as:

$$\sigma_c = \underbrace{-}_{\text{Compressive stress}} C_p \left[\frac{K_v W^t}{F\cos\emptyset}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right]^{1/2}$$

where, C_p: *AGMA* elastic coefficient $C_p = \left[\dfrac{1}{\pi\left(\dfrac{1-\nu_p^2}{E_p}+\dfrac{1-\nu_G^2}{E_G}\right)}\right]^{1/2}$

ν_p, ν_G: pinion & gear Poisson's ratio.
E_p, E_G: pinion & gear young's modulus.

W^t: Tangential (*transmitted*) load

F: Face width, $\emptyset$:Pressure angle

r_1 & r_2: Instantaneous radii of curvature of the pinion & gear teeth.

- This equation forms the basis of *AGMA* contact stress equation.

AGMA Stress Equations

In the *AGMA* methodology, there are two fundamental stress equations, one for bending stress and another for pitting resistance (*contact stress*).

- Bending stress

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F}\dfrac{K_m K_B}{J} & \textit{US Units} \\[2ex] W^t K_o K_v K_s \dfrac{1}{b m_t}\dfrac{K_H K_B}{Y_J} & \textit{(SI Units)} \end{cases}$$

where:

W^t: Tangential or transmitted load, *lb (N)*
K_o: Overload factor
K_v: Dynamic factor
K_s: Size factor

P_d:Transverse Diametral pitch, "*tooth per inch*"
m_t: Transverse metric module, *(mm)*
F,(b): Face width of the narrower member, *in (mm)*
$K_m, (K_H)$: Load-distribution factor
K_B: Rim thickness factor
$J, (Y_J)$: Geometry factor for bending stress

- Contact stress (*pitting resistance*)

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_p F} \dfrac{C_f}{I}} & \textit{US units} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \textit{(SI units)} \end{cases}$$

where:

$C_p(Z_E)$: *AGMA* elastic coefficient, $\sqrt{psi}$, $(\sqrt{MPa})$
$C_f(Z_R)$: Surface condition factor
$d_p(d_{w1})$: Pitch diameter of the pinion, *in (mm)*
$I(Z_I)$: Geometry factor for pitting resistance

EXAMPLE 14–1 A stock spur gear is available having a diametral pitch of 8 teeth/in, a $1\frac{1}{2}$-in face, 16 teeth, and a pressure angle of 20° with full-depth teeth. The material is AISI 1020 steel in as-rolled condition. Use a design factor of $n_d = 3$ to rate the horsepower output of the gear corresponding to a speed of 1200 rev/m and moderate applications.

Solution The term *moderate applications* seems to imply that the gear can be rated by using the yield strength as a criterion of failure. From Table A–20, we find $S_{ut} = 55$ kpsi and $S_y = 30$ kpsi. A design factor of 3 means that the allowable bending stress is $30/3 = 10$ kpsi. The pitch diameter is $N/P = 16/8 = 2$ in, so the pitch-line velocity is

$$V = \frac{\pi d n}{12} = \frac{\pi(2)1200}{12} = 628 \text{ ft/min}$$

The velocity factor from Eq. (14–4*b*) is found to be

$$K_v = \frac{1200 + V}{1200} = \frac{1200 + 628}{1200} = 1.52$$

Table 14–2 gives the form factor as $Y = 0.296$ for 16 teeth. We now arrange and substitute in Eq. (14–7) as follows:

$$W^t = \frac{F Y \sigma_{\text{all}}}{K_v P} = \frac{1.5(0.296)10\,000}{1.52(8)} = 365 \text{ lbf}$$

The horsepower that can be transmitted is

Answer

$$hp = \frac{W^t V}{33\,000} = \frac{365(628)}{33\,000} = 6.95 \text{ hp}$$

It is important to emphasize that this is a rough estimate, and that this approach must not be used for important applications. The example is intended to help you understand some of the fundamentals that will be involved in the AGMA approach.

EXAMPLE 14–2 Estimate the horsepower rating of the gear in the previous example based on obtaining an infinite life in bending.

Solution The rotating-beam endurance limit is estimated from Eq. (6–8)

$$S'_e = 0.5S_{ut} = 0.5(55) = 27.5 \text{ kpsi}$$

To obtain the surface finish Marin factor k_a we refer to Table 6–3 for machined surface, finding $a = 2.70$ and $b = -0.265$. Then Eq. (6–19) gives the surface finish Marin factor k_a as

$$k_a = aS_{ut}^b = 2.70(55)^{-0.265} = 0.934$$

The next step is to estimate the size factor k_b. From Table 13–1, the sum of the addendum and dedendum is

$$l = \frac{1}{P} + \frac{1.25}{P} = \frac{1}{8} + \frac{1.25}{8} = 0.281 \text{ in}$$

The tooth thickness t in Fig. 14–1b is given in Sec. 14–1 [Eq. (b)] as $t = (4lx)^{1/2}$ when $x = 3Y/(2P)$ from Eq. (14–3). Therefore, since from Ex. 14–1 $Y = 0.296$ and $P = 8$,

$$x = \frac{3Y}{2P} = \frac{3(0.296)}{2(8)} = 0.0555 \text{ in}$$

then

$$t = (4lx)^{1/2} = [4(0.281)0.0555]^{1/2} = 0.250 \text{ in}$$

We have recognized the tooth as a cantilever beam of rectangular cross section, so the equivalent rotating-beam diameter must be obtained from Eq. (6–25):

$$d_e = 0.808(hb)^{1/2} = 0.808(Ft)^{1/2} = 0.808[1.5(0.250)]^{1/2} = 0.495 \text{ in}$$

Then, Eq. (6–20) gives k_b as

$$k_b = \left(\frac{d_e}{0.30}\right)^{-0.107} = \left(\frac{0.495}{0.30}\right)^{-0.107} = 0.948$$

The load factor k_c from Eq. (6–26) is unity. With no information given concerning temperature and reliability we will set $k_d = k_e = 1$.

