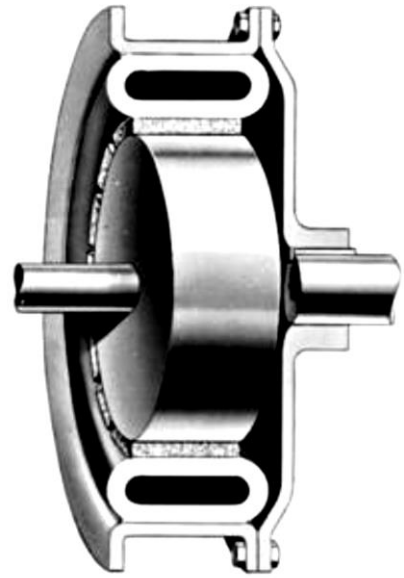
External Contracting Rim Clutches and Brakes

- The figure shows a Pneumatic external contracting clutch/brake system.
  Self-energizing or not?



❖ It should be clear that self-energizing or deenergizing condition only applies to pivoted-shoe brakes where the summation of moments about the pivoting pin should be zero. If the moment of the frictional force is in the same direction of the moment of the actuating force the brake will be self-energizing and if it is in the opposite direction the brake will be self-deenergizing.

- The figure shows the notation for pivoted external contracting rim brake system.
  - For the shoe configuration shown, a clockwise rotation will be self- deenergizing while a counterclockwise will be self-energizing.
  - The exact same analysis procedure of the internally expanding shoe applies here.
  - The moment of the normal and frictional forces are found using the same equations used before.
  - Torque also is found using the same equation as before.
  - The actuating force and pin reactions are found using:

$$\boxed{F = \frac{M_N + M_F}{c}}$$

$$\begin{cases} R_x = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(A + fB) - F_x \\ R_y = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(fA - B) + F_y \end{cases}$$

(Self-deenergizing)

Notation of External Contracting Shoes

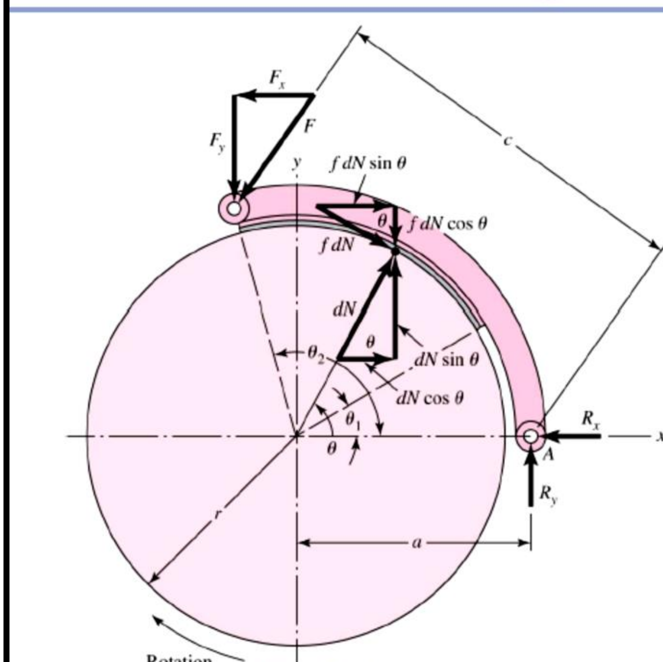


Fig. 16–11

$$\boxed{F = \frac{M_N - M_F}{c}}$$

$$\begin{cases} R_x = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(A - fB) - F_x \\ R_y = \dfrac{p_{all}\, b\, r}{\sin\theta_a}(-fA - B) + F_y \end{cases}$$

(Self-energizing)

- Fig. *16-12* shows a rim brake with symmetrical pivoted shoe.

❖ A special case arise there where the moment of the frictional forces about the pivot is zero (*i.e., the same braking capacity will be obtained for clockwise or counterclockwise rotation*).

