Energy Considerations

When rotating members are caused to stop (in the case of brake) or when members initially at rest are brought up to speed (in the case of clutch), slipping must occur between the mating surfaces causing some of the kinetic energy to transform into thermal energy (heat).

The capacity of a clutch/brake is limited by two factors, the characteristics of the material (*coefficient of friction and the max allowable pressure*) and the ability of the clutch to dissipate heat.

Friction materials

The most important characteristic of friction materials used in brakes/clutches includes:

- High and reproducible coefficient of friction.
- Resistance to environmental conditions, such as moisture.
- The ability to withstand high temperatures and pressure.
- High resistance to wear.

❖ *Table 16-3* gives the important characteristics of some friction materials used in clutches/brakes.

Characteristics of Friction Materials

Material	Friction Coefficient f	Maximum Pressure p_{max}, psi	Maximum Temperature Instantaneous, °F	Maximum Temperature Continuous, °F	Maximum Velocity V_{max}, ft/min	Applications
Cermet	0.32	150	1500	750		Brakes and clutches
Sintered metal (dry)	0.29–0.33	300–400	930–1020	570–660	3600	Clutches and caliper disk brakes
Sintered metal (wet)	0.06–0.08	500	930	570	3600	Clutches
Rigid molded asbestos (dry)	0.35–0.41	100	660–750	350	3600	Drum brakes and clutches
Rigid molded asbestos (wet)	0.06	300	660	350	3600	Industrial clutches
Rigid molded asbestos pads	0.31–0.49	750	930–1380	440–660	4800	Disk brakes
Rigid molded nonasbestos	0.33–0.63	100–150		500–750	4800–7500	Clutches and brakes
Semirigid molded asbestos	0.37–0.41	100	660	300	3600	Clutches and brakes
Flexible molded asbestos	0.39–0.45	100	660–750	300–350	3600	Clutches and brakes
Wound asbestos yarn and wire	0.38	100	660	300	3600	Vehicle clutches
Woven asbestos yarn and wire	0.38	100	500	260	3600	Industrial clutches and brakes
Woven cotton	0.47	100	230	170	3600	Industrial clutches and brakes
Resilient paper (wet)	0.09–0.15	400	300		$PV < 500\,000$ psi · ft/min	Clutches and transmission bands

Table 16–3

Flywheels

A flywheel is a mechanical energy storage/delivery device that stores energy in the Flywheels are used to regulate rotational speed/torque for systems having non-uniform torque supply/demand. This happens because a flywheel resists changes in its rotational speed.

- The equation of motion for a flywheel is:

$$\boxed{\sum M = 0 \quad \rightarrow \quad I\ddot{\theta} = T_i(\theta_i, \omega_i) - T_o(\theta_o, \omega_o)}$$

where: T_i & T_o are the input and output torque.
θ & ω are the angular position and angular velocity.

❖ In many cases, the torque depends on **θ** or ***ω*** or on both, for example electric motors torque depends on (ω) only.

- Assuming a rigid shaft $\theta_i = \theta_o = \theta$ and $\omega_i = \omega_o = \omega$. and , the equation of motion of the flywheel becomes:

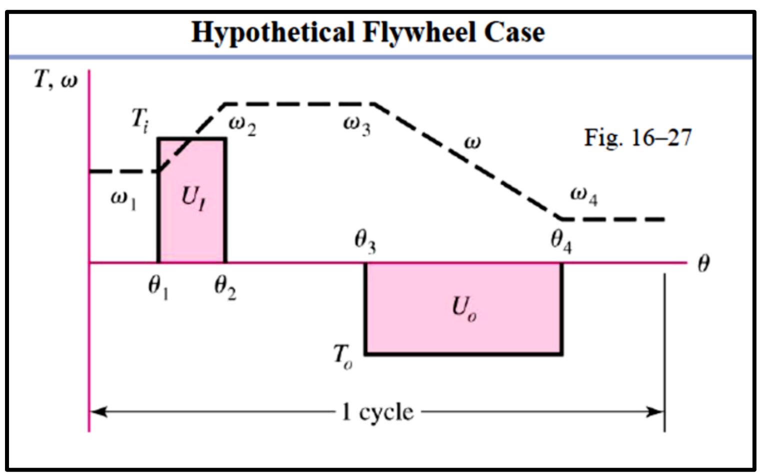
$$I\ddot{\theta} = T_i(\theta, \omega) - T_o(\theta, \omega)$$

- This equation can be solved for the instantaneous values of ω & $\ddot{\theta}$.
- However, those values are not of interest to us when designing a flywheel. We are interested in the overall performance of the flywheel:

✓ What should the moment of inertia be?

✓ How to match the power source to the load?

✓ What are the resulting performance characteristics of the system?



- The figure shows the performance of a flywheel under a hypothetical situation *(friction is neglected)*.
- The flywheel starts at angular velocity ω_1.
- Between $\theta_1 \rightarrow \theta_2$ the power source supplies a constant torque causing the flywheel to accelerate from ω_1 to ω_2.