Two effects are used to evaluate the miscellaneous-effects Marin factor k_f. The first of these is the effect of one-way bending. In general, a gear tooth is subjected only to one-way bending. Exceptions include idler gears and gears used in reversing mechanisms.

For one-way bending the steady and alternating stress components are $\sigma_a = \sigma_m = \sigma/2$ where σ is the largest repeatedly applied bending stress as given in Eq. (14–7). If a material exhibited a Goodman failure locus,

$$\frac{S_a}{S'_e} + \frac{S_m}{S_{ut}} = 1$$

Since S_a and S_m are equal for one-way bending, we substitute S_a for S_m and solve the preceding equation for S_a, giving

$$S_a = \frac{S'_e S_{ut}}{S'_e + S_{ut}}$$

Now replace S_a with $\sigma/2$, and in the denominator replace S'_e with $0.5S_{ut}$ to obtain

$$\sigma = \frac{2S'_e S_{ut}}{0.5S_{ut} + S_{ut}} = \frac{2S'_e}{0.5 + 1} = 1.33S'_e$$

Now $k_f = \sigma/S'_e = 1.33S'_e/S'_e = 1.33$. However, a Gerber fatigue locus gives mean values of

$$\frac{S_a}{S'_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

Setting $S_a = S_m$ and solving the quadratic in S_a gives

$$S_a = \frac{S_{ut}^2}{2S'_e}\left(-1 + \sqrt{1 + \frac{4S_e'^2}{S_{ut}^2}}\right)$$

Setting $S_a = \sigma/2$, $S_{ut} = S'_e/0.5$ gives

$$\sigma = \frac{S'_e}{0.5^2}\left[-1 + \sqrt{1 + 4(0.5)^2}\right] = 1.66S'_e$$

and $k_f = \sigma/S'_e = 1.66$. Since a Gerber locus runs in and among fatigue data and Goodman does not, we will use $k_f = 1.66$.

The second effect to be accounted for in using the miscellaneous-effects Marin factor k_f is stress concentration, for which we will use our fundamentals from Chap. 6. For a 20° full-depth tooth the radius of the root fillet is denoted r_f, where

$$r_f = \frac{0.300}{P} = \frac{0.300}{8} = 0.0375 \text{ in}$$

From Fig. A–15–6

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.0375}{0.250} = 0.15$$

Since $D/d = \infty$, we approximate with $D/d = 3$, giving $K_t = 1.68$. From Fig. 6–20, $q = 0.62$. From Eq. (6–32)

$$K_f = 1 + (0.62)(1.68 - 1) = 1.42$$

The miscellaneous-effects Marin factor for stress concentration can be expressed as

$$k_f = \frac{1}{K_f} = \frac{1}{1.42} = 0.704$$

The final value of k_f is the product of the two k_f factors, that is, 1.66(0.704) = 1.17. The Marin equation for the fully corrected endurance strength is

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$
$$= 0.934(0.948)(1)(1)(1)1.17(27.5) = 28.5 \text{ kpsi}$$

For a design factor of $n_d = 3$, as used in Ex. 14–1, applied to the load or strength, the allowable bending stress is

$$\sigma_{\text{all}} = \frac{S_e}{n_d} = \frac{28.5}{3} = 9.5 \text{ kpsi}$$

The transmitted load W^t is

$$W^t = \frac{FY\sigma_{\text{all}}}{K_v P} = \frac{1.5(0.296)9\,500}{1.52(8)} = 347 \text{ lbf}$$

and the power is, with $V = 628$ ft/min from Ex. 14–1,

$$hp = \frac{W^t V}{33\,000} = \frac{347(628)}{33\,000} = 6.6 \text{ hp}$$

Again, it should be emphasized that these results should be accepted *only* as preliminary estimates to alert you to the nature of bending in gear teeth.

EXAMPLE 14–3 The pinion of Examples 14–1 and 14–2 is to be mated with a 50-tooth gear manufactured of ASTM No. 50 cast iron. Using the tangential load of 382 lbf, estimate the factor of safety of the drive based on the possibility of a surface fatigue failure.

Solution From Table A–5 we find the elastic constants to be $E_P = 30$ Mpsi, $\nu_P = 0.292$, $E_G = 14.5$ Mpsi, $\nu_G = 0.211$. We substitute these in Eq. (14–13) to get the elastic coefficient as

$$C_p = \left\{ \frac{1}{\pi \left[\dfrac{1-(0.292)^2}{30(10^6)} + \dfrac{1-(0.211)^2}{14.5(10^6)} \right]} \right\}^{1/2} = 1817$$

From Example 14–1, the pinion pitch diameter is $d_P = 2$ in. The value for the gear is $d_G = 50/8 = 6.25$ in. Then Eq. (14–12) is used to obtain the radii of curvature at the pitch points. Thus

$$r_1 = \frac{2 \sin 20^\circ}{2} = 0.342 \text{ in} \qquad r_2 = \frac{6.25 \sin 20^\circ}{2} = 1.069 \text{ in}$$

The face width is given as $F = 1.5$ in. Use $K_v = 1.52$ from Example 14–1. Substituting all these values in Eq. (14–14) with $\phi = 20^\circ$ gives the contact stress as

$$\sigma_C = -1817 \left[\frac{1.52(380)}{1.5 \cos 20^\circ} \left(\frac{1}{0.342} + \frac{1}{1.069} \right) \right]^{1/2} = -72\,400 \text{ psi}$$

The surface endurance strength of cast iron can be estimated from

$$S_C = 0.32 H_B \text{ kpsi}$$

for 10^8 cycles, where S_C is in kpsi. Table A–24 gives $H_B = 262$ for ASTM No. 50 cast iron. Therefore $S_C = 0.32(262) = 83.8$ kpsi. Contact stress is not linear with transmitted load [see Eq. (14–14)]. If the factor of safety is defined as the loss-of-function load divided by the imposed load, then the ratio of loads is the ratio of stresses squared. In

other words,

$$n = \frac{\text{loss-of-function load}}{\text{imposed load}} = \frac{S_C^2}{\sigma_C^2} = \left(\frac{83.8}{72.4}\right)^2 = 1.34$$

