Flexible Mechanical Elements

Flexible mechanical elements (belts, chains, ropes) are used in conveying systems and to transmit power over long distances (*instead of using shafts and gears*).

- The use of flexible elements simplifies the design and reduces cost.
- Also, since these elements are elastic and usually long, they play a role in absorbing shock loads and reducing vibrations.
- Disadvantage, they have shorter life than gears, shafts, etc.

Belts

There are four basic types of belts (*Table 17-1*):

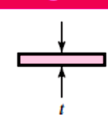
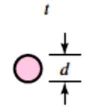
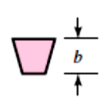
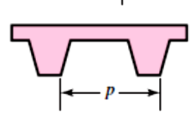
- Flat belts ~ *crowned pulleys*.
- Round belts ~ *grooved pulleys*.
- V-belts ~ *grooved pulleys*.
- Timing belts ~ *toothed pulleys*.

Characteristics of belt drives:

- Pulley axis must be separated by certain minimum distance. *Why?*
- Can be used for long center distances.
- Except for timing belts, there is some slipping between belt and pulley, thus angular velocity ratio is not constant or equal to the ratio of pulley diameters.
- A tension pulley can be used to maintain tension in the belt.

Table 17-1

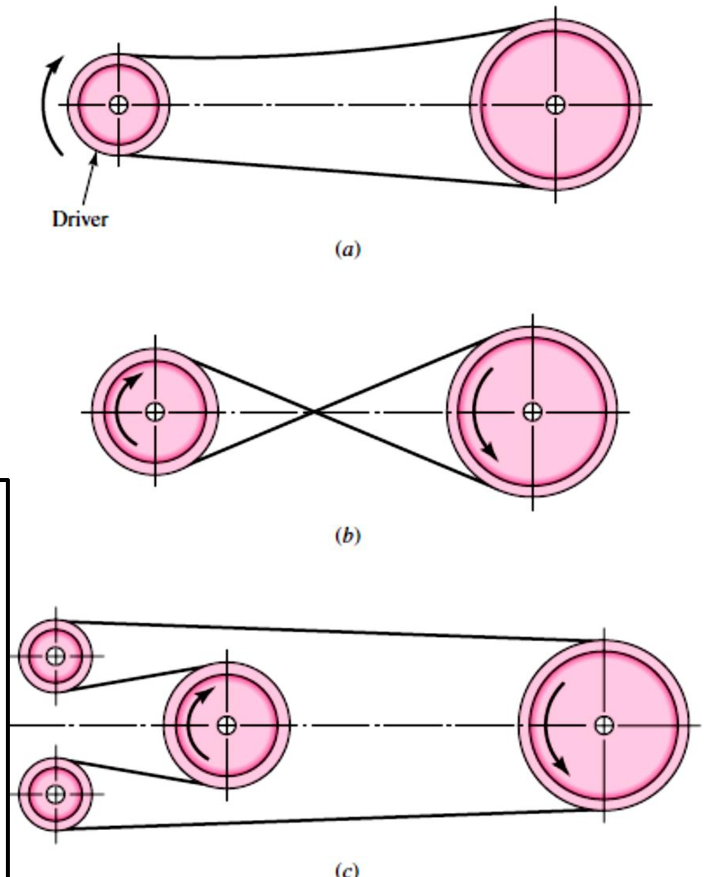
Characteristics of Some Common Belt Types. Figures are Cross Sections except for the Timing Belt, which is a Side View

Belt Type	Figure	Joint	Size Range	Center Distance
Flat		Yes	$t = \begin{cases} 0.03 \text{ to } 0.20 \text{ in} \\ 0.75 \text{ to } 5 \text{ mm} \end{cases}$	No upper limit
Round		Yes	$d = \frac{1}{8}$ to $\frac{3}{4}$ in	No upper limit
V		None	$b = \begin{cases} 0.31 \text{ to } 0.91 \text{ in} \\ 8 \text{ to } 19 \text{ mm} \end{cases}$	Limited
Timing		None	$p = 2$ mm and up	Limited

There are two main configurations for belt drives; open and crossed (*Fig 17-1*) where the direction of rotation will be reversed for the crossed belt drive. The figure shows reversing and non-reversing belt drives, always there is one *loose side* depending on the driver pulley and the direction of rotation.