- Between $\theta_2 \rightarrow \theta_3$ the shaft rotates with zero torque, thus there will be no acceleration $\omega_2 = \omega_3$.
- Between $\theta_3 \rightarrow \theta_4$ a constant output torque is applied causing the flywheel to slow down from ω_3 to ω_4.

- The work input to the flywheel is: $U_i = T_i(\theta_2 - \theta_1)$
- And the work output is: $U_o = T_o(\theta_4 - \theta_3)$

- If $U_o > U_i$ then $\omega_4 < \omega_1$.
- If $U_o = U_i$ then $\omega_4 = \omega_1$.
- If $U_o < U_i$ then $\omega_4 > \omega_1$

- The work done on the flywheel between θ_1 & θ_2 can be found as the difference in kinetic energy.

$$U_i = E_2 - E_1 = \frac{1}{2} I\,\omega_2^2 - \frac{1}{2} I\,\omega_1^2$$

$$\rightarrow \qquad E_2 - E_1 = \frac{1}{2} I(\omega_2^2 - \omega_1^2) = T(\theta_2 - \theta_1)$$

- Most of the torque-angular position functions encountered in engineering application are so complicated and they require using numerical integration to find the total work (*area under the curve*).
- *Fig 18-28* shows the torque of a one cylinder engine for one cycle.

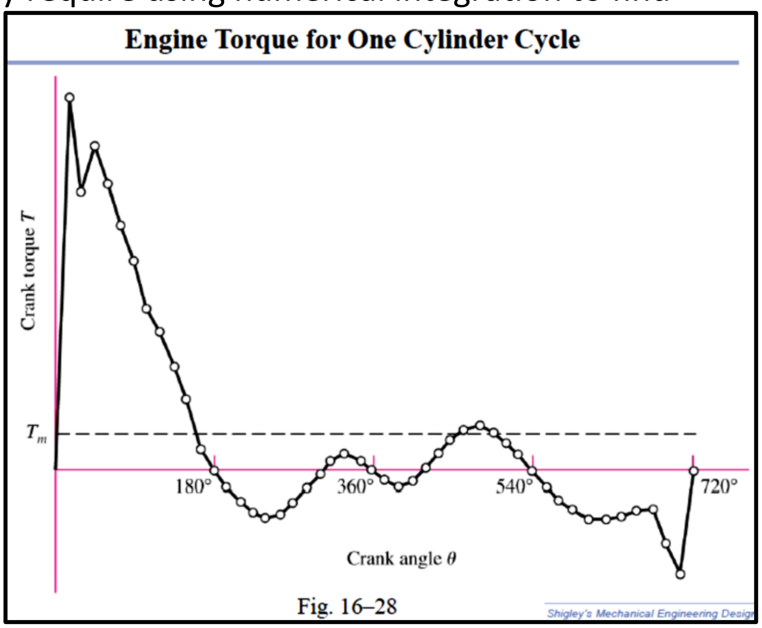


Fig. 16–28

✓ Integrating the curve gives the total energy supplied by the engine. Then dividing the result by the length of one-cycle (4π) gives the mean torque value of the engine.

- The allowable range of speed fluctuation is usually defined using the "*Coefficient of speed fluctuation*":

$$\boxed{C_s = \frac{\omega_2 - \omega_1}{\omega}}$$

where ω is the nominal angular velocity: $\omega = \frac{\omega_1+\omega_2}{2}$

- Substitute in the energy difference equation we get:

$$\boxed{E_2 - E_1 = C_s I \omega^2 = T(\theta_2 - \theta_1)}$$

where $T(\theta_2 - \theta_1)$ is the area under the torque curve

❖ This equation can be used to obtain the flywheel inertia " I " needed to satisfy the required C_s value.

Example 16–6

Table 16–6 lists values of the torque used to plot Fig. 16–28. The nominal speed of the engine is to be 250 rad/s.

(*a*) Integrate the torque-displacement function for one cycle and find the energy that can be delivered to a load during the cycle.

(*b*) Determine the mean torque T_m (see Fig. 16–28).

(*c*) The greatest energy fluctuation is approximately between $\theta = 15°$ and $\theta = 150°$ on the torque diagram; see Fig. 16–28 and note that $T_o = -T_m$. Using a coefficient of speed fluctuation $C_s = 0.1$, find a suitable value for the flywheel inertia.

(*d*) Find ω_2 and ω_1.

Solution

(*a*) Using $n = 48$ intervals of $\Delta\theta = 4\pi/48$, numerical integration of the data of Table 16–6 yields $E = 3368$ in · lbf. This is the energy that can be delivered to the load.