One is free to define factor of safety as S_C/σ_C. Awkwardness comes when one compares the factor of safety in bending fatigue with the factor of safety in surface fatigue for a particular gear. Suppose the factor of safety of this gear in bending fatigue is 1.20 and the factor of safety in surface fatigue is 1.34 as above. The threat, since 1.34 is greater than 1.20, is in bending fatigue since both numbers are based on load ratios. If the factor of safety in surface fatigue is based on $S_C/\sigma_C = \sqrt{1.34} = 1.16$, then 1.20 is greater than 1.16, but the threat is not from surface fatigue. The surface fatigue factor of safety can be defined either way. One way has the burden of requiring a squared number before numbers that instinctively seem comparable can be compared.

AGMA Strength Equations

When analyzing gear teeth, after the bending and contact stress values are found, they need to be compared with allowable values of stress (*also called strength*) to make sure the design is satisfactory.

The *AGMA* bending and contact strengths, , S_t & S_c, (*i.e., allowable stresses*) are obtained from charts or tables (for different materials) and then are modified by various factors to produce the limiting values of bending and contact stress.

- Allowable (*limiting*) bending stress

$$\boxed{\sigma_{all} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \textit{Us units} \\[2ex] \dfrac{\sigma_{FP}}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \textit{(SI units)} \end{cases}}$$

where,

$S_t(\sigma_{FP})$: *AGMA* bending strength, *psi (MPa)*

Y_N: Stress cycle life factor for bending

$K_T(Y_\theta)$: Temperature factor

$K_R(Y_Z)$: Reliability factor

S_F: The *AGMA* factor of safety

- Allowable (*limiting*) contact stress

$$\boxed{\sigma_{c,all} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \textit{Us units} \\[2ex] \dfrac{\sigma_{HP}}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \textit{(SI units)} \end{cases}}$$

where:

$S_C(\sigma_{HP})$: *AGMA* contact strength, *psi (MPa)*

Z_N: Stress cycle life factor for pitting

$C_H(Z_W)$: Hardness ratio factor for pitting resistance *(<u>only for the gear</u>)*

S_H: *AGMA* factor of safety

❖ The *AGMA* bending strength ( ) values are given in *Figures 14-2, 14-3, 14-4* and *Tables 14-3, 14-4 (note that it is termed as "allowable bending stress numbers").*

Figure 14-2

Allowable bending stress number for through-hardened steels. The SI equations are $S_t = 0.533H_B + 88.3$ MPa, grade 1, and $S_t = 0.703H_B + 113$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

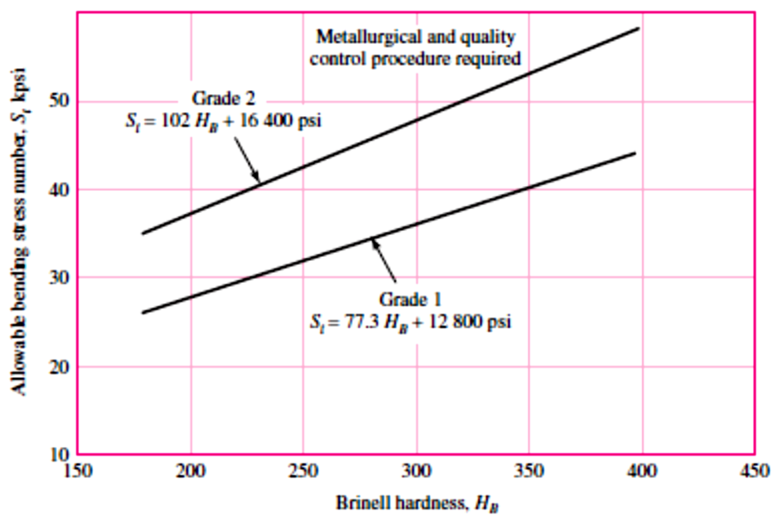


Figure 14-3

Allowable bending stress number for nitrided through-hardened steel gears (i.e., AISI 4140, 4340), S_t. The SI equations are $S_t = 0.568H_B + 83.8$ MPa, grade 1, and $S_t = 0.749H_B + 110$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

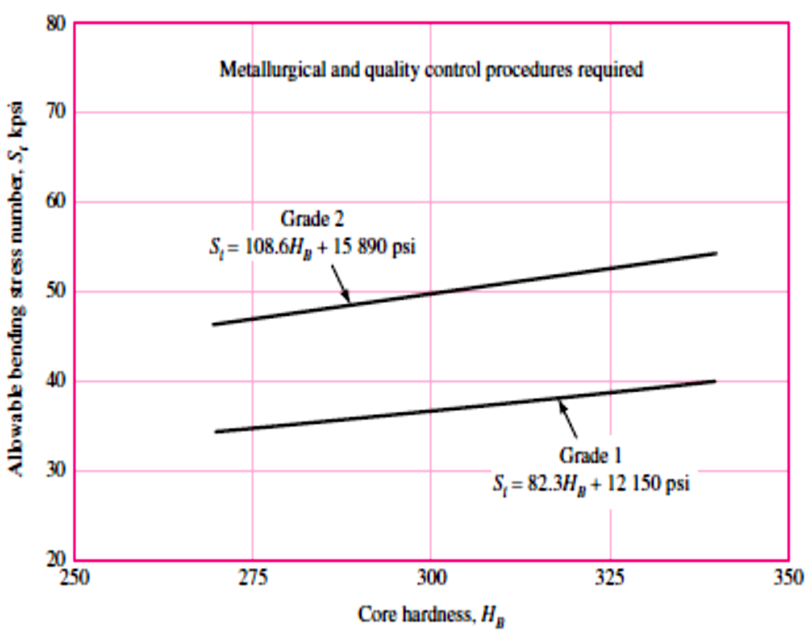


Figure 14–4

Allowable bending stress numbers for nitriding steel gears S_t. The SI equations are $S_t = 0.594H_B + 87.76$ MPa Nitralloy grade 1 $S_t = 0.784H_B + 114.81$ MPa Nitralloy grade 2 $S_t = 0.7255H_B + 63.89$ MPa 2.5% chrome, grade 1 $S_t = 0.7255H_B + 153.63$ MPa 2.5% chrome, grade 2 $S_t = 0.7255H_B + 201.91$ MPa 2.5% chrome, grade 3 *(Source: ANSI/AGMA 2001-D04, 2101-D04.)*

