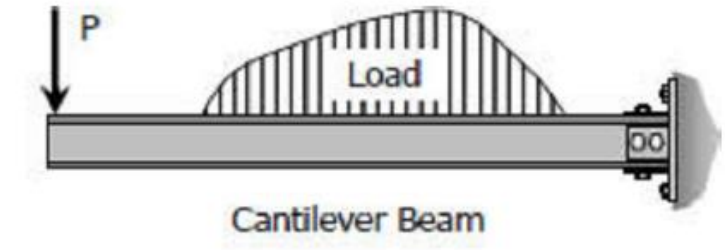
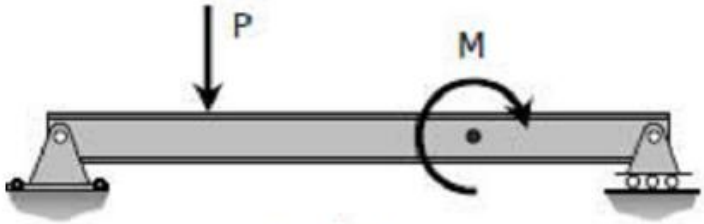
Shear and Moment in Beams:

A beam is a bar subject to forces or couples that lie in a plane containing the longitudinal section of the bar. According to determinacy, a beam may be determinate or indeterminate.

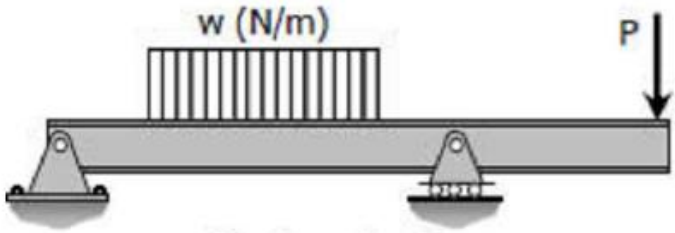
Statically Determinate Beams Statically determinate beams are those beams in which the reactions of the supports may be determined by the use of the equations of static equilibrium. The beams shown below are examples of statically determinate beams.