Figure 17–2

Nonreversing and reversing belt drives. (*a*) Nonreversing open belt. (*b*) Reversing crossed belt. Crossed belts must be separated to prevent rubbing if high-friction materials are used. (*c*) Reversing open-belt drive.



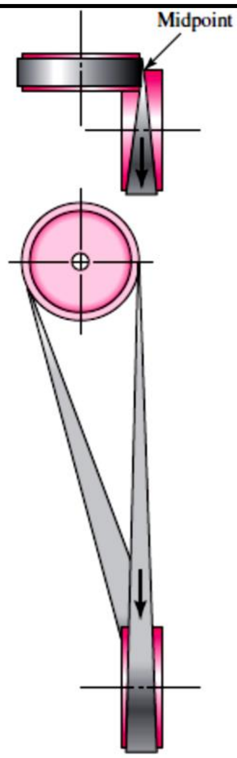


Figure 17–3

Quarter-twist belt drive; an idler guide pulley must be used if motion is to be in both directions.

- *Fig. (17-3)* shows flat belt drive for out of-plane pulleys.
- *Fig. (17-4)* shows how clutching action can be obtained by shifting the belt from loose to a tight pulley.
- *Fig. (17-5)* shows two types of variable-speed belt drives.

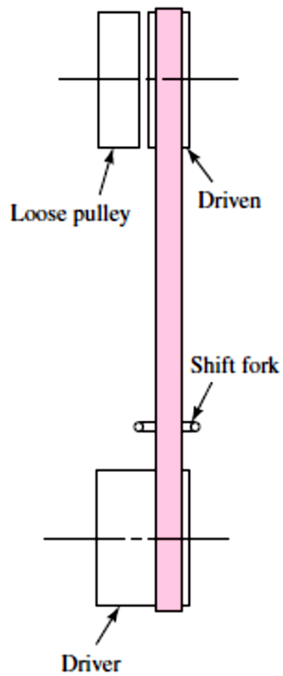


Figure 17–4

This drive eliminates the need for a clutch. Flat belt can be shifted left or right by use of a fork.

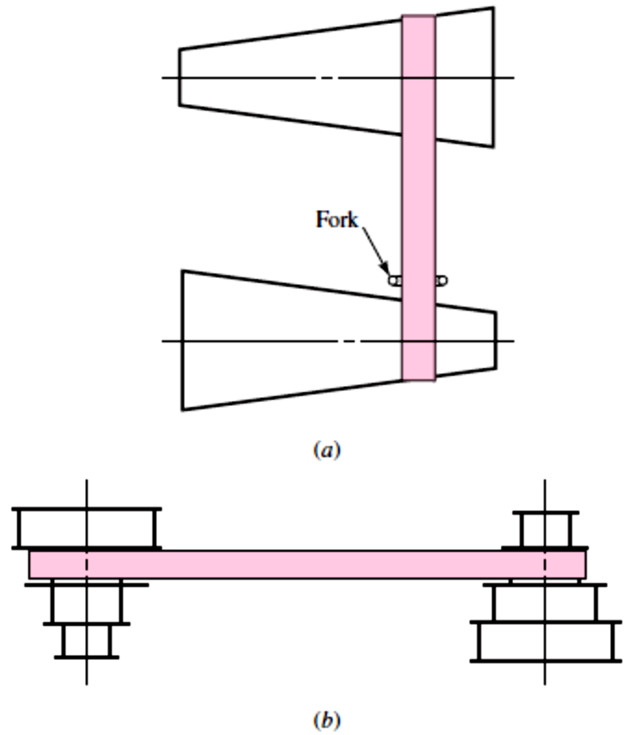


Figure 17–5

Variable-speed belt drives.

Flat and Round Belt Drives

Flat belt drivers produce very little noise and they absorb more vibration from the system than V-belts.

Also, flat belts drives have high efficiency of about *98 %* (*same as for gears*) compared to *70-96 %* for V-belts.