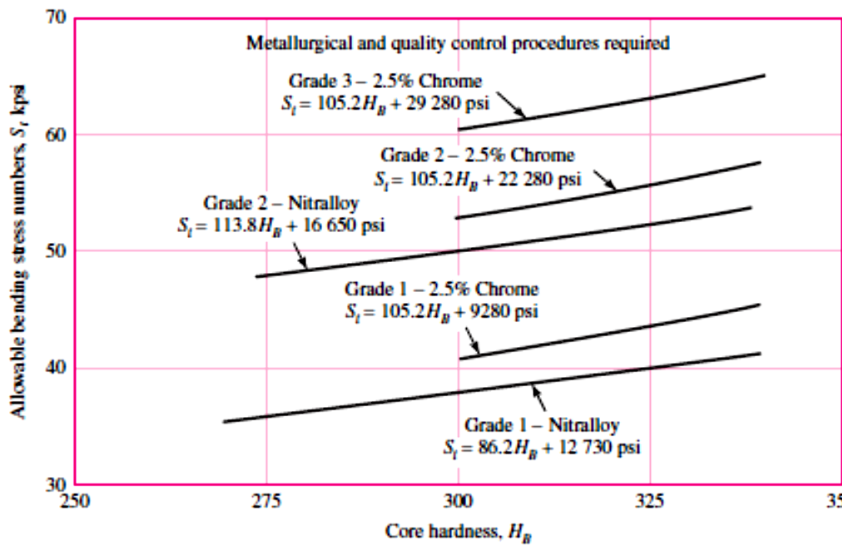


Table 14–3

Repeatedly Applied Bending Strength S_t at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Bending Stress Number S_t,[2] psi Grade 1	Grade 2	Grade 3
Steel[3]	Through-hardened	See Fig. 14–2	See Fig. 14–2	See Fig. 14–2	—
	Flame[4] or induction hardened[4] with type A pattern[5]	See Table 8*	45 000	55 000	—
	Flame[4] or induction hardened[4] with type B pattern[5]	See Table 8*	22 000	22 000	—
	Carburized and hardened	See Table 9*	55 000	65 000 or 70 000[6]	75 000
	Nitrided[4,7] (through-hardened steels)	83.5 HR15N	See Fig. 14–3	See Fig. 14–3	—
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminum)	Nitrided[4,7]	87.5 HR15N	See Fig. 14–4	See Fig. 14–4	See Fig. 14–4

Notes: See ANSI/AGMA 2001-D04 for references cited in notes 1–7.

[1]Hardness to be equivalent to that at the root diameter in the center of the tooth space and face width.

[2]See tables 7 through 10 for major metallurgical factors for each stress grade of steel gears.

[3]The steel selected must be compatible with the heat treatment process selected and hardness required.

[4]The allowable stress numbers indicated may be used with the case depths prescribed in 16.1.

[5]See figure 12 for type A and type B hardness patterns.

[6]If bainite and microcracks are limited to grade 3 levels, 70,000 psi may be used.

[7]The overload capacity of nitrided gears is low. Since the shape of the effective S-N curve is flat, the sensitivity to shock should be investigated before proceeding with the design. [7]

*Tables 8 and 9 of ANSI/AGMA 2001-D04 are comprehensive tabulations of the major metallurgical factors affecting S_t and S_c of flame-hardened and induction-hardened (Table 8) and carburized and hardened (Table 9) steel gears.

Table 14-4

Repeatedly Applied Bending Strength S_t for Iron and Bronze Gears at 10^7 Cycles and 0.99 Reliability

Source: ANSI/AGMA 2001-D04.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Bending Stress Number, S_t,[3] psi
ASTM A48 gray cast iron	Class 20	As cast	—	5000
	Class 30	As cast	174 HB	8500
	Class 40	As cast	201 HB	13 000
ASTM A536 ductile (nodular) Iron	Grade 60–40–18	Annealed	140 HB	22 000–33 000
	Grade 80–55–06	Quenched and tempered	179 HB	22 000–33 000
	Grade 100–70–03	Quenched and tempered	229 HB	27 000–40 000
	Grade 120–90–02	Quenched and tempered	269 HB	31 000–44 000
Bronze		Sand cast	Minimum tensile strength 40 000 psi	5700
	ASTM B-148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	23 600

Notes:

[1]See ANSI/AGMA 2004-B89, *Gear Materials and Heat Treatment Manual.*

[2]Measured hardness to be equivalent to that which would be measured at the root diameter in the center of the tooth space and face width.

[3]The lower values should be used for general design purposes. The upper values may be used when:

High quality material is used.

Section size and design allow maximum response to heat treatment.

Proper quality control is effected by adequate inspection.

Operating experience justifies their use.

