Design of Clutches, Brakes, Couplings and Flywheels

These types of elements are associated with rotation and they have in common the function of dissipating, transferring and/or storing rotational energy.

Clutches and Brakes perform the same function where two elements having different velocities are forced to have the same velocity (*zero in the case of brake*) by applying an actuating force.

When analyzing the performance of clutches/brakes, we will look at:

- Actuating force.
- Transmitted torque.
- Energy loss.
- Temperature rise.

❖ The transmitted torque is related to the actuating force, the coefficient of friction and geometry of the brake.

❖ Temperature rise is related to energy loss and geometry of heat-dissipation surfaces.

The basic types of clutches/brakes are:

- Rim types with internal or external shoes (*Fig. 16-3 & 16-10*).
- Band types (*Fig. 16-13*).
- Disk or axial types (*Fig. 16-14*).
- Cone types (*Fig. 16-21*).

Static analysis of Clutches and Brakes

The following general procedure can be used for analyzing many types of clutches or brakes:

1- Estimate the pressure distribution on friction surfaces.

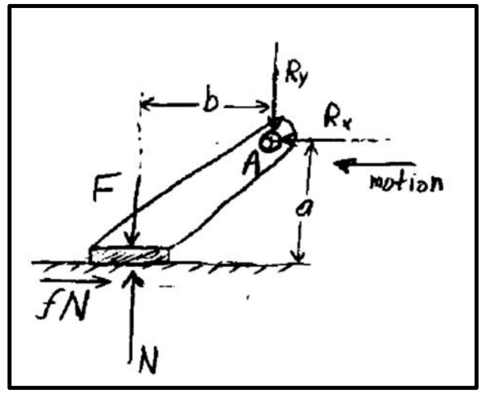
2- Find the relation between largest pressure and pressure at any point (*where the largest pressure will be set to be equal to the maximum allowable pressure for the frictional material*).

3- Use static analysis to find braking force or torque and reaction forces.

• Take for example the door-stop shown.

Applying the analysis procedure:

1- For simplicity since the shoe is short, we



assume the pressure to be uniformly distributed over the frictional area.

2- Since pressure is uniform, the pressure at any point is:

$$\boxed{P = P_{all}}$$

Where (P_{all}) is the max allowable pressure for a Given shoe material

3- Static analysis,

For uniform pressure, the normal force is:

$$\boxed{N = P_{all}\, A}$$

where (A) is the area of the shoe

$$\sum M_A = 0 \quad \rightarrow \quad Fb - Nb \pm fNa = 0$$

if the motion in the opposite direction

Substituting $N = P_{all} A$
and solving for the actuating force F we get:

$$F = \frac{P_{all}\, A\,(b \pm fa)}{b}$$

Solving for the reactions:

$$\sum F_x = 0 \quad \rightarrow \quad R_x = f P_{all} A$$
$$\sum F_y = 0 \quad \rightarrow \quad R_y = P_{all} A - F$$

- The actuating force "F" (*the maximum value of force that can be applied without exceeding* (P_{all}) *of the shoe material*) can be found from the equation above.
- The value of the term $(b \pm fa)$ needs to be larger than zero.
  - If the term zero $\rightarrow F=\mathbf{0}$ and the brake is called "self locking" (*i.e., no actuating force is needed*).

- It is not desired to have a negative value for the actuating force since it means that a force needs to be applied to prevent the maximum pressure from exceeding the material allowable pressure.

❖ The brake with the direction of motion shown is called “self energizing” (*i.e., the frictional force helps in reducing the needed actuating force* $(b \pm fa)$).

❖ If the motion is in the opposite direction, the brake is called “self deenergizing” (*i.e., the direction of the frictional force makes the needed actuating force to be larger* $(b \pm fa)$)

❖ In such example we made a good use of the max allowable pressure of the frictional material because we assumed the pressure to be uniform, thus we can reach the max allowable pressure at all points of contact. However, in reality the pressure distribution is not uniform.

❖ When designing a brake/clutch system, the designer has a required value of torque and makes the choice of the friction material to be used, the area of the friction surface, the geometry, and then finds the needed actuating force. Thus an iterative approach will be needed.

❖ Also the designer needs to ensure that the system is not “self locking” if that is not required.

Example 16–1

The doorstop depicted in Fig. 16–2*a* has the following dimensions: $a = 4$ in, $b = 2$ in, $c = 1.6$ in, $w_1 = 1$ in, $w_2 = 0.75$ in, where w_2 is the depth of the pad into the plane of the paper.