For open belt drives, the contact angles are:

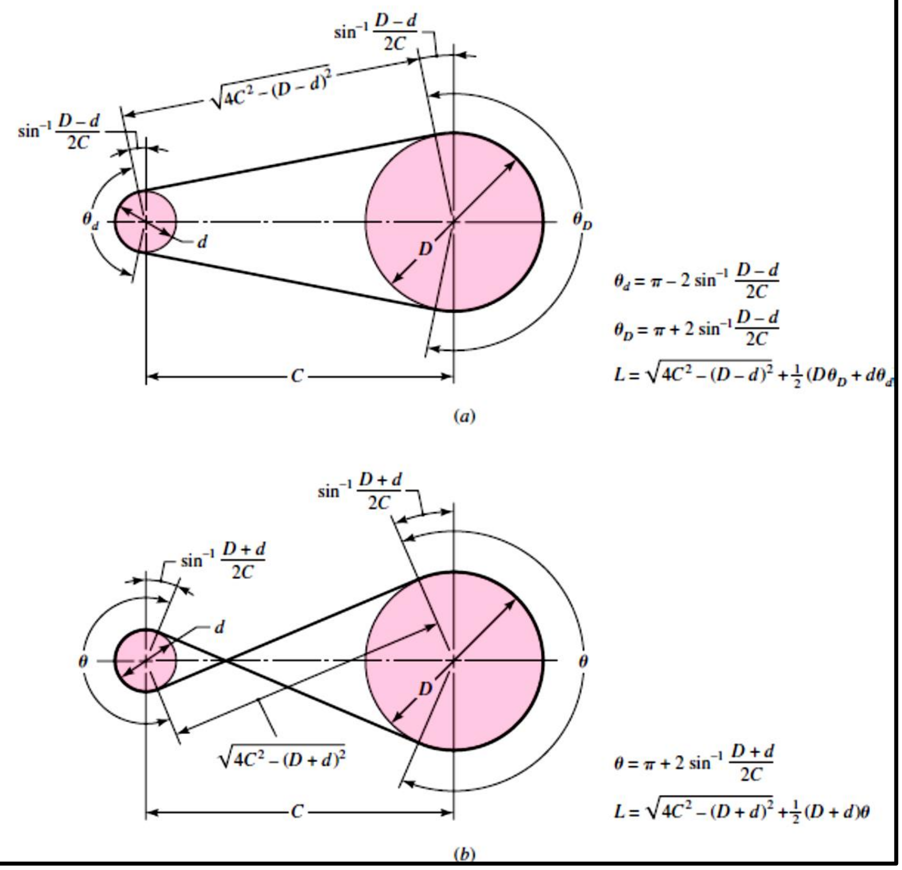
$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$
$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C} \qquad (17\text{–}1)$$

where D = diameter of large pulley
d = diameter of small pulley
C = center distance
θ = angle of contact

Larger contact-angle for the large pulley

Figure 17–1

Flat-belt geometry. (*a*) Open belt. (*b*) Crossed belt.



- And the length of the belt is:

$$L = [4C^2 - (D-d)^2]^{1/2} + \frac{1}{2}(D\theta_D + d\theta_d) \qquad (17\text{–}2)$$

- For crossed belt drives, the contact angle is the same for both pulleys: (Fig.17-2b)

$$\boxed{\theta = \pi + 2\sin^{-1}\frac{D+d}{2C} \qquad (17\text{–}3)}$$

- And the belt length is:

$$\boxed{L = [4C^2 - (D+d)^2]^{1/2} + \frac{1}{2}(D+d)\theta \qquad (17\text{–}4)}$$

• ***Force Analysis***:

Figure 17–7 shows a free body of a pulley and part of the belt. The tight side tension $F1$ and the loose side tension $F2$ have the following additive components:

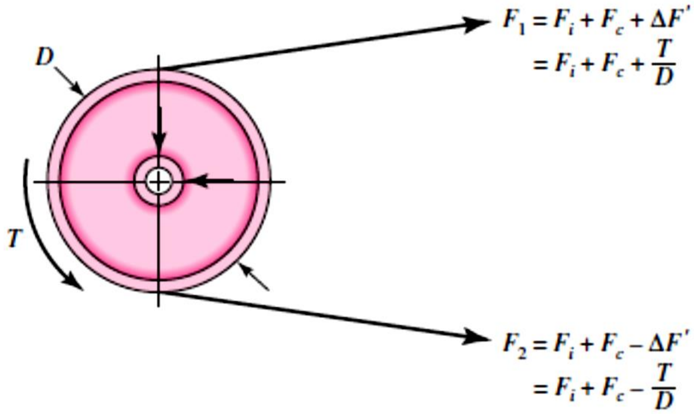


Figure 17–7
Forces and torques on a pulley.

