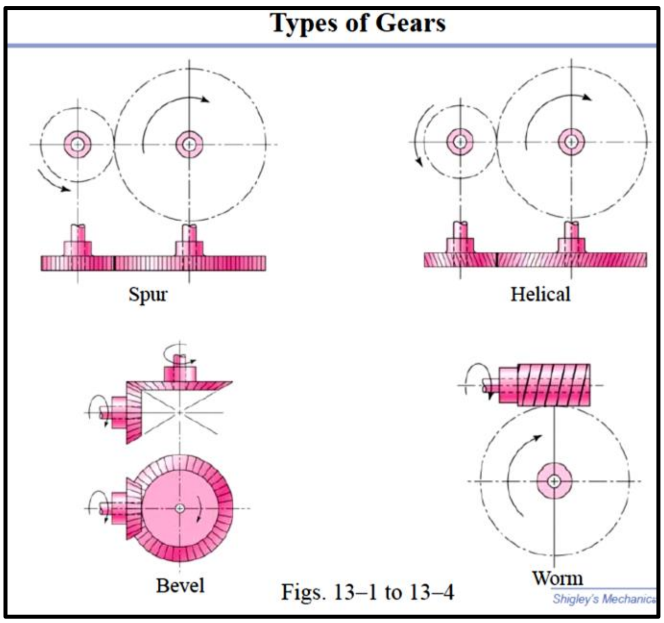
# Gears-General

## Types of Gears

There are four principal types of gears:

- *Spur gears*: (*Fig. 13-1*) the simplest type of gears. The teeth are parallel to the axis of rotation. It transmits rotation between parallel shafts.
- *Helical gears*: (*Fig. 13-2*) the teeth are inclined with respect to the axis of rotation. It transmits rotation between parallel shafts, same as spur gears, but it is less noisy than spur gears because of the more gradual engagement of the teeth during meshing. Because of the inclined teeth, thrust load is developed.
- *Bevel gears*: (*Fig. 13-3*) the teeth are formed on conical surfaces. It transmits rotation between intersecting shafts. The gear shown in figure 13-3 has straight teeth. In another type, the teeth form circular arcs and it is called spiral bevel gears.
- *Worms and worm gears*: (*Fig. 13-4*) transmits rotation between intersecting shafts. The worm resembles a screw (it can be right handed or left handed). Worm gear sets are used when speed ratios are high (*3 or more*). It transmits rotation from the worm to the worm gear, but not the opposite.

## Nomenclature



Figs. 13–1 to 13–4

Since spur gears are the simplest type, it will be used for illustration to develop the primary kinematic relations.
The figure illustrates spur-gears.

- *Pitch circle*: the theoretical circle upon which calculations are based and its diameter is called the "pitch diameter". Pitch circles of mating gears are tangent to each other. The "smaller" of the mating gears is called the pinion and the "larger" is called the gear.
- *Circular pitch "p"*: the distance measured on the pitch circle from point on one tooth to a corresponding point on adjacent tooth. The circular pitch is equal to the sum of tooth thickness and width of space.

- *Dedendum and Addendum circles*: the circles defining the bottom and top faces of the teeth.
- *Addendum " $a$ "*: the radial distance from the pitch circle to the top surface of the teeth.
- *Dedendum " $b$ "*: the radial distance from the pitch circle to the bottom surface of the teeth.
- *Clearance circle*: the circle tangent to the addendum circle of the mating gear.
- *Clearance " $c$ "*: the distance between the tooth top surface and the bottom surface of a mating gear.