Cantilever Beam



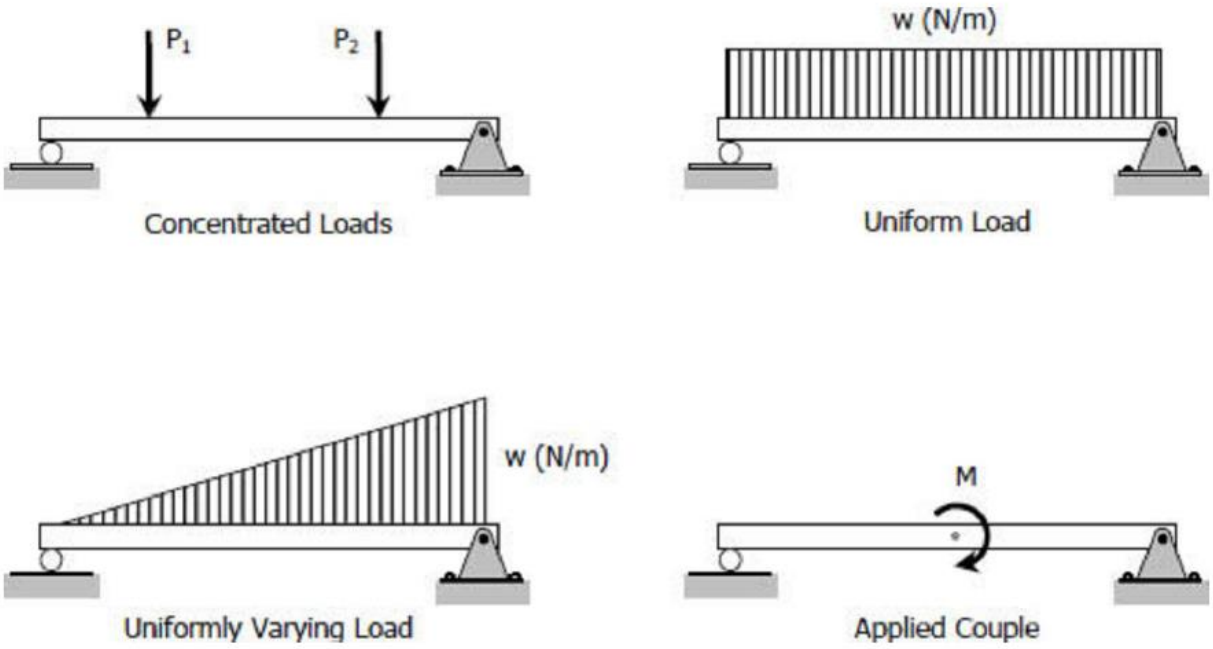
Simple Beam



Overhanging Beam

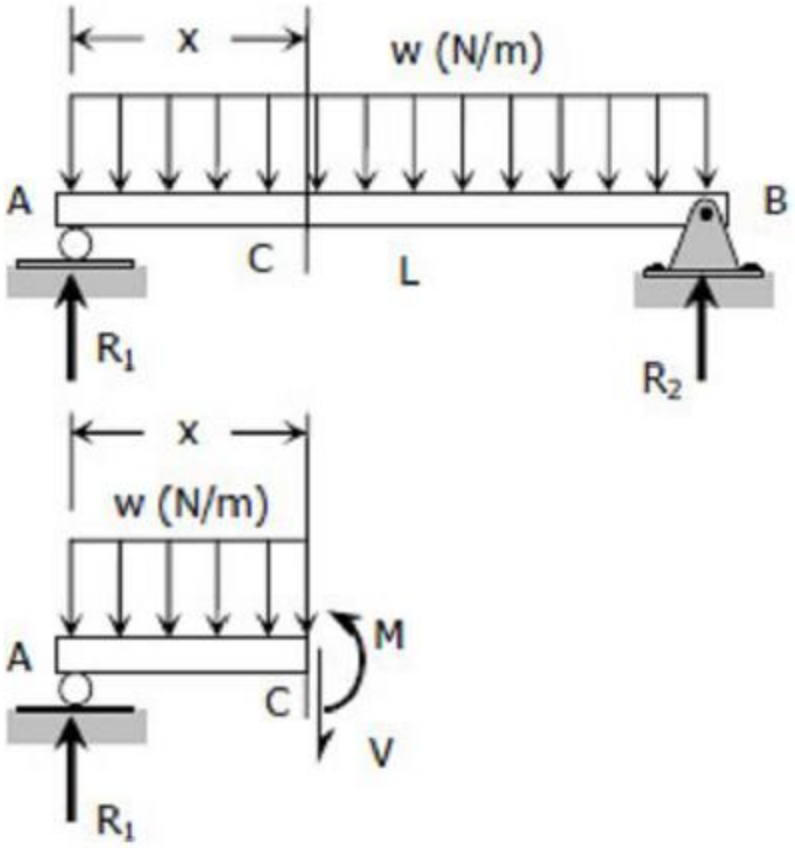
Types of Loading:

Loads applied to the beam may consist of a concentrated load (load applied at a point), uniform load, uniformly varying load, or an applied couple or moment. These loads are shown in the following figures.



Shear and Moment Diagrams:

Consider a simple beam shown of length L that carries a uniform load of w (N/m) throughout its length and is held in equilibrium by reactions R1 and R2. Assume that the beam is cut at point C a distance of x from he left support and the portion of the beam to the right of C be removed. The portion removed must then be replaced by vertical shearing force V together with a couple M to hold the left portion of the bar in equilibrium under the action of R1 and wx.



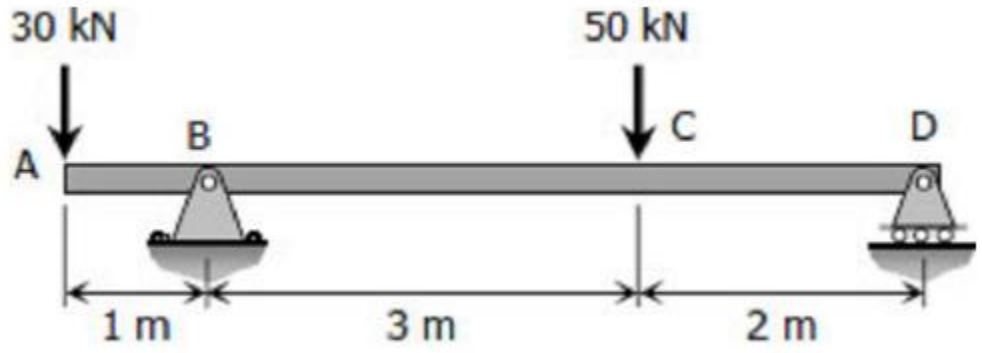
The couple M is called the resisting moment or moment and the force V is called the resisting shear or shear. The sign of V and M are taken to be positive if they have the senses indicated above.