θ, deg	T, lbf · in	θ, deg	T, lbf · in	θ, deg	T, lbf · in	θ, deg	T, lbf · in
0	0	195	−107	375	−85	555	−107
15	2800	210	−206	390	−125	570	−206
30	2090	225	−260	405	−89	585	−292
45	2430	240	−323	420	8	600	−355
60	2160	255	−310	435	126	615	−371
75	1840	270	−242	450	242	630	−362
90	1590	285	−126	465	310	645	−312
105	1210	300	−8	480	323	660	−272
120	1066	315	89	495	280	675	−274
135	803	330	125	510	206	690	−548
150	532	345	85	525	107	705	−760
165	184	360	0	540	0	720	0
180	0						

Table 16–6

(*b*)
$$T_m = \frac{3368}{4\pi} = 268 \text{ lbf} \cdot \text{in}$$

(*c*) The largest positive loop on the torque-displacement diagram occurs between $\theta = 0°$ and $\theta = 180°$. We select this loop as yielding the largest speed change. Subtracting 268 lbf · in from the values in Table 16–6 for this loop gives, respectively, −268, 2532, 1822, 2162, 1892, 1572, 1322, 942, 798, 535, 264, −84, and −268 lbf · in. Numerically integrating $T - T_m$ with respect to θ yields $E_2 - E_1 = 3531$ lbf · in. We now solve Eq. (16–64) for *I*. This gives

$$I = \frac{E_2 - E_1}{C_s\omega^2} = \frac{3531}{0.1(250)^2} = 0.565 \text{ lbf} \cdot \text{s}^2 \text{ in}$$

(*d*) Equations (16–62) and (16–63) can be solved simultaneously for ω_2 and ω_1. Substituting appropriate values in these two equations yields

$$\omega_2 = \frac{\omega}{2}(2 + C_s) = \frac{250}{2}(2 + 0.1) = 262.5 \text{ rad/s}$$

$$\omega_1 = 2\omega - \omega_2 = 2(250) - 262.5 = 237.5 \text{ rad/s}$$

These two speeds occur at $\theta = 180°$ and $\theta = 0°$, respectively.

Characteristics of Friction Materials

Material	Friction Coefficient f	Maximum Pressure p_{max}, psi	Maximum Temperature Instantaneous, °F	Maximum Temperature Continuous, °F	Maximum Velocity V_{max}, ft/min	Applications
Cermet	0.32	150	1500	750		Brakes and clutches
Sintered metal (dry)	0.29–0.33	300–400	930–1020	570–660	3600	Clutches and caliper disk brakes
Sintered metal (wet)	0.06–0.08	500	930	570	3600	Clutches
Rigid molded asbestos (dry)	0.35–0.41	100	660–750	350	3600	Drum brakes and clutches
Rigid molded asbestos (wet)	0.06	300	660	350	3600	Industrial clutches
Rigid molded asbestos pads	0.31–0.49	750	930–1380	440–660	4800	Disk brakes
Rigid molded nonasbestos	0.33–0.63	100–150		500–750	4800–7500	Clutches and brakes
Semirigid molded asbestos	0.37–0.41	100	660	300	3600	Clutches and brakes
Flexible molded asbestos	0.39–0.45	100	660–750	300–350	3600	Clutches and brakes
Wound asbestos yarn and wire	0.38	100	660	300	3600	Vehicle clutches
Woven asbestos yarn and wire	0.38	100	500	260	3600	Industrial clutches and brakes
Woven cotton	0.47	100	230	170	3600	Industrial clutches and brakes
Resilient paper (wet)	0.09–0.15	400	300		$PV < 500\,000$ psi · ft/min	Clutches and transmission bands

Table 16–3

Friction Materials for Clutches

Table 16–5

Friction Materials for Clutches

Material	Friction Coefficient Wet	Friction Coefficient Dry	Max. Temperature °F	Max. Temperature °C	Max. Pressure psi	Max. Pressure kPa
Cast iron on cast iron	0.05	0.15–0.20	600	320	150–250	1000–1750
Powdered metal* on cast iron	0.05–0.1	0.1–0.4	1000	540	150	1000
Powdered metal* on hard steel	0.05–0.1	0.1–0.3	1000	540	300	2100
Wood on steel or cast iron	0.16	0.2–0.35	300	150	60–90	400–620
Leather on steel or cast iron	0.12	0.3–0.5	200	100	10–40	70–280
Cork on steel or cast iron	0.15–0.25	0.3–0.5	200	100	8–14	50–100
Felt on steel or cast iron	0.18	0.22	280	140	5–10	35–70
Woven asbestos* on steel or cast iron	0.1–0.2	0.3–0.6	350–500	175–260	50–100	350–700
Molded asbestos* on steel or cast iron	0.08–0.12	0.2–0.5	500	260	50–150	350–1000
Impregnated asbestos* on steel or cast iron	0.12	0.32	500–750	260–400	150	1000
Carbon graphite on steel	0.05–0.1	0.25	700–1000	370–540	300	2100