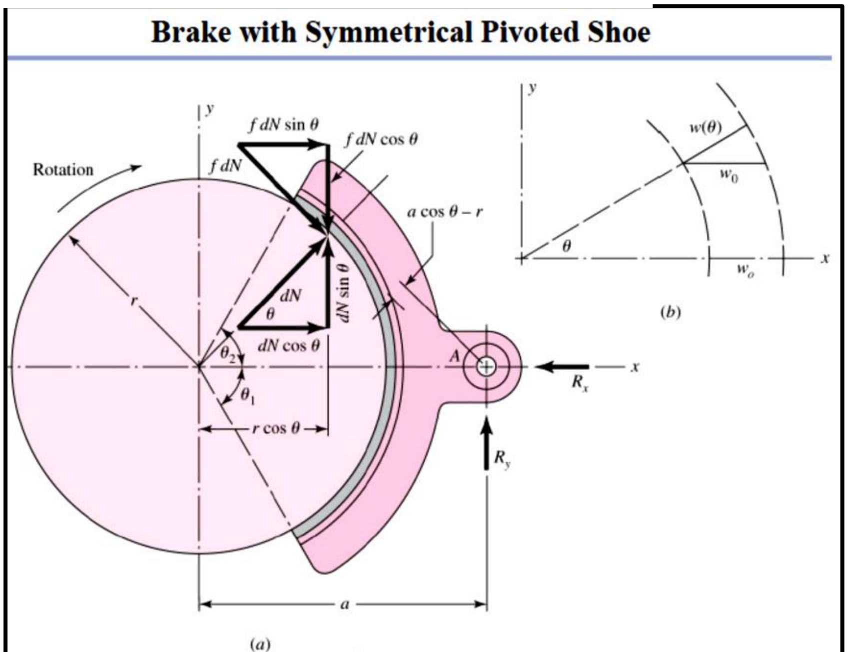


Fig. 16–12

Frictional Contact Axial Clutches

In axial clutches the mating frictional members are moved in a direction parallel to the shaft.

❖ The advantages of disk clutches over rim clutches include:

- Freedom from centrifugal effects.
- The large frictional area that can be installed in a small space.
- More effective heat dissipation surfaces.
- More uniform pressure distribution.

- Disk clutches can have single-plate (*friction on two surfaces*), *fig. 16-14*, or multiple-disks *fig. 16-15*.

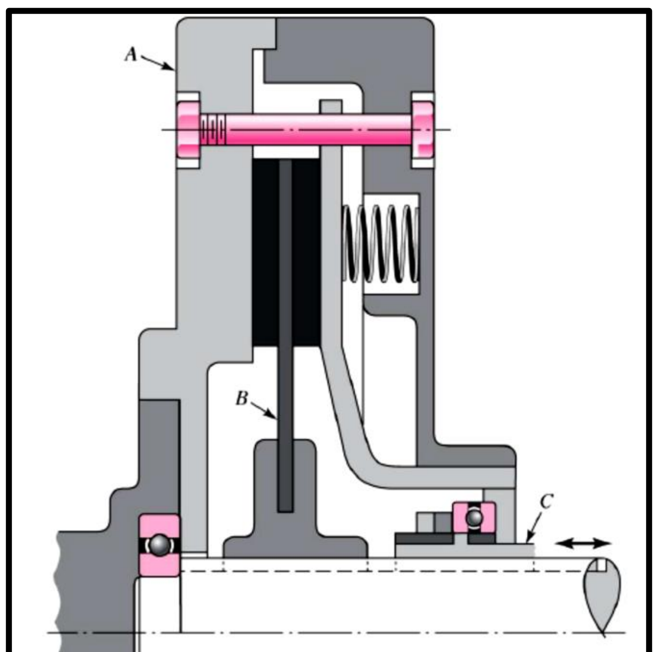


Fig. 16–14

❖ There are two methods for analyzing disk clutches, *uniform wear* and *uniform pressure*.

▪ If the disk is rigid (*or if springs are used*) a uniform pressure will be applied over the frictional surfaces. This will cause more wear in the outer areas since more work is done at the outer areas. Uniform pressure is usually the case for new clutches.

▪ After certain amount of wear has taken place (*more wear at the outer areas*), the pressure distribution will change (*less pressure at the outer areas*) and that makes the wear to become more uniform, this is usually the case with old clutches.

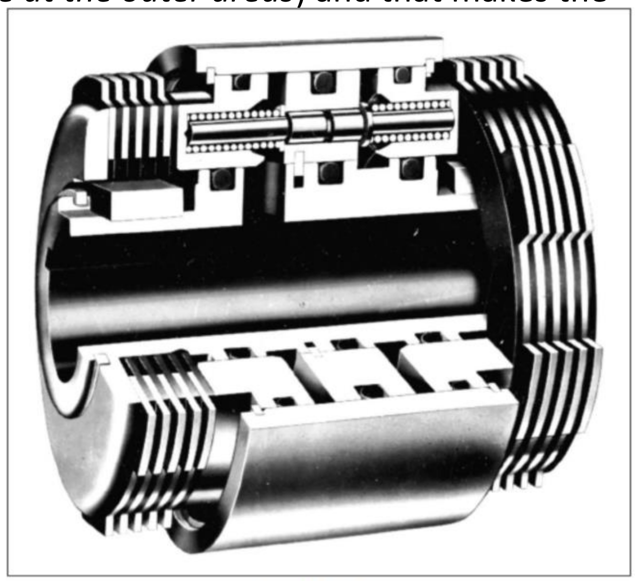


Fig. 16–15

Uniform Wear *(old clutches)*

After uniform wear condition has been reached, the axial wear can be expressed as:

$$\boxed{w = f_1 f_2 KPVt \quad \textit{*from chapter 12*}}$$

▪ Since wear is uniform and *P* & V are the only variables then (*PV*) needs to be constant;

$$PV = Pr\omega = constant, \qquad \text{since } \omega \text{ is constant}$$

$$\boxed{\rightarrow Pr = constant = P_{max} r_i = P_{all} r_i = P_{all}\frac{d}{2}}$$

▪ Taking a differential ring of radius " *r* " and thickness"*dr* "

▪ The force applied over this area is: $dF = PdA = P\,2\pi\,r\,dr$

▪ The actuating force over the whole area is:

$$F = \int dF = \int 2\pi\,P\,r\,dr = \int_{\frac{d}{2}}^{\frac{D}{2}} \pi\,P_{all}\,d\,dr = \boxed{\frac{\pi\,P_{all}\,d}{2}(D-d)} \qquad (1)$$

▪ The torque is found by integrating the product of frictional force and radius.

$$\boxed{T = \int r\, f\, dF = \int 2\pi\, f\, P\, r^2\, dr = \pi\, f\, P_{all}\, d \int_{\frac{d}{2}}^{\frac{D}{2}} r\, dr = \frac{\pi\, f\, P_{all}\, d}{8}(D^2 - d^2)}$$

Substituting the value of ***F*** we get:

$$\boxed{T = \frac{F\,f}{4}(D + d)} \qquad (2)$$

- ❖ Equation (1) gives the actuating force required for the max pressure to reach the allowable pressure and it holds true for any number of friction surfaces.
- ❖ Equation (2) gives the torque capacity associated with for one friction surface.

Uniform pressure (new clutches)

When uniform pressure is assumed, the force is simply the product of pressure and area.

- For $P = P_{all}$, the actuating force is:

$$\boxed{F = \frac{\pi\, P_{all}}{4}(D^2 - d^2)} \qquad \textit{for any number of frictional surfaces}$$

- The torque can be obtained as before and it is found to be:

$$\boxed{T = \frac{F\,f}{3}\left(\frac{D^3 - d^3}{D^2 - d^2}\right)} \qquad \textit{for one frictional surface only}$$

- From these equations the advantage of using multiple disks can be seen. For example if two disks are used (*four friction surfaces*), we can get four times the torque for the same actuating force compared to a single friction surface.
- Comparing the results obtained using uniform wear and uniform pressure equations (*see text*) it can be seen that the difference is not that big. Since new clutches get old anyway, it is suggested to use the uniform wear equations always.

Disk Brakes

There is no fundamental difference between clutches and disk brakes. The same analysis procedure applies to both. *Fig. 16-18* shows a typical automotive disk brake.

- We have seen that rim brakes can be design to be self-energizing (*of course disk brakes cannot*). While self-energization has an advantage in reducing the required braking force, it also has a big disadvantage when the coefficient of friction is reduced

(*due to shoes getting wet for example*) where the braking torque will be decreased severely (*because the moment of the frictional forces will be decreased thus decreasing the actual force applied to the shoe*). This is not the case with disk brakes, where a reduction in the coefficient of friction will reduce the braking torque only by the same percentage by which the friction was reduced.