INSTRUCTION:

Write shear and moment equations for the beams in the following problems. In each problem, let x be the distance measured from left end of the beam. Also, draw shear and moment diagrams, specifying values at all change of loading positions and at points of zero shear. Neglect the mass of the beam in each problem.

Example 1:

Draw the shear force and the bending moment diagrams for the beam shown in the figure.

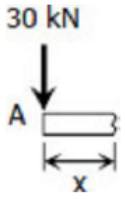


Sol. :

From the load diagram:

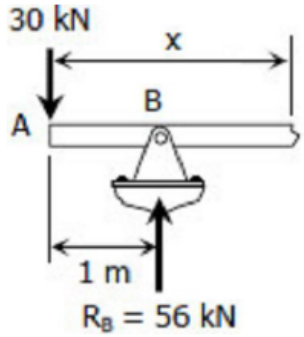
$$\Sigma M_B = 0$$
$$5R_D + 1(30) = 3(50)$$
$$R_D = 24\,\text{kN}$$

$$\Sigma M_D = 0$$
$$5R_B = 2(50) + 6(30)$$
$$R_B = 56\,\text{kN}$$



Segment AB:

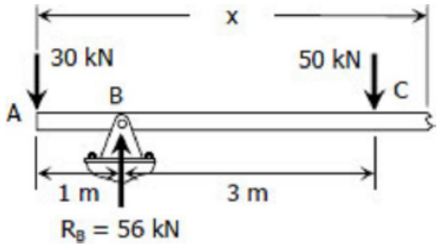
$$V_{AB} = -30\,\text{kN}$$
$$M_{AB} = -30x\,\text{kN}\cdot\text{m}$$



Segment BC:

$$V_{BC} = -30 + 56$$
$$V_{BC} = 26\ \text{kN}$$

$$M_{BC} = -30x + 56(x-1)$$
$$M_{BC} = 26x - 56\ \text{kN} \cdot \text{m}$$

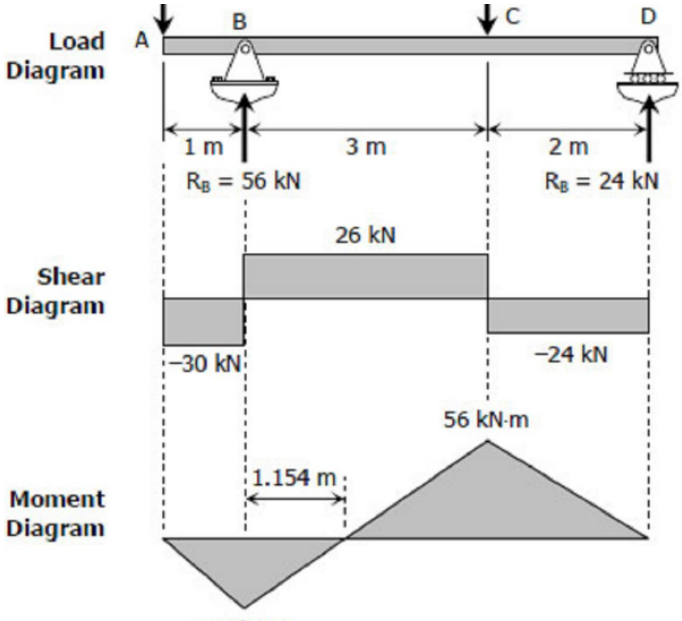


Segment CD:

$$V_{CD} = -30 + 56 - 50$$
$$V_{CD} = -24\ \text{kN}$$
$$M_{CD} = -30x + 56(x-1) - 50(x-4)$$
$$M_{CD} = -30x + 56x - 56 - 50x + 200$$
$$M_{CD} = -24x + 144\ \text{kN} \cdot \text{m}$$

To draw the Shear Diagram:

1. In segment AB, the shear is uniformly distributed over the segment at a magnitude of –30 kN.
2. In segment BC, the shear is uniformly distributed at a magnitude of 26 kN.
3. In segment CD, the shear is uniformly distributed at a magnitude of –24 kN.