- Tight side tension:

$$\boxed{\begin{aligned} F_1 &= F_i + F_c + \Delta F' \\ &= F_i + F_c + T/D \end{aligned}} \quad \text{(f)}$$

- Loose side tension:

$$\boxed{\begin{aligned} F_2 &= F_i + F_c - \Delta F' \\ &= F_i + F_c - T/D \end{aligned}} \quad \text{(g)}$$

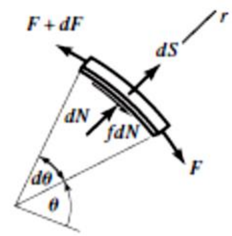


Figure 17–6
Free body of an infinitesimal element of a flat belt in contact with a pulley.

where F_i = initial tension
F_c = hoop tension due to centrifugal force
$\Delta F'$ = tension due to the transmitted torque T
D = diameter of the pulley

The total transmitted force is the difference between F_1 & F_2 is related to the pulley torque

$$F_1 - F_2 = \frac{2T}{D} = \frac{T}{D/2} \quad (h)$$

- The centrifugal tension F_C can be found as:

$$\boxed{F_c = mr^2\omega^2}$$

where ω : is the angular velocity, & m: is the mass per unit length.
It also can be written as:

$$\boxed{F_c = \frac{w}{g}V^2}$$

where $g = 9.81\ m/s^2$, : w is weight per unit length, $V = \pi Dn$

- The initial tension can be expressed as:

$$\boxed{F_i = \frac{F_1 + F_2}{2} - F_c} \qquad (1)$$

- The belting equation relates the possible belt tension values with the coefficient of friction and it is defined as:

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{f\phi}$$

Note that ϕ is the __smallest__ value of the contact angle

where f: coefficient of friction, ϕ: contact angle.

- Substituting in *eqn.(1)* we find the relation between F_i and T

$$\boxed{F_i = \frac{T}{D}\frac{e^{f\phi} + 1}{e^{f\phi} - 1}}$$

Minimum value of F_i needed to transmit a certain value of torque without slipping

➢ This equation shows that if F_i is zero; then T is zero (*i.e. there is no transmitted torque*).

- Substituting in F_1 & F_2 equations we get:

$$\boxed{F_1 = F_c + F_i \frac{2\, e^{f\phi}}{e^{f\phi}+1}}$$

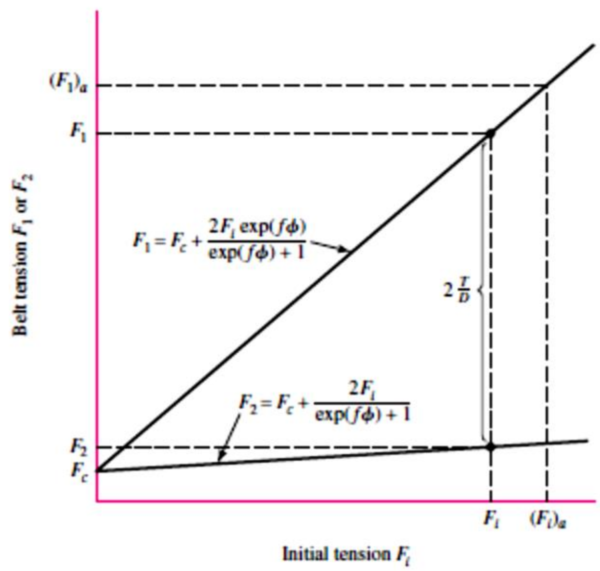
$$\boxed{F_2 = F_c + F_i \frac{2}{e^{f\phi}+1}}$$

Used to find the F_1 & F_2 values when the belt is on the verge of slipping *<u>or</u>* to find F_1 & F_2 for small F_i values where slipping is occurring (*note that the kinetic coefficient of friction should be used in such case*)

➢ Plotting F_1 & F_2 vs. F_i we can see that the initial tension needs to be sufficient so that the difference between F_1 & F_2 curves is $2T/D$.