**Nomenclature of Spur-Gear Teeth**

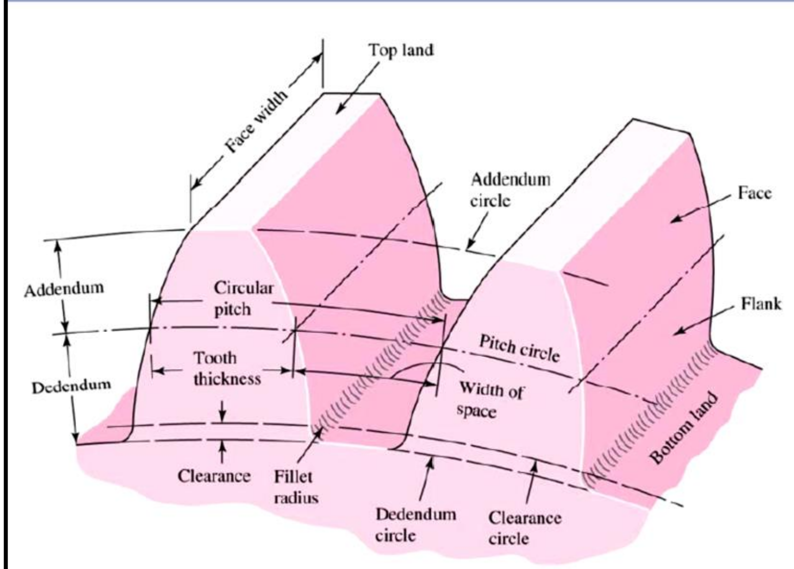



Fig. 13–5

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- *Diametral pitch "$P$ "*: the ratio of the number of teeth of a gear to the pitch diameter.

Diametral pitch *(teeth per inch)* → $P = \frac{N}{d}$ ← Number of teeth ($N$), Pitch diameter ($d$)

Circular pitch → $p = \frac{\pi d}{N} = \frac{\pi}{P}$

*It should be clear that gears meshing with each others must have the same diametral pitch.*

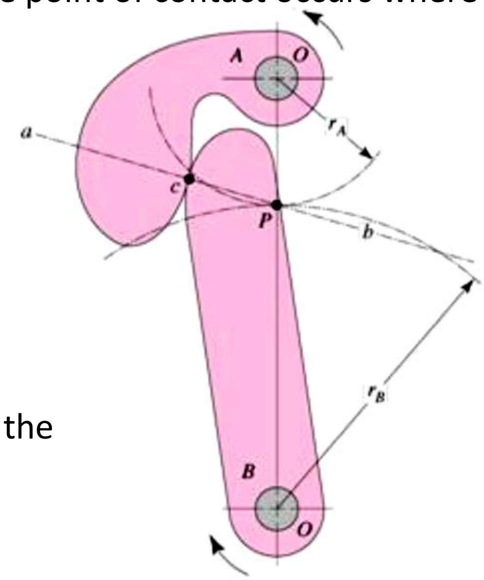
- *Module " m "*: is the ratio of pitch diameter to the number of teeth *(it is the inverse of the diametral pitch and it is commonly used with the SI system of units)*.

$$m = \frac{d}{N} = \frac{1}{P} \quad \rightarrow \quad p = \pi m$$

**Conjugate Action**

The profile of gear teeth are designed such that they will produce constant angular velocity ratio during meshing, and this is called conjugate action. When one curved

surface pushes against another, as seen in the figure, the point of contact occurs where the two surfaces are tangent to each other (Point *c*) and the forces will be directed along the common normal (line *ab*) which is also called the "line of action" or the "pressure line".




The line of action will intersect the line of centers at point "*P*" which also defines the point of tangency of the pitch circles of the two mating gears and it is called the pitch point.

The angular-velocity ratio is inversely proportional to the ratio of radii of the pitch circles of mating gear.

To transmit rotation at constant angular velocity the pitch point must remain fixed, meaning that all lines of action must pass through the same point "*P*".

To satisfy that, the profile of gear teeth are shaped as "involute profile".
With involute profile, all points of contact occur along the same line which is the line of action.

**Involute Properties**

The circle on which involute is generated is called "Base circle". An involute curve may be generated using a cord wrapped around the base circle,

- The most common conjugate profile is the involute profile.
- Can be generated by unwrapping a string from a cylinder, keeping the string taut and tangent to the cylinder.
- Circle is called base circle.