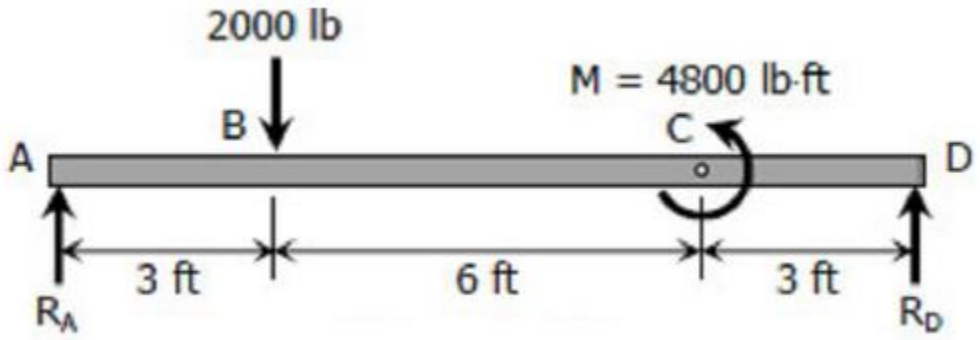


To draw the Moment Diagram:

1. The equation $M_{AB} = -30x$ is linear, at x = 0, $M_{AB} = 0$ and at x = 1 m, $M_{AB} = -30$ kN·m.
2. $M_{BC} = 26x - 56$ is also linear. At x = 1 m, $M_{BC} = -30$ kN·m; at x = 4 m, $M_{BC} = 48$ kN·m. When $M_{BC} = 0$, x = 2.154 m, thus the moment is zero at 1.154 m from B.
3. $M_{CD} = -24x + 144$ is again linear. At x = 4 m, $M_{CD} = 48$ kN·m; at x = 6 m, $M_{CD} = 0$.

Example 2:

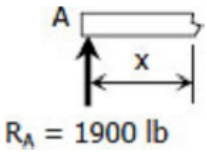
Draw the shear force and the bending moment diagrams for the beam shown in the figure.



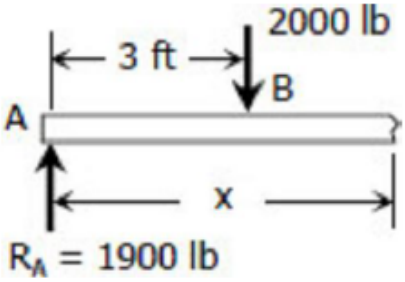
Sol.:

$$\Sigma M_A = 0$$
$$12R_D + 4800 = 3(2000)$$
$$R_D = 100\ \text{lb}$$

$\Sigma M_D = 0$
$12R_A = 9(2000) + 4800$
$R_A = 1900 \text{ lb}$

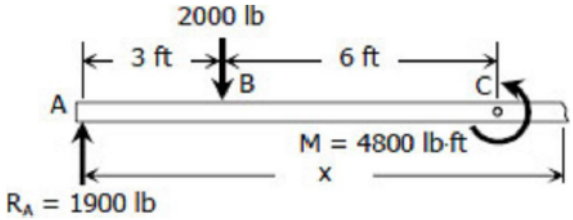


Segment AB:
$V_{AB} = 1900 \text{ lb}$
$M_{AB} = 1900x \text{ lb} \cdot \text{ft}$