Figure 17-8

Plot of initial tension F_i against belt tension F_1 or F_2, showing the intercept F_c, the equations of the curves, and where $2T/D$ is to be found.



❖ *Table 17-2* gives the manufacturers specifications for the allowable tension for each type of belts.

• When a belt is selected, the tension in the tight side is set to be equal to the max allowable tension for that belt type.

➢ However, severity of flexing at the pulley, and the belt speed affect the belt life, thus they need to be accounted for.

▪ Therefore the max allowable tension is found as:

$$\boxed{(F_1)_a = bF_aC_PC_V}$$

where:

F_a : allowable tension per unit width for a specific belt material (kN/m) (*Table 17-2*)

b : belt width (m)

C_P : pulley correction factor (*for the severity of flexing*), it is found from (*Table17-4*) for the small pulley diameter.

Use C_P=1 for urethane belts

- The transmitted horsepower can be found as:

$$\boxed{H=(F_1-F_2)V=Tn}$$

Table 17-2

Properties of Some Flat- and Round-Belt Materials. (Diameter = d, thickness = t, width = w)

Material	Specification	Size, in	Minimum Pulley Diameter, in	Allowable Tension per Unit Width at 600 ft/min, lbf/in	Specific Weight, lbf/in³	Coefficient of Friction
Leather	1 ply	$t=\frac{11}{64}$	3	30	0.035–0.045	0.4
		$t=\frac{13}{64}$	$3\frac{1}{2}$	33	0.035–0.045	0.4
	2 ply	$t=\frac{18}{64}$	$4\frac{1}{2}$	41	0.035–0.045	0.4
		$t=\frac{20}{64}$	6^a	50	0.035–0.045	0.4
		$t=\frac{23}{64}$	9^a	60	0.035–0.045	0.4
Polyamide[b]	F–0[c]	$t=0.03$	0.60	10	0.035	0.5
	F–1[c]	$t=0.05$	1.0	35	0.035	0.5
	F–2[c]	$t=0.07$	2.4	60	0.051	0.5
	A–2[c]	$t=0.11$	2.4	60	0.037	0.8
	A–3[c]	$t=0.13$	4.3	100	0.042	0.8
	A–4[c]	$t=0.20$	9.5	175	0.039	0.8
	A–5[c]	$t=0.25$	13.5	275	0.039	0.8
Urethane[d]	$w=0.50$	$t=0.062$	See	5.2^e	0.038–0.045	0.7
	$w=0.75$	$t=0.078$	Table	9.8^e	0.038–0.045	0.7
	$w=1.25$	$t=0.090$	17–3	18.9^e	0.038–0.045	0.7
	Round	$d=\frac{1}{4}$	See	8.3^e	0.038–0.045	0.7
		$d=\frac{3}{8}$	Table	18.6^e	0.038–0.045	0.7
		$d=\frac{1}{2}$	17–3	33.0^e	0.038–0.045	0.7
		$d=\frac{3}{4}$		74.3^e	0.038–0.045	0.7

[a]Add 2 in to pulley size for belts 8 in wide or more.

[b]*Source: Habasit Engineering Manual*, Habasit Belting, Inc., Chamblee (Atlanta), Ga.

[c]Friction cover of acrylonitrile-butadiene rubber on both sides.

[d]*Source:* Eagle Belting Co., Des Plaines, Ill.

[e]At 6% elongation; 12% is maximum allowable value.

A moderate variety of belt materials, with some of their properties, are listed in Table 17–2. These are sufficient for solving a large variety of design and analysis problems. The design equation to be used is Eq. (*j*).

$$H = \frac{(F_1 - F_2)V}{33\,000} \qquad (j)$$

➢ However, when designing a design factor n_d needs to be included to account for unquantifiable effects. Also another correction factor K_S, is included

The values given in Table 17–2 for the allowable belt tension are based on a belt speed of 600 ft/min. For higher speeds, use Fig. 17–9 to obtain C_v values for leather belts. For polyamide and urethane belts, use $C_v = 1.0$.