(*a*) For a leftward relative movement of the floor, an actuating force F of 10 lbf, a coefficient of friction of 0.4, use a uniform pressure distribution p_{av}, find R_x, R_y, p_{av}, and the largest pressure p_a.

(*b*) Repeat part *a* for rightward relative movement of the floor.

(*c*) Model the normal pressure to be the “crush” of the pad, much as if it were composed of many small helical coil springs. Find R_x, R_y, p_{av}, and p_a for leftward relative movement of the floor and other conditions as in part *a*.

(*d*) For rightward relative movement of the floor, is the doorstop a self-acting brake?

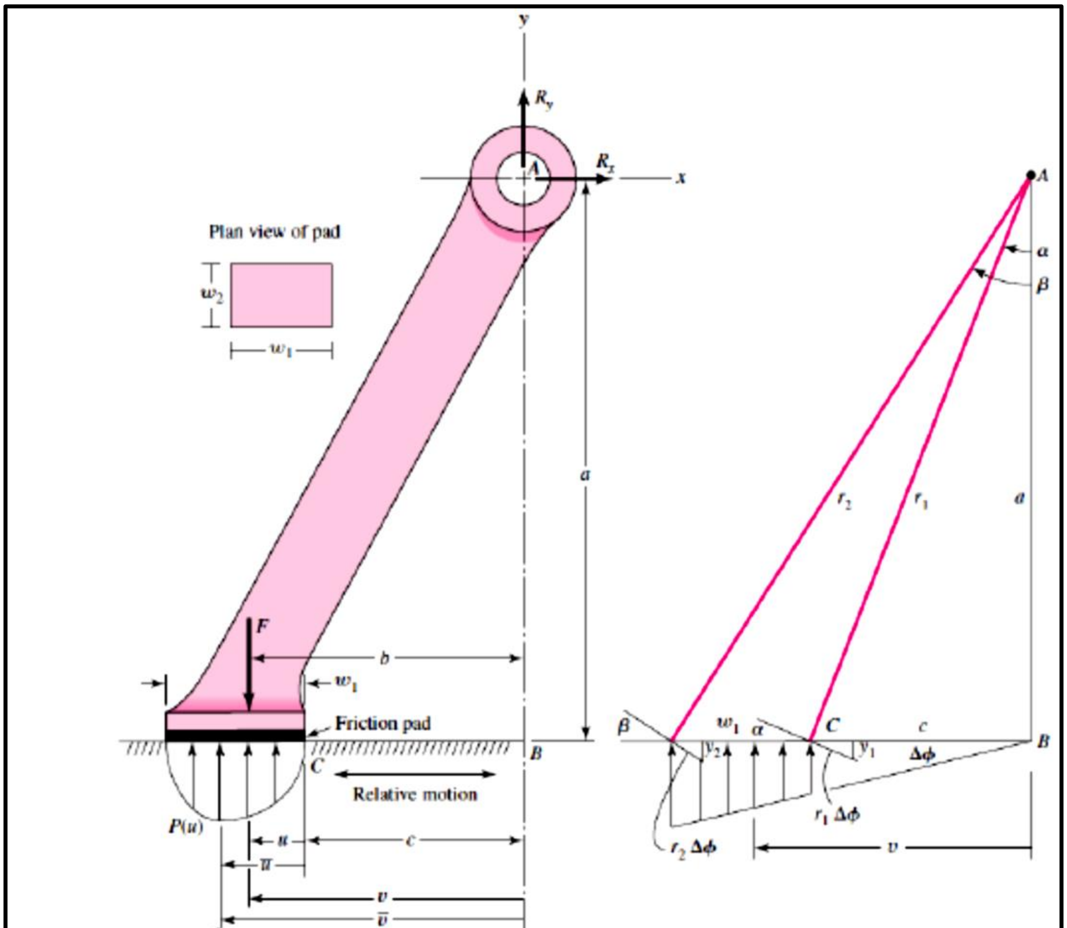


Fig. 16–2

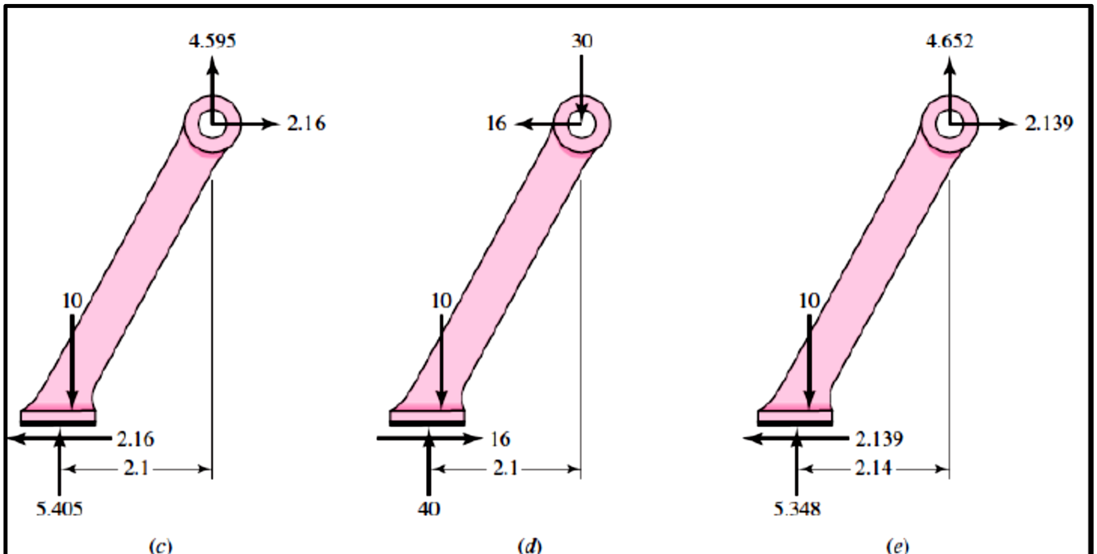


Fig. 16–2

(*a*)

Eq. (*c*): $R_x = f p_{av} w_1 w_2 = 0.4(1)(0.75) p_{av} = 0.3 p_{av}$

Eq. (*d*): $R_y = F - p_{av} w_1 w_2 = 10 - p_{av}(1)(0.75) = 10 - 0.75 p_{av}$

Eq. (*e*):
$$F = \frac{w_2}{b}\left[\int_0^1 p_{av}(c+u)\,du + af\int_0^1 p_{av}\,du\right]$$
$$= \frac{w_2}{b}\left(p_{av}c\int_0^1 du + p_{av}\int_0^1 u\,du + afp_{av}\int_0^1 du\right)$$