❖ The *AGMA* contact strength (S_C) values are given in *Figure 14-5* and *Tables 14-5, 14-6, 14-7* (*note that it is termed as "allowable contact stress numbers").*

❖ The values given in *AGMA* charts and tables are based on:

- Unidirectional loading
- *10^7* stress cycles
- *99* percent reliability

▪ When two-way loading occurs, such as in idler gears, *AGMA* recommends multiplying the bending strength (S_t value by *0.7)*

- *But this is not used for the S_C value, why?*

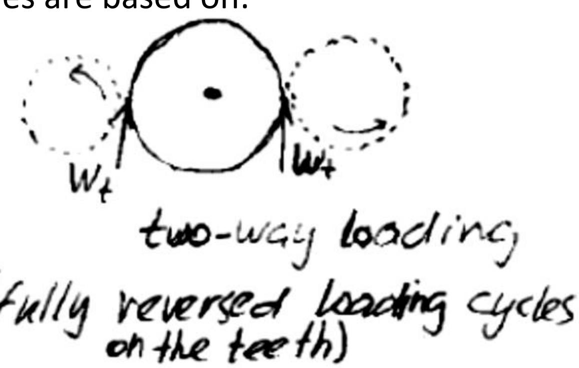


Figure 14–5

Contact-fatigue strength S_c at 10^7 cycles and 0.99 reliability for through-hardened steel gears. The SI equations are $S_c = 2.22H_B + 200$ MPa, grade 1, and $S_c = 2.41H_B + 237$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

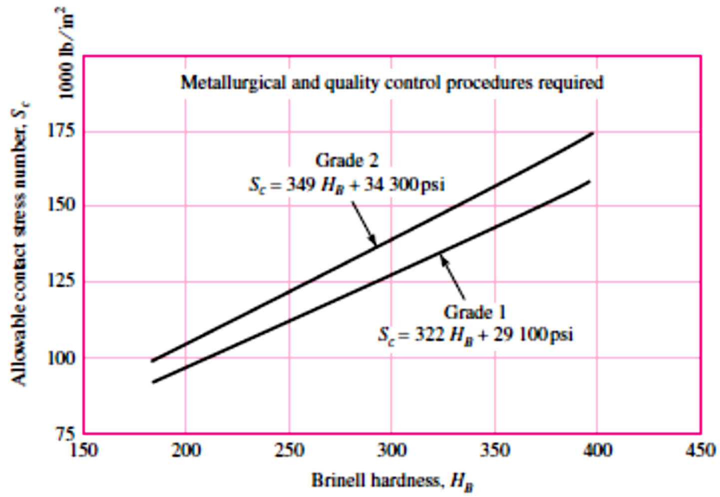


Table 14–5

Nominal Temperature Used in Nitriding and Hardnesses Obtained

Source: Darle W. Dudley, *Handbook of Practical Gear Design*, rev. ed., McGraw-Hill, New York, 1984.

Steel	Temperature before nitriding, °F	Nitriding, °F	Hardness, Rockwell C Scale: Case	Hardness, Rockwell C Scale: Core
Nitralloy 135*	1150	975	62–65	30–35
Nitralloy 135M	1150	975	62–65	32–36
Nitralloy N	1000	975	62–65	40–44
AISI 4340	1100	975	48–53	27–35
AISI 4140	1100	975	49–54	27–35
31 Cr Mo V 9	1100	975	58–62	27–33

*Nitralloy is a trademark of the Nitralloy Corp., New York.

Table 14–6

Repeatedly Applied Contact Strength S_c at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Contact Stress Number,[2] S_c, psi: Grade 1	Grade 2	Grade 3
Steel[3]	Through hardened[4]	See Fig. 14–5	See Fig. 14–5	See Fig. 14–5	—
	Flame[5] or induction hardened[5]	50 HRC	170 000	190 000	—
		54 HRC	175 000	195 000	—
	Carburized and hardened[5]	See Table 9*	180 000	225 000	275 000
	Nitrided[5] (through hardened steels)	83.5 HR15N	150 000	163 000	175 000
		84.5 HR15N	155 000	168 000	180 000
2.5% chrome (no aluminum)	Nitrided[5]	87.5 HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided[5]	90.0 HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided[5]	90.0 HR15N	172 000	188 000	205 000
2.5% chrome (no aluminum)	Nitrided[5]	90.0 HR15N	176 000	196 000	216 000

Notes: See ANSI/AGMA 2001-D04 for references cited in notes 1–5.
[1]Hardness to be equivalent to that at the start of active profile in the center of the face width.
[2]See Tables 7 through 10 for major metallurgical factors for each stress grade of steel gears.
[3]The steel selected must be compatible with the heat treatment process selected and hardness required.
[4]These materials must be annealed or normalized as a minimum.
[5]The allowable stress numbers indicated may be used with the case depths prescribed in 16.1.
*Table 9 of ANSI/AGMA 2001-D04 is a comprehensive tabulation of the major metallurgical factors affecting S_t and S_c of carburized and hardened steel gears.

Table 14–7

Repeatedly Applied Contact Strength S_c 10^7 Cycles and 0.99 Reliability for Iron and Bronze Gears
Source: ANSI/AGMA 2001-D04.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Contact Stress Number,[3] S_c, psi
ASTM A48 gray cast iron	Class 20	As cast	—	50 000–60 000
	Class 30	As cast	174 HB	65 000–75 000
	Class 40	As cast	201 HB	75 000–85 000
ASTM A536 ductile (nodular) iron	Grade 60–40–18	Annealed	140 HB	77 000–92 000
	Grade 80–55–06	Quenched and tempered	179 HB	77 000–92 000
	Grade 100–70–03	Quenched and tempered	229 HB	92 000–112 000
	Grade 120–90–02	Quenched and tempered	269 HB	103 000–126 000
Bronze	—	Sand cast	Minimum tensile strength 40 000 psi	30 000
	ASTM B-148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	65 000

Notes:
[1]See ANSI/AGMA 2004-B89, *Gear Materials and Heat Treatment Manual.*
[2]Hardness to be equivalent to that at the start of active profile in the center of the face width.
[3]The lower values should be used for general design purposes. The upper values may be used when:
High-quality material is used.
Section size and design allow maximum response to heat treatment.
Proper quality control is effected by adequate inspection.
Operating experience justifies their use.