The service factors K_s for V-belt drives, given in Table 17–15 in Sec. 17–3, are also recommended here for flat- and round-belt drives.

Figure 17–9

Velocity correction factor C_v for leather belts for various thicknesses. *(Data source: Machinery's Handbook, 20th ed., Industrial Press, New York, 1976, p. 1047.)*

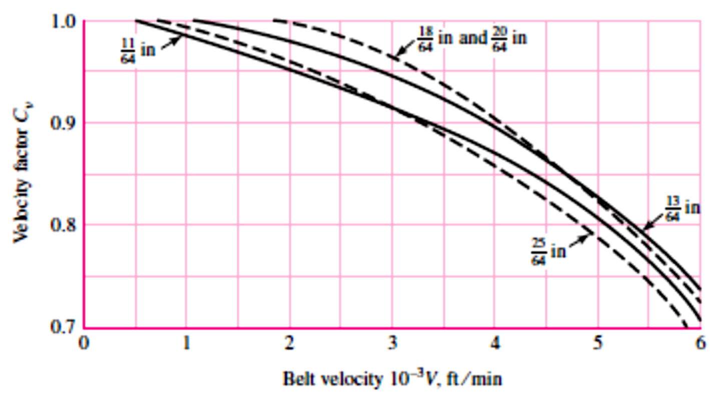


Table 17–4

Pulley Correction Factor C_p for Flat Belts*

	Small-Pulley Diameter, in					
Material	**1.6 to 4**	**4.5 to 8**	**9 to 12.5**	**14, 16**	**18 to 31.5**	**Over 31.5**
Leather	0.5	0.6	0.7	0.8	0.9	1.0
Polyamide, F–0	0.95	1.0	1.0	1.0	1.0	1.0
F–1	0.70	0.92	0.95	1.0	1.0	1.0
F–2	0.73	0.86	0.96	1.0	1.0	1.0
A–2	0.73	0.86	0.96	1.0	1.0	1.0
A–3	—	0.70	0.87	0.94	0.96	1.0
A–4	—	—	0.71	0.80	0.85	0.92
A–5	—	—	—	0.72	0.77	0.91

*Average values of C_p for the given ranges were approximated from curves in the *Habasit Engineering Manual*, Habasit Belting, Inc., Chamblee (Atlanta), Ga.

to account for load deviations from the nominal value (*i.e., over loads*).

- Thus the design horsepower is:

$$H_d = H_{nom} K_S n_d$$

• Steps for analyzing flat belts include:

1. Find ϕ for the smallest pulley from geometry (*find $e^{f\phi}$ if needed*).
2. From belt material and speed find F_c.

$$F_C = \frac{w}{g} V^2$$

3. Find the transmitted torque.

$$T = H_d / n = (H_{nom} K_S n_d)/n$$

4. From torque T, find the transmitted load.

$$(F_1)_a - F_2 = 2T/D$$

5. From belt material, drive geometry & speed, find (F_{1a}) .

$$(F_1)_a = bF_a C_P C_v$$

6. Find F_2

$$F_2 = (F_1)_a - ((F_1)_a - F_2)$$

Note that F_2 must be larger than zero

7. From (F_{1a}), F_2 & F_C find F_i .

$$F_i = \frac{(F_1)_a + F_2}{2} - F_C$$

8. Check if the friction of the belt material is sufficient to transmit the torque.

$$\grave{f} < f$$

where $$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_C}{F_2 - F_C}$$

Minimum friction needed to transmit the load without slipping

9. Find the factor of safety

$$\boxed{n_{fs} = H_a / H_{nom} K_S}$$

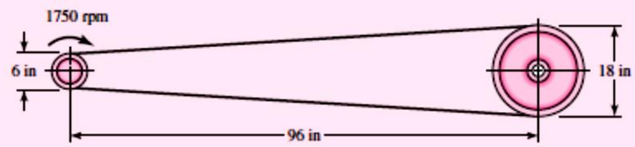
EXAMPLE 17–1 A polyamide A-3 flat belt 6 in wide is used to transmit 15 hp under light shock conditions where $K_s = 1.25$, and a factor of safety equal to or greater than 1.1 is appropriate. The pulley rotational axes are parallel and in the horizontal plane. The shafts are 8 ft apart. The 6-in driving pulley rotates at 1750 rev/min in such a way that the loose side is on top. The driven pulley is 18 in in diameter. See Fig. 17–10. The factor of safety is for unquantifiable exigencies.