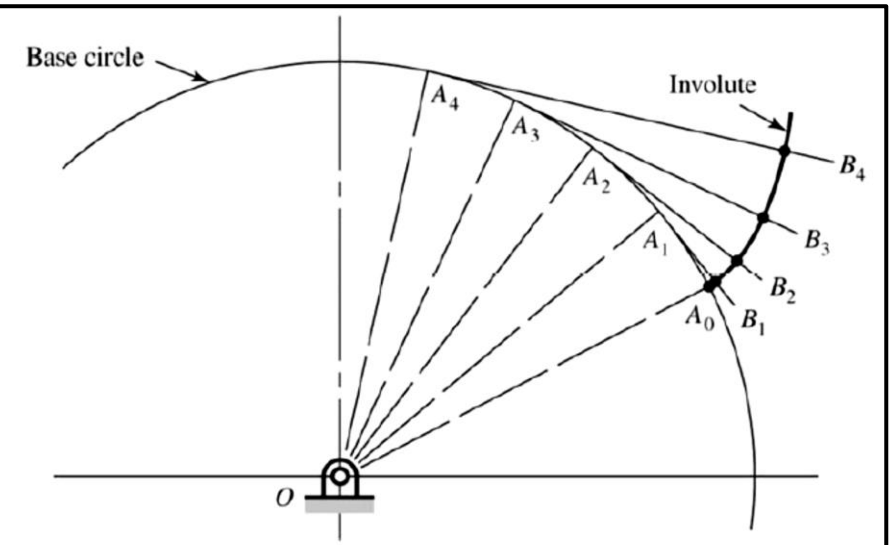
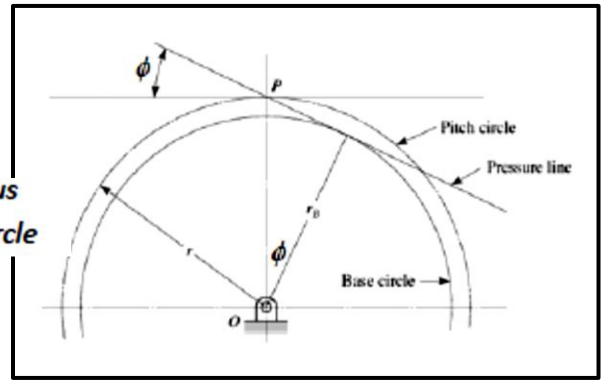



Fig. 13–8

**Fundamentals**

This section illustrates how to draw the teeth on a pair of meshing gears.

Pressure Angle "ϕ " is the angle between the pressure line (*line of action*) and the common tangent of the pitch circles of mating gears.

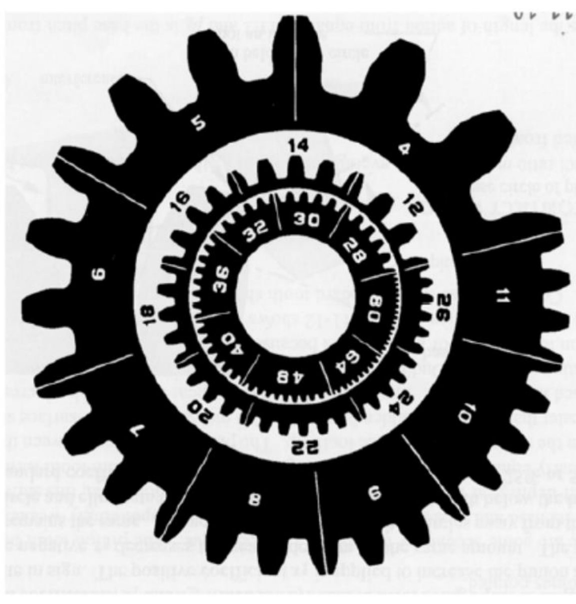



- Common values of ϕ are *20°* or *25°*
- Radius of base $\boxed{r_b = r\cos\emptyset}$ ***" r ": radius of pitch circle***
- Addendum: $a = \frac{1}{P}$
- Dedendum: $b = \frac{1.25}{P}$