Geometry Factors, (J and I (Z_J and Z_I))

The purpose of geometry factors to account for the tooth form in the stress equations.

- Bending-stress geometry factor , $J\ (Y_J)$.

▪ This factor depends on the shape of the tooth and the distance from the tooth root to the highest point of single-tooth contact.

▪ It also includes the effect of stress concentration in the tooth and the ratio of face width upon which load is applied (*i.e., the length of line of contact in helical gears*).

❖ The value of for spur gears with 20° pressure angle and full-depth teeth is found from *Fig. 14-6*.

❖ The value of for helical gears with 20° normal pressure angle is found from *Figs. 14-7 & 14-8*.

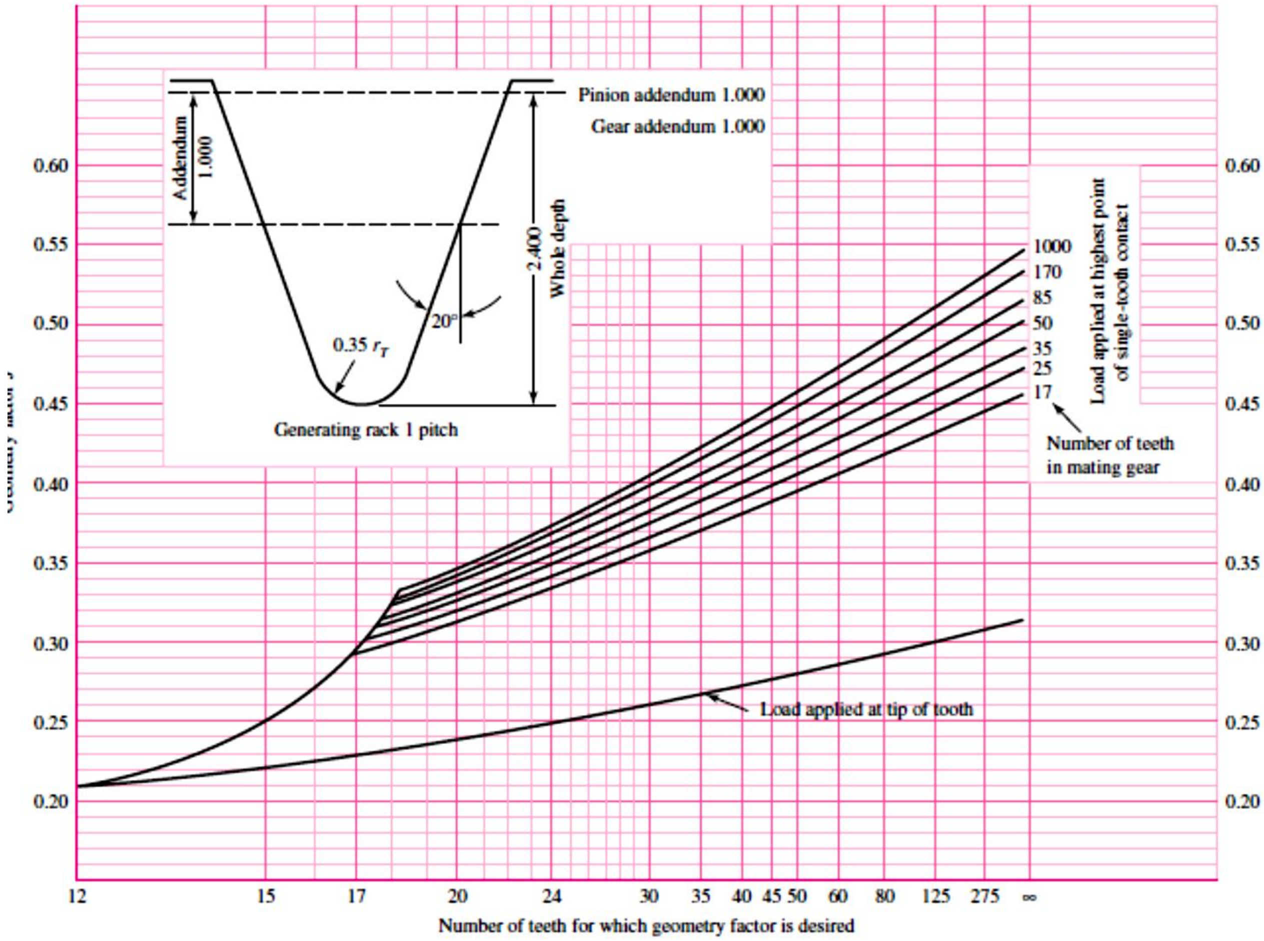
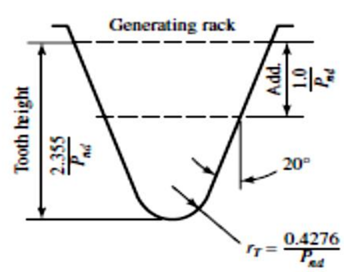


Figure 14–6

Spur-gear geometry factors *J*. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.