$$= \frac{w_2 p_{av}}{b}(c + 0.5 + af) = \frac{0.75}{2}[1.6 + 0.5 + 4(0.4)]p_{av}$$
$$= 1.3875 p_{av}$$

Solving for p_{av} gives

$$p_{av} = \frac{F}{1.3875} = \frac{10}{1.3875} = 7.207 \text{ psi}$$

We evaluate R_x and R_y as

$$R_x = 0.3(7.207) = 2.162 \text{ lbf}$$

$$R_y = 10 - 0.75(7.207) = 4.595 \text{ lbf}$$

The normal force N on the pad is $F - R_y = 10 - 4.595 = 5.405$ lbf, upward. The line of action is through the center of pressure, which is at the center of the pad. The friction force is $fN = 0.4(5.405) = 2.162$ lbf directed to the left. A check of the moments about A gives

$$\sum M_A = Fb - fNa - N(w_1/2 + c)$$
$$= 10(2) - 0.4(5.405)4 - 5.405(1/2 + 1.6) \doteq 0$$

The maximum pressure $p_a = p_{av} = 7.207$ psi.

(*b*)

Eq. (*c*): $R_x = -f p_{av} w_1 w_2 = -0.4(1)(0.75)p_{av} = -0.3p_{av}$

Eq. (*d*): $R_y = F - p_{av} w_1 w_2 = 10 - p_{av}(1)(0.75) = 10 - 0.75p_{av}$

Eq. (*e*):
$$F = \frac{w_2}{b}\left[\int_0^1 p_{av}(c+u)\,du + af\int_0^1 p_{av}\,du\right]$$
$$= \frac{w_2}{b}\left(p_{av}c\int_0^1 du + p_{av}\int_0^1 u\,du + afp_{av}\int_0^1 du\right)$$
$$= \frac{0.75}{2}p_{av}[1.6 + 0.5 - 4(0.4)] = 0.1875p_{av}$$

