Internal Expanding Rim Clutches and Brakes

An internal-shoe rim clutch/brake consists mainly of three elements:

- The mating frictional surfaces.
- The means of transmitting the torque to and from the surfaces.
- The actuating mechanisms.

According to the actuating mechanism, clutches/brakes are further classified as:

- Expanding ring.
- Centrifugal.
- Magnetic.
- Hydraulic & Pneumatic.

An Internal Expanding Centrifugal-acting Rim Clutch

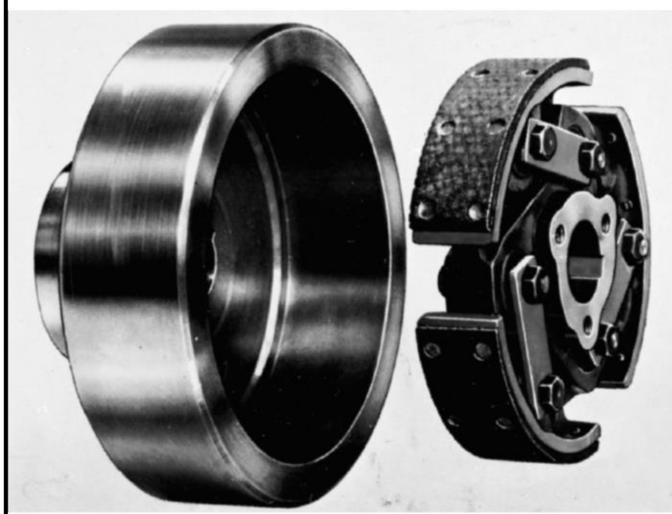


Fig. 16–3 Shigley's Mechanical Engin

Consider the shown internal expanding rim brake with a single shoe and the rim rotating clockwise.

- For such configuration we cannot make the assumption that the pressure distribution is uniform. But rather the pressure distribution has the following characteristics:
  - Pressure distribution is sinusoidal with respect to angle θ.
  - For short shoe, the largest pressure occurs at the end of the shoe θ_2.
  - For long shoe, the largest pressure occurs at $\theta_a = 90°$

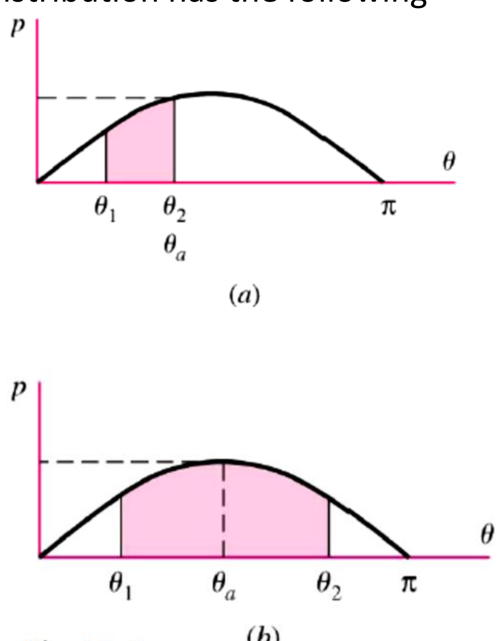


Fig. 16–6

- Using static analysis, the characteristic equation for such brake configuration can be found to be *(see derivation in the text)*:

- The actuating force:

For clockwise rotation (self-energizing)