(*a*) Estimate the centrifugal tension F_c and the torque T.

(*b*) Estimate the allowable F_1, F_2, F_i and allowable power H_a.

(*c*) Estimate the factor of safety. Is it satisfactory?

Figure 17–10
The flat-belt drive of Ex. 17–1.



Solution (*a*) Eq. (17–1): $\phi = \theta_d = \pi - 2\sin^{-1}\left[\dfrac{18-6}{2(8)12}\right] = 3.0165$ rad

$$\exp(f\phi) = \exp[0.8(3.0165)] = 11.17$$

$$V = \pi(6)1750/12 = 2749 \text{ ft/min}$$

Table 17–2: $w = 12\gamma bt = 12(0.042)6(0.130) = 0.393$ lbf/ft

Answer Eq. (*e*): $F_c = \dfrac{w}{g}\left(\dfrac{V}{60}\right)^2 = \dfrac{0.393}{32.17}\left(\dfrac{2749}{60}\right)^2 = 25.6$ lbf

$$T = \frac{63\,025 H_{nom} K_s n_d}{n} = \frac{63\,025(15)1.25(1.1)}{1750}$$

Answer $= 742.8$ lbf · in

(*b*) The necessary $(F_1)_a - F_2$ to transmit the torque T, from Eq. (*h*), is

$$(F_1)_a - F_2 = \frac{2T}{d} = \frac{2(742.8)}{6} = 247.6 \text{ lbf}$$

From Table 17–2 $F_a = 100$ lbf. For polyamide belts $C_v = 1$, and from Table 17–4 $C_p = 0.70$. From Eq. (17–12) the allowable largest belt tension $(F_1)_a$ is

Answer $(F_1)_a = bF_aC_pC_v = 6(100)0.70(1) = 420$ lbf

then

Answer $F_2 = (F_1)_a - [(F_1)_a - F_2] = 420 - 247.6 = 172.4$ lbf

and from Eq. (*i*)

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{420 + 172.4}{2} - 25.6 = 270.6 \text{ lbf}$$

Answer The combination $(F_1)_a$, F_2, and F_i will transmit the design power of $15(1.25)(1.1) = 20.6$ hp and protect the belt. We check the friction development by solving Eq. (17–7) for f':

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.0165} \ln \frac{420 - 25.6}{172.4 - 25.6} = 0.328$$

From Table 17–2, $f = 0.8$. Since $f' < f$, that is, $0.328 < 0.80$, there is no danger of slipping.

(*c*)

Answer
$$n_{fs} = \frac{H}{H_{\text{nom}} K_s} = \frac{20.6}{15(1.25)} = 1.1 \qquad \text{(as expected)}$$

Answer The belt is satisfactory and the maximum allowable belt tension exists. If the initial tension is maintained, the capacity is the design power of 20.6 hp.

EXAMPLE 17–2 Design a flat-belt drive to connect horizontal shafts on 16-ft centers. The velocity ratio is to be 2.25:1. The angular speed of the small driving pulley is 860 rev/min, and the nominal power transmission is to be 60 hp under very light shock.

Solution
- Function: $H_{\text{nom}} = 60$ hp, 860 rev/min, 2.25:1 ratio, $K_s = 1.15$, $C = 16$ ft
- Design factor: $n_d = 1.05$
- Initial tension maintenance: catenary
- Belt material: polyamide
- Drive geometry, d, D
- Belt thickness: t
- Belt width: b

The last four could be design variables. Let's make a few more a priori decisions.

Decision $d = 16$ in, $D = 2.25d = 2.25(16) = 36$ in.