## Diametral Pitch vs. Module

Table 11-2 Standard Diametral Pitches

| Coarse ($p_d$ < 20) | Fine ($p_d$ ≥ 20) |
|---|---|
| 1 | 20 |
| 1.25 | 24 |
| 1.5 | 32 |
| 1.75 | 48 |
| 2 | 64 |
| 2.5 | 72 |
| 3 | 80 |
| 4 | 96 |
| 5 | 120 |
| 6 | |
| 8 | |
| 10 | |
| 12 | |
| 14 | |
| 16 | |
| 18 | |



Tooth sizes for various $p_d$

Table 11-3 Standard Metric Modules

| Metric Module (mm) | Equivalent $p_d$ (in$^{-1}$) |
|---|---|
| 0.3 | 84.67 |
| 0.4 | 63.50 |
| 0.5 | 50.80 |
| 0.8 | 31.75 |
| 1 | 25.40 |
| 1.25 | 20.32 |
| 1.5 | 16.93 |
| 2 | 12.70 |
| 3 | 8.47 |
| 4 | 6.35 |
| 5 | 5.08 |
| 6 | 4.23 |
| 8 | 3.18 |
| 10 | 2.54 |
| 12 | 2.12 |
| 16 | 1.59 |
| 20 | 1.27 |
| 25 | 1.02 |

## Rack

- A *rack* is a spur gear with an pitch diameter of infinity.

- The sides of the teeth are straight lines making an angle to the line of centers equal to the pressure angle.
- The *base pitch* and *circular pitch*, shown in Fig. 13–13, are related by

$$p_b = p_c \cos\phi \qquad (13\text{–}7)$$

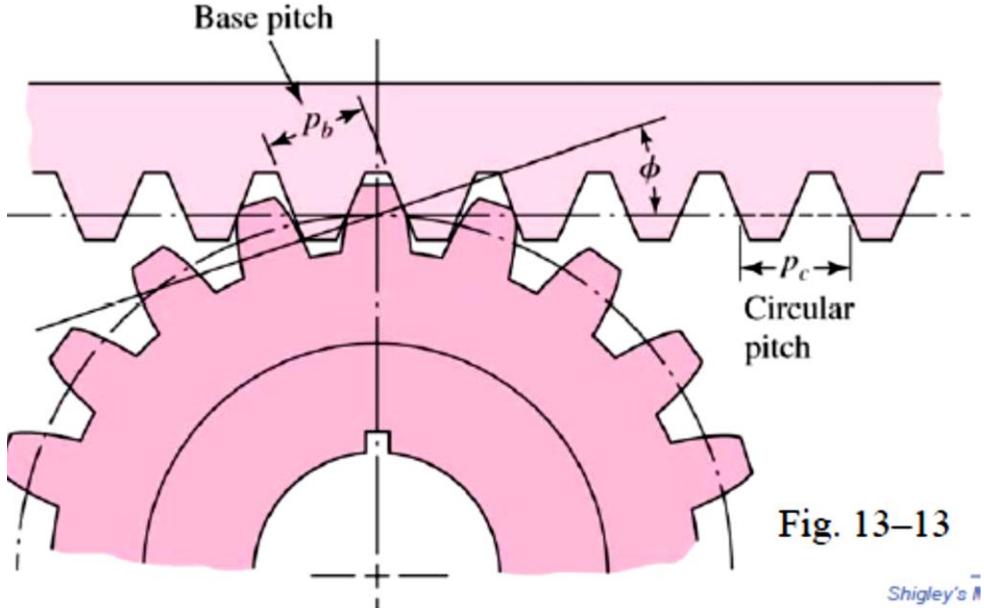



Fig. 13–13

## Example 13–1

A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The diametral pitch is 2, and the addendum and dedendum are 1/*P* and 1.25/*P*, respectively. The gears are cut using a pressure angle of 20°.

(*a*) Compute the circular pitch, the center distance, and the radii of the base circles.

(*b*) In mounting these gears, the center distance was incorrectly made $\frac{1}{4}$ in larger. Compute the new values of the pressure angle and the pitch-circle diameters.