$$\boxed{F = \frac{M_N - M_f}{c}}$$

✓ To avoid self locking we should have

$$M_N > M_f$$

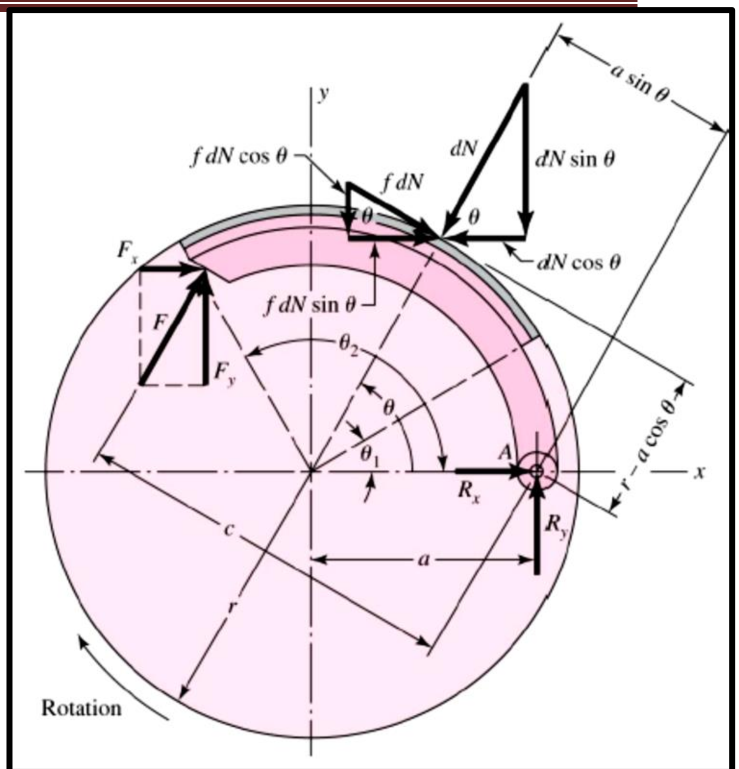
or

For counterclockwise rotation (self-deenergizing) $\boxed{F = \frac{M_N + M_f}{c}}$

Where:

C: is the moment arm for the actuating force.

M_N: is the moment of normal forces.



$$\boxed{M_N = \frac{P_{all}\, b\, r\, a}{\sin\theta_a}\int_{\theta_1}^{\theta_2}(\sin\theta)^2\, d\theta = \frac{P_{all}\, b\, r\, a}{\sin\theta_a}\left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right|_{\theta_1}^{\theta_2}}$$

M_f: is the moment of the frictional force.

$$\boxed{M_f = \frac{f\, P_{all}\, b\, r}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin\theta\,(r - a\cos\theta)\, d\theta = \frac{f\, P_{all}\, b\, r}{\sin\theta_a}\left[-r\cos\theta - \frac{a}{2}\sin^2\theta\right|_{\theta_1}^{\theta_2}}$$

where b: is the face width.

θ_a is the angle defining the location of max pressure "P_{all}".

- The torque applied to the drum by the brake shoe is:

$$\boxed{T = \frac{f\, P_{all}\, b\, r^2\,(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}}$$

- The reaction forces are found as:

$$\begin{cases} R_x = \dfrac{P_{all}\, b\, r}{\sin\theta_a}(A - fB) - F_x \\[2ex] R_y = \dfrac{P_{all}\, b\, r}{\sin\theta_a}(B + fA) - F_y \end{cases}$$

Clockwise rotation (self-energizing)

$$\begin{cases} R_x = \dfrac{P_{all}\, b\, r}{\sin\theta_a}(A + fB) - F_x \\ R_y = \dfrac{P_{all}\, b\, r}{\sin\theta_a}(B - fA) - F_y \end{cases}$$

Counterclockwise rotation (self-deenergizing)

Where:

$$\begin{cases} A = \left(\frac{1}{2}(\sin\theta)^2 \Big|_{\theta_2}^{\theta_1}\right. \\ B = \left(\frac{\theta}{2} - \frac{1}{4}(\sin 2\theta)^2 \Big|_{\theta_2}^{\theta_1}\right. \end{cases}$$

- It is important to note that these reactions equations can be used only when: the origin of the axis is at the center of the drum, the positive *x* axis passes through the hinge pin, and the positive *y* axis is in the direction of the shoe.

Example 16–2

The brake shown in Fig. 16–8 is 300 mm in diameter and is actuated by a mechanism that exerts the same force F on each shoe. The shoes are identical and have a face width of 32 mm. The lining is a molded asbestos having a coefficient of friction of 0.32 and a pressure limitation of 1000 kPa. Estimate the maximum

(*a*) Actuating force F.
(*b*) Braking capacity.
(*c*) Hinge-pin reactions.