(*a*)

$m_N = \frac{p_N}{0.95Z}$

Value for Z is for an element of indicated numbers of teeth and a 75-tooth mate

Normal tooth thickness of pinion and gear tooth each reduced 0.024 in to provide 0.048 in total backlash for one normal diametral pitch

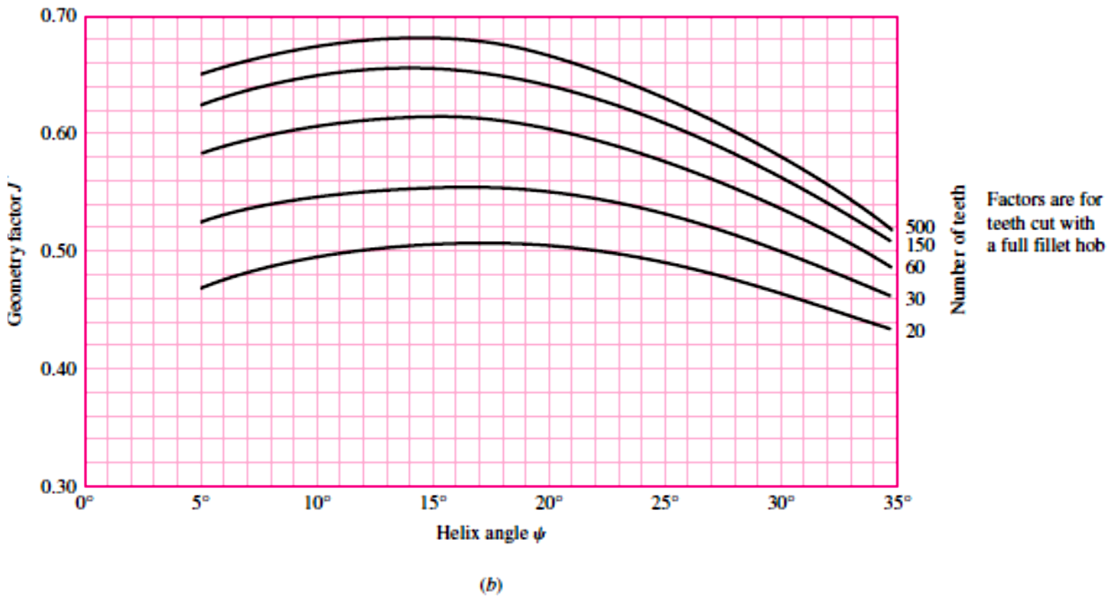
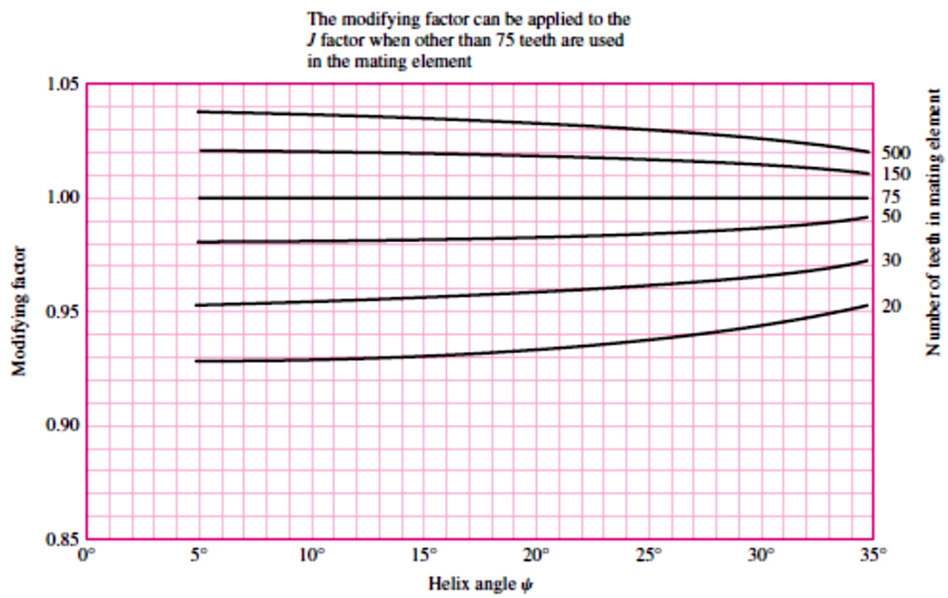


Figure 14–7

Helical-gear geometry factors J'. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

Figure 14–8

J'-factor multipliers for use with Fig. 14–7 to find J. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.



- Contact-stress geometry factor, $I\ (Y_I)$

Also called by *AGMA* as the *pitting-resistance* geometry factor.

- It accounts for the values of the instantaneous radius of curvature of the two teeth at the point of contact (*and for the length of contact line for "helical gears"*).
- Its value can be found as:

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G+1} & \text{External Gears} \\[2ex] \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G-1} & \text{Internal Gears} \end{cases}$$

where:

ϕ_t: Pressure angle for <u>spur</u> gears
<u>or</u> Transverse pressure angle for <u>helical</u> gears.

m_G: Speed ratio, $m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P}$

m_N: Load-sharing ratio

- $m_N = 1$ *for <u>Spur</u> gears*
- $m_N = \frac{p_N}{0.95Z}$ *for <u>Helical</u> gears*

where;

"p_N" is the <u>normal base pitch</u> $p_N = p_n \cos \emptyset_n$

Normal circular pitch measured on the <u>base circle</u>

Normal circular pitch

"Z" is the <u>length of line of action</u> in the transverse plane,

$$Z = \left[\left(r_p + a\right)^2 - r_{bp}^2\right]^{1/2} + [(r_G + a)^2 - r_{bG}^2]^{1/2} - \left(r_p + r_G\right)\sin\emptyset_t$$

- Where "r_p & r_G" are the pitch radii of pinion and gear, "a" is the addendum and "r_{bp}& r_{bG}" are the base-circle radii of pinion and gear.

Remember that $r_b = r\cos\emptyset_t$

- *<u>Note:</u>* in the "Z" equation, if any of the first two terms is <u>larger</u> than the third term, then it should be replaced by the third term.

<u>The Elastic Coefficient,</u> $C_p(Z_E)$

The coefficient combines the elastic constants of the gear and pinion.

- The value of "C_p" (Z_E) can be found as:

$$C_p = \sqrt{\frac{1}{\pi\left(\frac{1-v_p^2}{E_P} + \frac{1-v_G^2}{E_G}\right)}}$$

❖ Or "*easier*" from <u>*Table 14-8*</u>.