**Solution**

(*a*)

$$p = \frac{\pi}{P} = \frac{\pi}{2} = 1.57 \text{ in}$$

The pitch diameters of the pinion and gear are, respectively,

$$d_P = \frac{16}{2} = 8 \text{ in} \qquad d_G = \frac{40}{2} = 20 \text{ in}$$

Therefore the center distance is

$$\frac{d_P + d_G}{2} = \frac{8+20}{2} = 14 \text{ in}$$

Since the teeth were cut on the 20° pressure angle, the base-circle radii are found to be, using $r_b = r\cos\phi$,

$$r_b\ (\text{pinion}) = \frac{8}{2}\cos 20^\circ = 3.76\ \text{in}$$

$$r_b\ (\text{gear}) = \frac{20}{2}\cos 20^\circ = 9.40\ \text{in}$$

(*b*) Designating $d'_P$ and $d'_G$ as the new pitch-circle diameters, the $\frac{1}{4}$-in increase in the center distance requires that

$$\frac{d'_P + d'_G}{2} = 14.250 \qquad (1)$$

Also, the velocity ratio does not change, and hence

$$\frac{d'_P}{d'_G} = \frac{16}{40} \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously yields

$$d'_P = 8.143\ \text{in} \qquad d'_G = 20.357\ \text{in}$$

Since $r_b = r\cos\phi$, the new pressure angle is

$$\phi' = \cos^{-1}\frac{r_b\ (\text{pinion})}{d'_P/2} = \cos^{-1}\frac{3.76}{8.143/2} = 22.56^\circ$$

**Contact Ratio**

The contact ratio "$m_C$" defines the average number of teeth pairs in contact during meshing.

- If ($m_C$=1) it means that only one pair of teeth is in contact at a time.
- To reduce possibility of impact it is recommended that $m_C \geq 1.2$

**Interference**

The contact of portions of tooth profiles that are not conjugate is called interference.

- Interference happens in gear pairs because the Dedendum circle is smaller than the base circle and thus the involute portion of teeth profile is small.
- Thus, to reduce interference larger pressure angles 20° or 25° are used (*i.e., making the base circle smaller*).

- On a spur pinion and gear with one-to-one gear ratio, smallest number of teeth which will not have interference is

$$N_P = \frac{2k}{3\sin^2\phi}\left(1+\sqrt{1+3\sin^2\phi}\right) \qquad (13\text{–}10)$$

- k =1 for full depth teeth. k = 0.8 for stub teeth
- On spur meshed with larger gear with gear ratio mG = NG/NP = m, the smallest number of teeth which will not have interference is

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right) \qquad (13\text{–}11)$$

- Largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2\sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi} \qquad (13\text{–}12)$$

- Smallest spur pinion that is interference-free with a rack is

$$N_G = \frac{N_P^2\sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi} \qquad (13\text{–}12)$$

- For 20º pressure angle, the most useful values from Eq. (13–12) are calculated and shown in the table below.

| Minimum $N_P$ | Max $N_G$ | Integer Max $N_G$ | Max Gear Ratio $m_G = N_G/N_P$ |
|---|---|---|---|
| 13 | 16.45 | 16 | 1.23 |
| 14 | 26.12 | 26 | 1.86 |
| 15 | 45.49 | 45 | 3 |
| 16 | 101.07 | 101 | 6.31 |
| 17 | 1309.86 | 1309 | 77 |