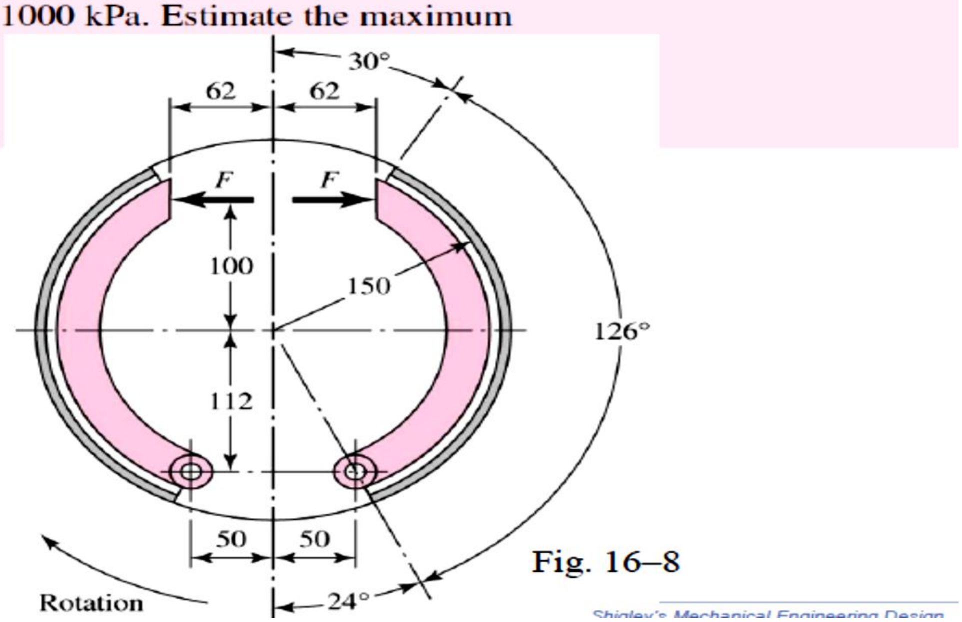


Fig. 16–8

- It should be noted that in *Example 16-2*, the braking capacity of the right-hand shoe is much larger than that of the left-hand shoe (*because the right-hand shoe is self-energizing*).
- If the left hand shoe is turned over such that the hinge is at top, it will become self-energizing as well and the braking capacity will increase. However, if the rim is to turn in the opposite direction, both shoes will be self-deenergizing and the

braking capacity will be small. If the same braking capacity is to be obtained, a larger actuating force needs to be applied.

(*a*) The right-hand shoe is self-energizing, and so the force F is found on the basis that the maximum pressure will occur on this shoe. Here $\theta_1 = 0°$, $\theta_2 = 126°$, $\theta_a = 90°$, and $\sin\theta_a = 1$. Also,

$$a = \sqrt{(112)^2 + (50)^2} = 122.7 \text{ mm}$$

Integrating Eq. (16–2) from 0 to θ_2 yields

$$M_f = \frac{f p_a b r}{\sin\theta_a}\left[\left(-r\cos\theta\right)_0^{\theta_2} - a\left(\frac{1}{2}\sin^2\theta\right)_0^{\theta_2}\right]$$

$$= \frac{f p_a b r}{\sin\theta_a}\left(r - r\cos\theta_2 - \frac{a}{2}\sin^2\theta_2\right)$$

Changing all lengths to meters, we have

$$M_f = (0.32)[1000(10)^3](0.032)(0.150)$$

$$\times\left[0.150 - 0.150\cos 126° - \left(\frac{0.1227}{2}\right)\sin^2 126°\right]$$

$$= 304 \text{ N}\cdot\text{m}$$

The moment of the normal forces is obtained from Eq. (16–3). Integrating from 0 to θ_2 gives

$$M_N = \frac{p_a b r a}{\sin\theta_a}\left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)_0^{\theta_2}$$

$$= \frac{p_a b r a}{\sin\theta_a}\left(\frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta_2\right)$$

$$= [1000(10)^3](0.032)(0.150)(0.1227)\left\{\frac{\pi}{2}\frac{126}{180} - \frac{1}{4}\sin[(2)(126°)]\right\}$$

$$= 788 \text{ N}\cdot\text{m}$$

From Eq. (16–4), the actuating force is

$$F = \frac{M_N - M_f}{c} = \frac{788 - 304}{100 + 112} = 2.28 \text{ kN}$$

(b) From Eq. (16–6), the torque applied by the right-hand shoe is

$$T_R = \frac{f p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

$$= \frac{0.32[1000(10)^3](0.032)(0.150)^2(\cos 0^\circ - \cos 126^\circ)}{\sin 90^\circ} = 366\ \text{N}\cdot\text{m}$$