Decision Use polyamide A-3 belt; therefore $t = 0.13$ in and $C_v = 1$.
Now there is one design decision remaining to be made, the belt width b.

Table 17–2: $\gamma = 0.042$ lbf/in^3 $\quad f = 0.8 \quad F_a = 100$ lbf/in at 600 rev/min

Table 17–4: $C_p = 0.94$

Eq. (17–12): $F_{1a} = b(100)0.94(1) = 94.0b$ lbf **(1)**

$$H_d = H_{\text{nom}} K_s n_d = 60(1.15)1.05 = 72.5 \text{ hp}$$

$$T = \frac{63\,025 H_d}{n} = \frac{63\,025(72.5)}{860} = 5310 \text{ lbf} \cdot \text{in}$$

Estimate $\exp(f\phi)$ for full friction development:

Eq. (17–1): $$\phi = \theta_d = \pi - 2\sin^{-1}\frac{36 - 16}{2(16)12} = 3.037 \text{ rad}$$

$$\exp(f\phi) = \exp[0.80(3.037)] = 11.35$$

Estimate centrifugal tension F_c in terms of belt width b:

$$w = 12\gamma bt = 12(0.042)b(0.13) = 0.0655b \text{ lbf/ft}$$

$$V = \pi dn/12 = \pi(16)860/12 = 3602 \text{ ft/min}$$

Eq. (*e*): $$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.0655b}{32.17}\left(\frac{3602}{60}\right)^2 = 7.34b \text{ lbf} \qquad (2)$$

For design conditions, that is, at H_d power level, using Eq. (*h*) gives

$$(F_1)_a - F_2 = 2T/d = 2(5310)/16 = 664 \text{ lbf} \qquad (3)$$

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 94.0b - 664 \text{ lbf} \qquad (4)$$

Using Eq. (*i*) gives

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{94.0b + 94.0b - 664}{2} - 7.34b = 86.7b - 332 \text{ lbf} \qquad (5)$$

Place friction development at its highest level, using Eq. (17–7):

$$f\phi = \ln\frac{(F_1)_a - F_c}{F_2 - F_c} = \ln\frac{94.0b - 7.34b}{94.0b - 664 - 7.34b} = \ln\frac{86.7b}{86.7b - 664}$$

Solving the preceding equation for belt width b at which friction is fully developed gives

$$b = \frac{664}{86.7}\frac{\exp(f\phi)}{\exp(f\phi) - 1} = \frac{664}{86.7}\frac{11.38}{11.38 - 1} = 8.40 \text{ in}$$

A belt width greater than 8.40 in will develop friction less than $f = 0.80$. The manufacturer's data indicate that the next available larger width is 10-in.

Decision Use 10-in-wide belt.

It follows that for a 10-in-wide belt

Eq. (2): $$F_c = 7.34(10) = 73.4 \text{ lbf}$$

Eq. (1): $$(F_1)_a = 94(10) = 940 \text{ lbf}$$

Eq. (4): $$F_2 = 94(10) - 664 = 276 \text{ lbf}$$

Eq. (5): $$F_i = 86.7(10) - 332 = 535 \text{ lbf}$$

The transmitted power, from Eq. (3), is

$$H_t = \frac{[(F_1)_a - F_2]V}{33\,000} = \frac{664(3602)}{33\,000} = 72.5 \text{ hp}$$

and the level of friction development f', from Eq. (17–7) is

$$f' = \frac{1}{\phi}\ln\frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.037}\ln\frac{940 - 73.4}{276 - 73.4} = 0.479$$

which is less than $f = 0.8$, and thus is satisfactory. Had a 9-in belt width been available, the analysis would show $(F_1)_a = 846$ lbf, $F_2 = 182$ lbf, $F_i = 448$ lbf, and $f' = 0.63$. With a figure of merit available reflecting cost, thicker belts (A-4 or A-5) could be examined to ascertain which of the satisfactory alternatives is best. From Eq. (17–13) the catenary dip is

$$d = \frac{3L^2w}{2F_i} = \frac{3(15^2)0.0655(10)}{2(535)} = 0.413 \text{ in}$$