**Cutting of Gear Teeth**

- Common ways of cutting gear teeth

◦ Milling
◦ Shaping
◦ Hobbing

**Shaping with Pinion Cutter**

Fig. 13–17

**Hobbing a Worm Gear**

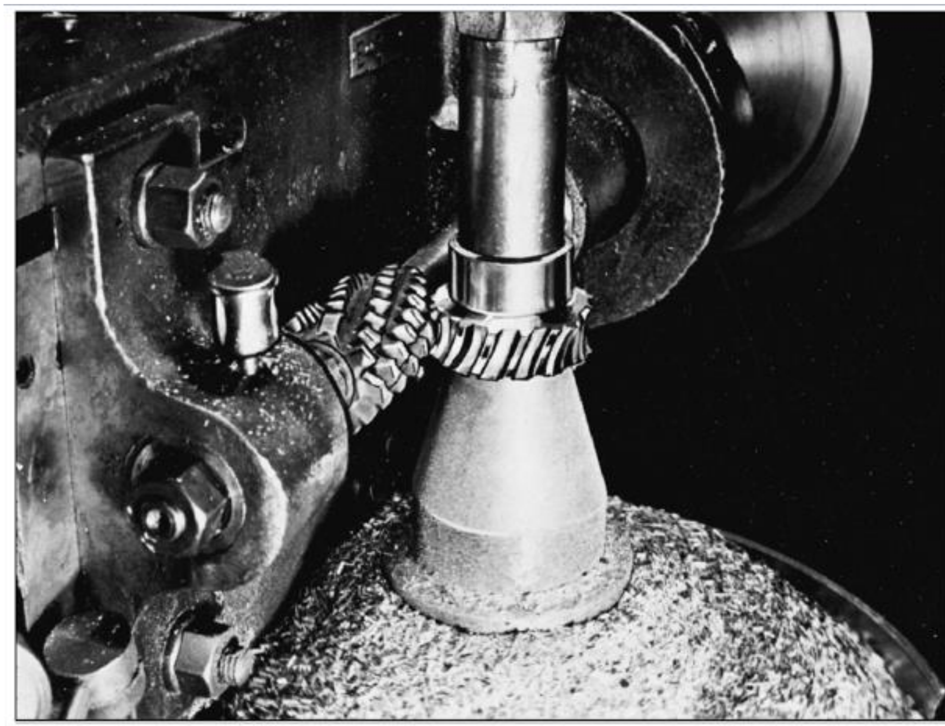


Fig. 13–19

**Parallel Helical Gears**

Helical gears are mostly used to transmit motion between parallel shafts *(it can be used for intersecting shafts as well)*.

- Both gears have the same helix angle, but one gear must have a right-hand helix and the other a left-hand helix.
- The shape of the teeth is an involute helicoid, and it can be produced by wrapping a piece of paper shaped as parallelogram on the base cylinder *(see fig. 13-21)*.

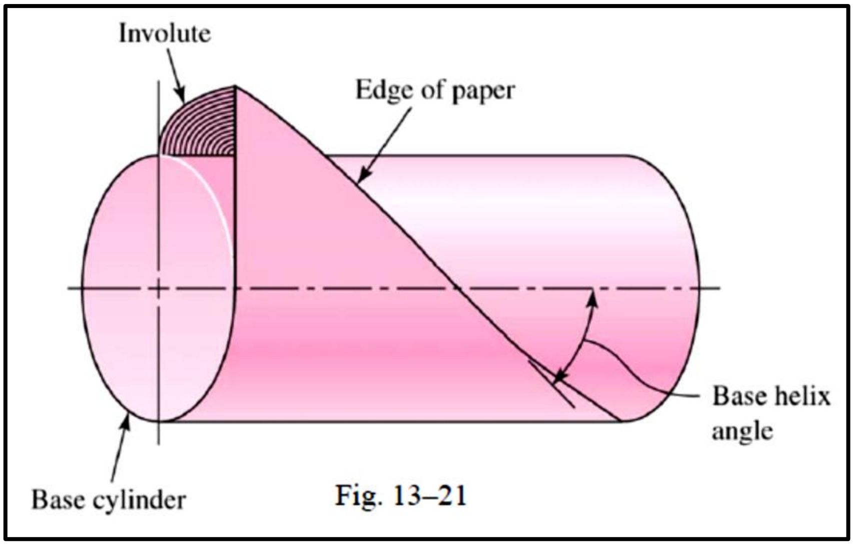


Fig. 13–21

- Contact of the teeth starts as a point then extends into a diagonal line across the face of the tooth as teeth come into more engagement.
- Because of the gradual engagement of the teeth, helical gears can transmit heavy loads at high speeds.
- Helical gears produce both radial and thrust loads on the shaft.
- Two opposite-hand helical gears mounted side by side on each shaft can be used to cancel the thrust load.