The torque contributed by the left-hand shoe cannot be obtained until we learn its maximum operating pressure. Equations (16–2) and (16–3) indicate that the frictional and normal moments are proportional to this pressure. Thus, for the left-hand shoe,

$$M_N = \frac{788 p_a}{1000} \qquad M_f = \frac{304 p_a}{1000}$$

Then, from Eq. (16–7),

$$F = \frac{M_N + M_f}{c}$$

or

$$2.28 = \frac{(788/1000)p_a + (304/1000)p_a}{100 + 112}$$

Solving gives $p_a = 443$ kPa. Then, from Eq. (16–6), the torque on the left-hand shoe is

$$T_L = \frac{f p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

Since $\sin\theta_a = \sin 90^\circ = 1$, we have

$$T_L = 0.32[443(10)^3](0.032)(0.150)^2(\cos 0^\circ - \cos 126^\circ) = 162\ \text{N}\cdot\text{m}$$

The braking capacity is the total torque:

$$T = T_R + T_L = 366 + 162 = 528\ \text{N}\cdot\text{m}$$

(c) In order to find the hinge-pin reactions, we note that $\sin\theta_a = 1$ and $\theta_1 = 0$. Then Eq. (16–8) gives

$$A = \frac{1}{2}\sin^2\theta_2 = \frac{1}{2}\sin^2 126^\circ = 0.3273$$

$$B = \frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta_2 = \frac{\pi(126)}{2(180)} - \frac{1}{4}\sin[(2)(126^\circ)] = 1.3373$$

Also, let

$$D = \frac{p_a br}{\sin\theta_a} = \frac{1000(0.032)(0.150)}{1} = 4.8 \text{ kN}$$

where $p_a = 1000$ kPa for the right-hand shoe. Then, using Eq. (16–9), we have

$$R_x = D(A - fB) - F_x = 4.8[0.3273 - 0.32(1.3373)] - 2.28 \sin 24^\circ$$
$$= -1.410 \text{ kN}$$
$$R_y = D(B + fA) - F_y = 4.8[1.3373 + 0.32(0.3273)] - 2.28 \cos 24^\circ$$
$$= 4.839 \text{ kN}$$

The resultant on this hinge pin is

$$R = \sqrt{(-1.410)^2 + (4.839)^2} = 5.04 \text{ kN}$$

The reactions at the hinge pin of the left-hand shoe are found using Eqs. (16–10) for a pressure of 443 kPa. They are found to be $R_x = 0.678$ kN and $R_y = 0.538$ kN. The resultant is

$$R = \sqrt{(0.678)^2 + (0.538)^2} = 0.866 \text{ kN}$$

The reactions for both hinge pins, together with their directions, are shown in Fig. 16–9.

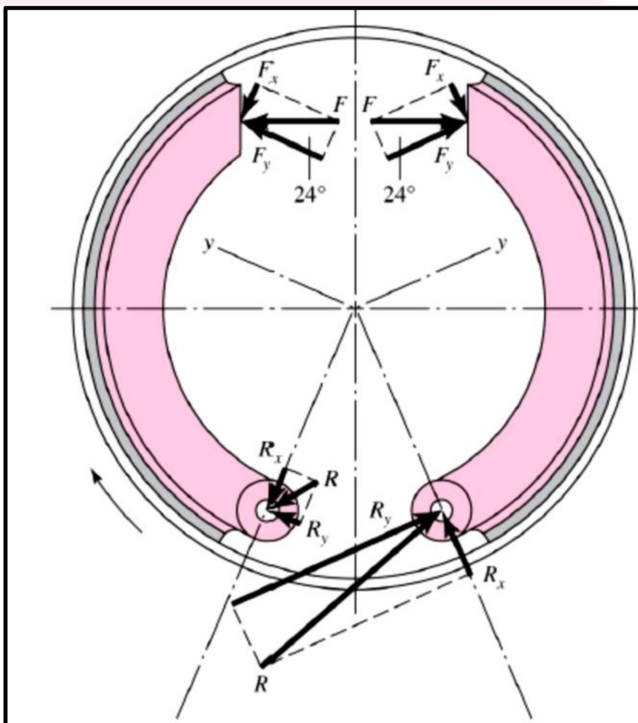


Fig. 16–9