Table 14-8

Elastic Coefficient C_p (Z_E), $\sqrt{psi}$ $(\sqrt{MPa})$ *Source:* AGMA 218.01

Pinion Material	Pinion Modulus of Elasticity E_p psi (MPa)*	Gear Material and Modulus of Elasticity E_G, lbf/in^2 (MPa)*: Steel 30×10^6 (2×10^5)	Malleable Iron 25×10^6 (1.7×10^5)	Nodular Iron 24×10^6 (1.7×10^5)	Cast Iron 22×10^6 (1.5×10^5)	Aluminum Bronze 17.5×10^6 (1.2×10^5)	Tin Bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.

*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

Dynamic Factor, K_V

This factor is used to account for inaccuracies in the manufacture and meshing of gear teeth in action, which cause deviation from the uniform angular speed a gear pair is supposed to have.

AGMA uses a "*transmission accuracy-level numbers* ", Q_v , to quantify gears into different classes according to manufacturing accuracy (*tolerances*).

- Q_v From 3 to 7 is for commercial quality gears.
- Q_v From 8 to 12 is for precision quality gears.

❖ The value of K_v can be found using *Eqn.(14-27)* in the text or from *Fig. 14-9* where it gives K_v as a function of pitch-line speed for different Q_V classes.

$$(V_t)_{\max} = \begin{cases} [A + (Q_v - 3)]^2 & \text{ft/min} \\ \dfrac{[A + (Q_v - 3)]^2}{200} & \text{m/s} \end{cases} \qquad (14\text{-}29)$$

Overload Factor, K_O

This factor is used to account for external loads exceeding the nominal tangential load W^t (*such as variations in torque due to firing of cylinders in internal combustion engines*).

- ➢ The values of K_o are based on field experience in a particular application.
- ❖ Values of K_o can be found from the table given in *Fig. 14-17*.

Figure 14–9

Dynamic factor K_v. The equations to these curves are given by Eq. (14–27) and the end points by Eq. (14–29). *(ANSI/AGMA 2001-D04, Annex A)*

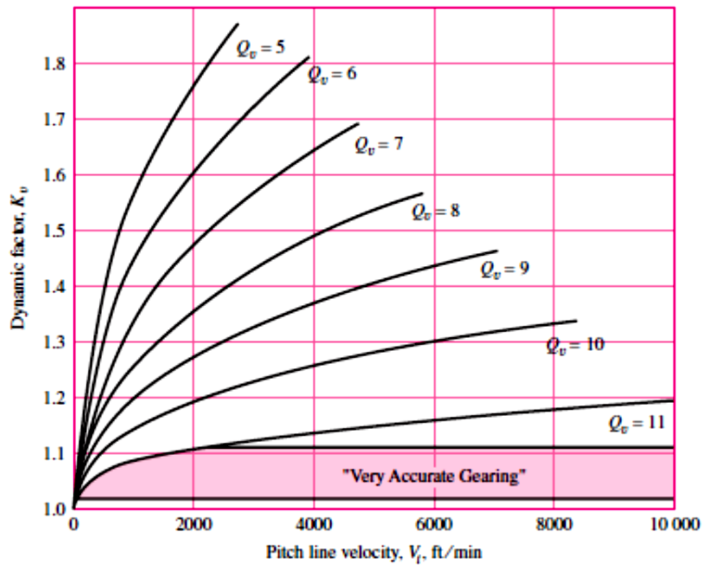


Figure 14–9 is a graph of K_v, the dynamic factor, as a function of pitch-line speed for graphical estimates of K_v.

Size Factor, K_S

This factor is used to account for non-uniform of material properties due to size.

- Standard size factors for gear teeth have not been established yet, thus, *AGMA* suggests using K_S=1 (*if the size effect is known, $K_S > 1$*).
- However, from the formulation given in *Chapter 7* for the endurance limit size correction factor, an expression can developed for computing *size* factor "K_S" for gear teeth which is:

$$\boxed{K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535}}$$

where, ***F*: Face width**
***P*: Diametral pitch**
***Y*: Lewis from factor (Table 14-2)**

- ➢ *Note:* If $\boldsymbol{K_S}$ was found from the equation to be less than one, then we will use $\boldsymbol{K_S}$ =1

SPUR GEAR BENDING
BASED ON ANSI/AGMA 2001-D04

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ H}{V}$$

1 [or Eq. (a), Sec. 14–10]; p. 739

Eq. (14–30); p. 739

Eq. (14–40); p. 744

Gear bending stress equation Eq. (14–15)

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

Fig. 14–6; p. 733

Eq. (14–27); p. 736

Table below

$_{0.99}(S_t)_{10^7}$ Tables 14–3, 14–4; pp. 728, 729

Fig. 14–14; p. 743

Gear bending endurance strength equation Eq. (14–17)

$$\sigma_{\text{all}} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

Table 14–10, Eq. (14–38); pp. 744, 743

1 if $T < 250°F$

Bending factor of safety Eq. (14–41)

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Figure 14–17

Roadmap of gear bending equations based on AGMA standards. (ANSI/AGMA 2001-D04.)

SPUR GEAR WEAR
BASED ON ANSI/AGMA 2001-D04

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

1 [or Eq. (a), Sec. 14–10]; p. 739

Eq. (14–30); p. 739

1

Gear contact stress equation Eq. (14–16)

$$\sigma_c = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)^{1/2}$$

Eq. (14–23); p. 735

Eq. (14–27); p. 736

Eq. (14–13), Table 14–8; pp. 724, 737

Table below

${}_{0.99}(S_c)_{10^7}$ Tables, 14–6, 14–7; pp. 731, 732

Fig. 14–15; p. 743

Section 14–12, gear only; pp. 741, 742

Gear contact endurance strength Eq. (14–18)

$$\sigma_{c,\text{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

Table 14–10, Eqs. (14–38); pp. 744, 743

1 if $T < 250°$ F

Gear only

Wear factor of safety Eq. (14–42)

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Figure 14–18

Roadmap of gear wear equations based on AGMA standards. (ANSI/AGMA 2001-D04.)