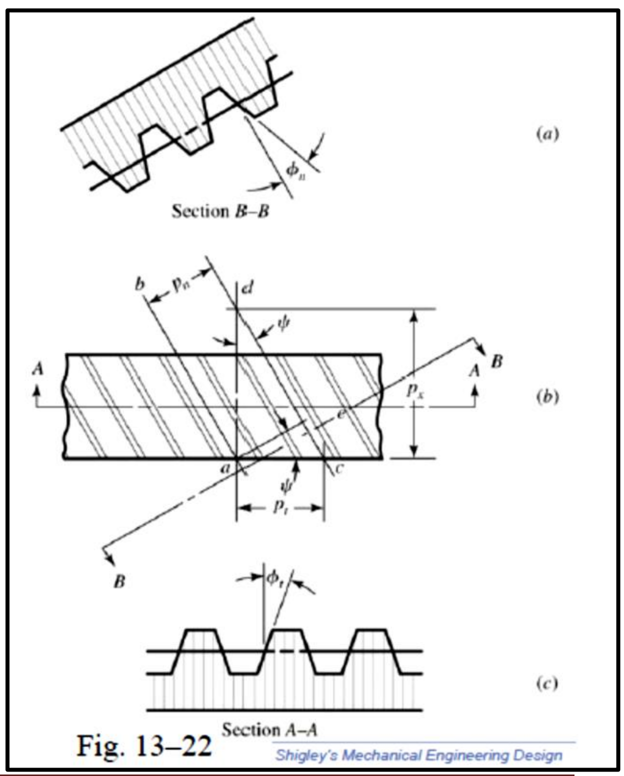


Fig. 13–22

The figure shows the nomenclature of helical gears.

- Helix angle: $\Psi$
- Transverse circular pitch: $P_t$

- Normal circular pitch: $P_n$

$$\boxed{p_n = p_t \cos\psi}$$

- Normal and transverse pressure angles $\phi_n$ & $\phi_t$ are

related to the helix angle as:

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$$

Axial pitch $p_x$ is along the direction of the shaft axis

$$p_x = \frac{p_t}{\tan\psi} \qquad (13\text{–}17)$$

- Normal diametral pitch

$$P_n = \frac{P_t}{\cos\psi} \qquad (13\text{–}18)$$

$$p_n P_n = \pi$$

**Parallel Helical Gears**

- Viewing along the teeth, the apparent pitch radius is greater than when viewed along the shaft.
- The greater virtual ***R*** has a greater virtual number of teeth ***N***'

$$N' = \frac{N}{\cos^3\psi} \qquad (13\text{–}20)$$

- Allows fewer teeth on helical gears without undercutting.

**Tooth Systems**

A tooth system is a standard that specifies the relationships between the gear parameters such as addendum, dedendum, tooth thickness, pressure angle, etc.

- The standards were developed to attain interchangeability of gears having the same pressure angle and diametral pitch regardless of the number of teeth.

❖ *Tables 13-1 to 13-5* give the standards for different types of gears.

## Standard and Commonly Used Tooth Systems for Spur Gears

| Tooth System | Pressure Angle $\phi$, deg | Addendum $a$ | Dedendum $b$ |
|---|---|---|---|
| Full depth | 20 | $1/P_d$ or $1m$ | $1.25/P_d$ or $1.25m$ |
| | | | $1.35/P_d$ or $1.35m$ |
| | $22\frac{1}{2}$ | $1/P_d$ or $1m$ | $1.25/P_d$ or $1.25m$ |
| | | | $1.35/P_d$ or $1.35m$ |
| | 25 | $1/P_d$ or $1m$ | $1.25/P_d$ or $1.25m$ |
| | | | $1.35/P_d$ or $1.35m$ |
| Stub | 20 | $0.8/P_d$ or $0.8m$ | $1/P_d$ or $1m$ |

Table 13–1

## Tooth Sizes in General Use

| Diametral Pitch | |
|---|---|
| Coarse | 2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16 |
| Fine | 20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200 |

| Modules | |
|---|---|
| Preferred | 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50 |
| Next Choice | 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45 |