from which

$$p_{av} = \frac{F}{0.1875} = \frac{10}{0.1875} = 53.33 \text{ psi}$$

which makes

$$R_x = -0.3(53.33) = -16 \text{ lbf}$$

$$R_y = 10 - 0.75(53.33) = -30 \text{ lbf}$$

The normal force N on the pad is $10 + 30 = 40$ lbf upward. The friction shearing force is $fN = 0.4(40) = 16$ lbf to the right. We now check the moments about A:

$$M_A = fNa + Fb - N(c + 0.5) = 16(4) + 10(2) - 40(1.6 + 0.5) = 0$$

Note the change in average pressure from 7.207 psi in part *a* to 53.3 psi. Also note how directions of forces have changed. The maximum pressure p_a is the same as p_{av}, which has changed from 7.207 psi to 53.3 psi.

(*c*) We will model the deformation of the pad as follows. If the doorstop rotates $\Delta\phi$ counterclockwise, the right and left edges of the pad will deform down y_1 and y_2, respectively (Fig. 16–2*b*). From similar triangles, $y_1/(r_1\,\Delta\phi) = c/r_1$ and $y_2/(r_2\,\Delta\phi) = (c + w_1)/r_2$. Thus, $y_1 = c\,\Delta\phi$ and $y_2 = (c + w_1)\,\Delta\phi$. This means that y is directly proportional to the horizontal distance from the pivot point A; that is, $y = C_1 v$, where C_1 is a constant (see Fig. 16–2*b*). Assuming the pressure is directly proportional to deformation, then $p(v) = C_2 v$, where C_2 is a constant. In terms of u, the pressure is $p(u) = C_2(c + u) = C_2(1.6 + u)$.

Eq. (e):

$$F = \frac{w_2}{b}\left[\int_0^{w_1} p(u)c\,du + \int_0^{w_1} p(u)u\,du + af\int_0^{w_1} p(u)\,du\right]$$

$$= \frac{0.75}{2}\left[\int_0^{1} C_2(1.6+u)1.6\,du + \int_0^{1} C_2(1.6+u)\,u\,du + af\int_0^{1} C_2(1.6+u)du\right]$$

$$= 0.375C_2[(1.6+0.5)1.6 + (0.8+0.3333) + 4(0.4)(1.6+0.5)] = 2.945C_2$$

Since $F = 10$ lbf, then $C_2 = 10/2.945 = 3.396$ psi/in, and $p(u) = 3.396(1.6+u)$. The average pressure is given by

$$p_{av} = \frac{1}{w_1}\int_0^{w_1} p(u)\,du = \frac{1}{1}\int_0^{1} 3.396(1.6+u)\,du = 3.396(1.6+0.5) = 7.132 \text{ psi}$$

The maximum pressure occurs at $u = 1$ in, and is

$$p_a = 3.396(1.6+1) = 8.83 \text{ psi}$$

Equations (c) and (d) of Sec. 16–1 are still valid. Thus,

$$R_x = 0.3p_{av} = 0.3(7.131) = 2.139 \text{ lbf}$$

$$R_y = 10 - 0.75p_{av} = 10 - 0.75(7.131) = 4.652 \text{ lbf}$$

The average pressure is $p_{av} = 7.13$ psi and the maximum pressure is $p_a = 8.83$ psi, which is approximately 24 percent higher than the average pressure. The presumption that the pressure was uniform in part a (because the pad was small, or because the arithmetic would be easier?) underestimated the peak pressure. Modeling the pad as a one-dimensional springset is better, but the pad is really a three-dimensional continuum. A theory of elasticity approach or a finite element modeling may be overkill, given uncertainties inherent in this problem, but it still represents better modeling.

(d) To evaluate $\bar{u}$ we need to evaluate two integrations

$$\int_0^{c} p(u)u\,du = \int_0^{1} 3.396(1.6+u)u\,du = 3.396(0.8+0.3333) = 3.849 \text{ lbf}$$

$$\int_0^{c} p(u)\,du = \int_0^{1} 3.396(1.6+u)\,du = 3.396(1.6+0.5) = 7.132 \text{ lbf/in}$$

Thus $\bar{u} = 3.849/7.132 = 0.5397$ in. Then, from Eq. (f) of Sec. 16–1, the critical coefficient of friction is

$$f_{cr} \geq \frac{c + \bar{u}}{a} = \frac{1.6 + 0.5397}{4} = 0.535$$

The doorstop friction pad does not have a high enough coefficient of friction to make the doorstop a self-acting brake. The configuration must change and/or the pad material specification must be changed to sustain the function of a doorstop.