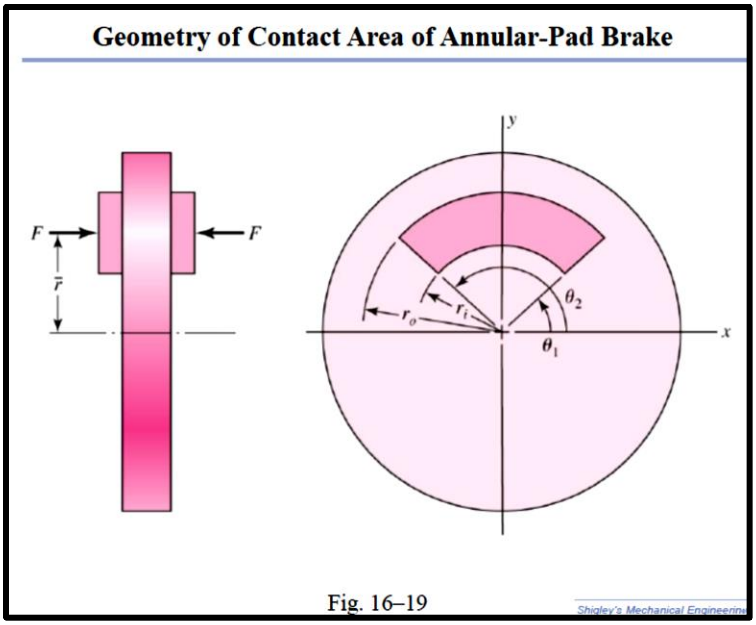


Fig. 16–19

- The equations for disk brakes are developed the same way as for disk clutches, but since the pads are not full circles, double integration (*from* r_i *to* r_o *and from* θ_1 *to* θ_2) is used.

❖ Note that the shoe is symmetric with respect to the *y* axis and that the angles θ_1 & θ_2 are measured from the positive *x* axis.

Uniform wear:

- Actuating force: $\boxed{F = (\theta_2 - \theta_1) P_{all}\, r_i (r_o - r_i)}$
- Braking torque $\boxed{T = \frac{1}{2}(\theta_2 - \theta_1) f\, P_{all}\, r_i (r_o^2 - r_i^2)}$ *single friction surface*
- Torque equivalent radius:
  (*obtained from* $T = f\,F\,r_e$) $\boxed{r_e = \frac{r_o + r_i}{2}}$
- Location of actuating force: $\boxed{\bar{r} = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1}\frac{r_o + r_i}{2}}$

Uniform pressure:

- Actuating force: $\boxed{F = \frac{1}{2}(\theta_2 - \theta_1) P_{all}\, (r_o^2 - r_i^2)}$

- Braking torque: $\boxed{T = \frac{1}{3}(\theta_2 - \theta_1) f\, P_{all}\, ({r_o}^3 - {r_i}^3)}$ *single friction surface*
- Torque equivalent radius: $\boxed{r_e = \frac{2}{3}\frac{{r_o}^3 - {r_i}^3}{{r_o}^2 - {r_i}^2}}$
- Location of "***F***" : $\boxed{\bar{r} = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1}\frac{2}{3}\frac{{r_o}^3 - {r_i}^3}{{r_o}^2 - {r_i}^2}}$

❖ Note that the uniform wear or uniform pressure conditions in this case are controlled by controlling the location of the actuating force (i.e., by changing).

Example 16–3

Two annular pads, $r_i = 3.875$ in, $r_o = 5.50$ in, subtend an angle of 108°, have a coefficient of friction of 0.37, and are actuated by a pair of hydraulic cylinders 1.5 in in diameter. The torque requirement is 13 000 lbf · in. For uniform wear

(*a*) Find the largest normal pressure p_a.

(*b*) Estimate the actuating force F.

(*c*) Find the equivalent radius r_e and force location $\bar{r}$.

(*d*) Estimate the required hydraulic pressure.

Solution

(*a*) From Eq. (16–34), with $T = 13\,000/2 = 6500$ lbf · in for each pad,

$$p_a = \frac{2T}{(\theta_2 - \theta_1) f r_i \left(r_o^2 - r_i^2\right)}$$

$$= \frac{2(6500)}{(144° - 36°)(\pi/180)0.37(3.875)(5.5^2 - 3.875^2)} = 315.8 \text{ psi}$$

(*b*) From Eq. (16–33),

$$F = (\theta_2 - \theta_1) p_a r_i (r_o - r_i) = (144^\circ - 36^\circ)(\pi/180)315.8(3.875)(5.5 - 3.875)$$
$$= 3748 \text{ lbf}$$

(*c*) From Eq. (16–35),

$$r_e = \frac{r_o + r_i}{2} = \frac{5.50 + 3.875}{2} = 4.688 \text{ in}$$

From Eq. (16–36),

$$\bar{r} = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1}\frac{r_o + r_i}{2} = \frac{\cos 36^\circ - \cos 144^\circ}{(144^\circ - 36^\circ)(\pi/180)}\frac{5.50 + 3.875}{2}$$
$$= 4.024 \text{ in}$$

(*d*) Each cylinder supplies the actuating force, 3748 lbf.

$$p_{\text{hydraulic}} = \frac{F}{A_P} = \frac{3748}{\pi(1.5^2/4)} = 2121 \text{ psi}$$