Table 13–2

## Tooth Proportions for 20° Straight Bevel-Gear Teeth

| Item | Formula |
|---|---|
| Working depth | $h_k = 2.0/P$ |
| Clearance | $c = (0.188/P) + 0.002$ in |

| | |
|---|---|
| Addendum of gear | $a_G = \dfrac{0.54}{P} + \dfrac{0.460}{P(m_{90})^2}$ |
| Gear ratio | $m_G = N_G/N_P$ |
| Equivalent 90° ratio | $m_{90} = m_G$ when $\Gamma = 90^\circ$ |
| | $m_{90} = \sqrt{m_G \dfrac{\cos\gamma}{\cos\Gamma}}$ when $\Gamma \neq 90^\circ$ |
| Face width | $F = 0.3A_0$ or $F = \dfrac{10}{P}$, whichever is smaller |

Minimum number of teeth

| Pinion | 16 | 15 | 14 | 13 |
|---|---|---|---|---|
| Gear | 16 | 17 | 20 | 30 |

Table 13–3

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## Standard Tooth Proportions for Helical Gears

| Quantity* | Formula | Quantity* | Formula |
|---|---|---|---|
| Addendum | $\dfrac{1.00}{P_n}$ | External gears: | |
| Dedendum | $\dfrac{1.25}{P_n}$ | Standard center distance | $\dfrac{D+d}{2}$ |
| Pinion pitch diameter | $\dfrac{N_P}{P_n \cos\psi}$ | Gear outside diameter | $D + 2a$ |
| Gear pitch diameter | $\dfrac{N_G}{P_n \cos\psi}$ | Pinion outside diameter | $d + 2a$ |
| Normal arc tooth thickness† | $\dfrac{\pi}{P_n} - \dfrac{B_n}{2}$ | Gear root diameter | $D - 2b$ |
| Pinion base diameter | $d \cos\phi_t$ | Pinion root diameter | $d - 2b$ |
| | | Internal gears: | |
| Gear base diameter | $D \cos\phi_t$ | Center distance | $\dfrac{D-d}{2}$ |
| Base helix angle | $\tan^{-1}(\tan\psi \cos\phi_t)$ | Inside diameter | $D - 2a$ |
| | | Root diameter | $D + 2b$ |

Table 13–4

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**Recommended Pressure Angles and Tooth Depths for Worm Gearing**

| Lead Angle λ, deg | Pressure Angle $\phi_n$, deg | Addendum $a$ | Dedendum $b_G$ |
|---|---|---|---|
| 0–15 | $14\frac{1}{2}$ | $0.3683p_x$ | $0.3683p_x$ |
| 15–30 | 20 | $0.3683p_x$ | $0.3683p_x$ |
| 30–35 | 25 | $0.2865p_x$ | $0.3314p_x$ |
| 35–40 | 25 | $0.2546p_x$ | $0.2947p_x$ |
| 40–45 | 30 | $0.2228p_x$ | $0.2578p_x$ |

Table 13–5

**Gear Trains**

A gear train consists of more than two gears. In general, there can be several driving gears *(input)* and several driven gears *(output)*. Any gear that is not giving any input torque nor taking any output torque is called an "idler" gear.

Consider a pinion "2" driving a gear "3", the speed of the gear is:

$$n_3 = \left|\frac{N_2}{N_3}n_2\right| = \left|\frac{d_2}{d_3}n_2\right| \qquad (13\text{–}29)$$

where

***n***: Angular velocity (rev./min.), ***N***: Number of teeth, ***d***: Pitch diameter

This equation applies to all types of gears (*spur, helical, bevel and worm*)

❖ *but note that* $n_3 \neq \frac{d_2}{d_3}n_2$ *r worm gear sets*

The absolute-value sign is to give freedom in choosing positive or negative directions.

❖ The sign convention follows the right-hand rule: + Counter clockwise, – Clockwise

For the gear train shown, speed of gear "6" is:

The sign is decided manually.

$$n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6}n_2$$