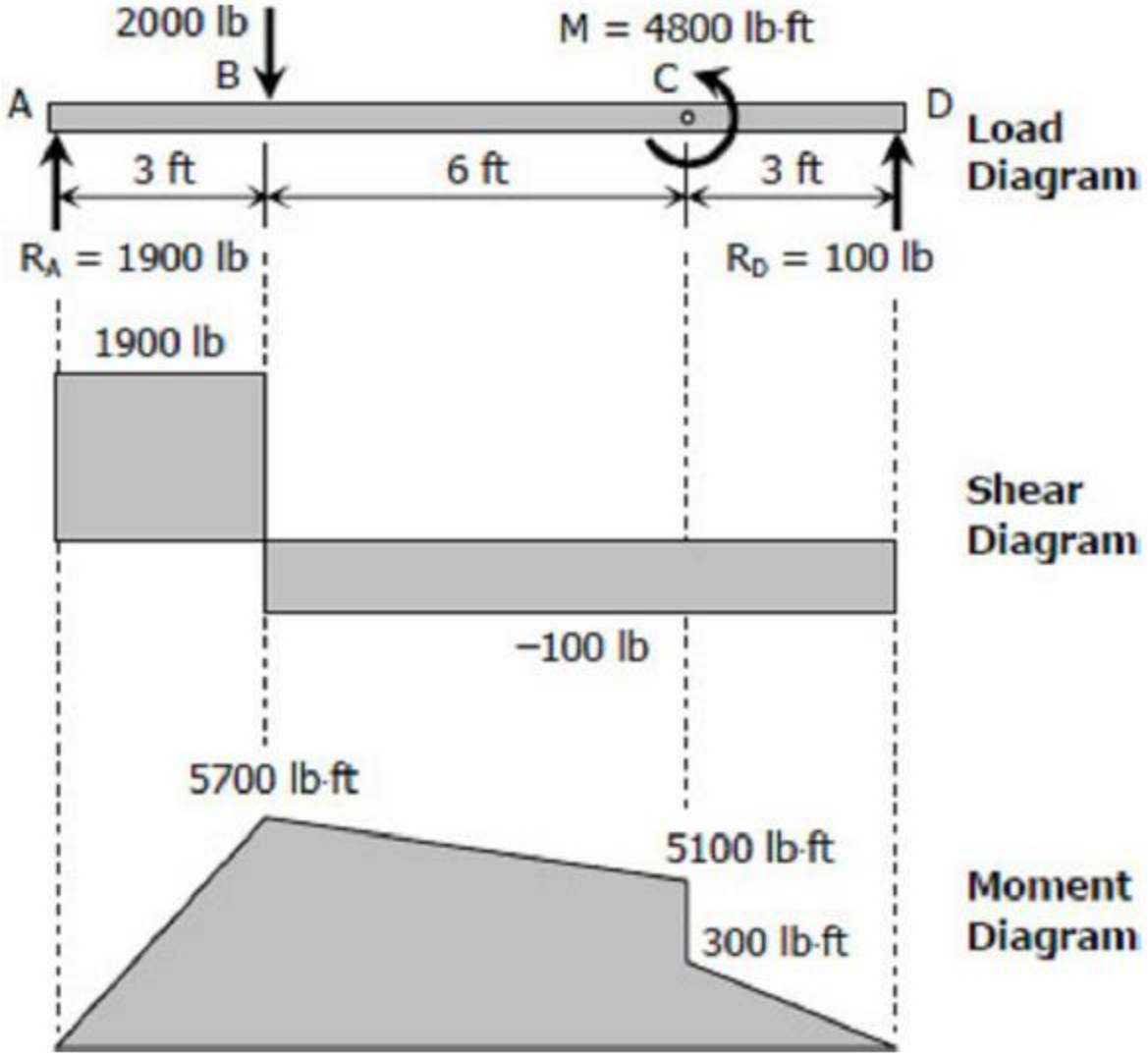
Segment BC:
$V_{BC} = 1900–2000$
$V_{BC} = –100 \text{ lb}$

$M_{BC} = 1900x–2000(x–3)$
$M_{BC} = 1900x–2000x + 6000$
$M_{BC} = –100x + 6000 \text{ lb} \cdot \text{ft}$



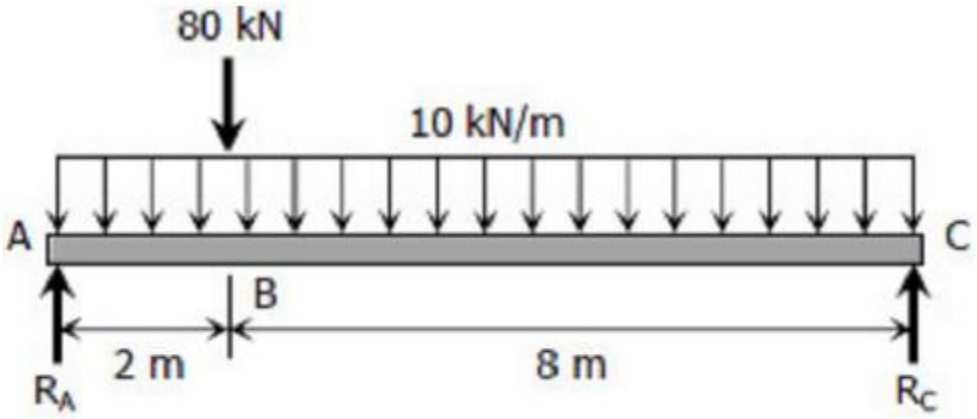
Segment CD:
$V_{CD} = 1900–2000$
$V_{CD} = –100 \text{ lb}$

$M_{CD} = 1900x–2000(x–3)–4800$
$M_{CD} = 1900x–2000x + 6000–4800$
$M_{CD} = –100x + 1200 \text{ lb} \cdot \text{ft}$


2000 lb
B
M = 4800 lb·ft
C
A
D
3 ft
6 ft
3 ft
R_A = 1900 lb
R_D = 100 lb
Load Diagram
1900 lb
−100 lb
Shear Diagram
5700 lb·ft
5100 lb·ft
300 lb·ft
Moment Diagram

Example 3:

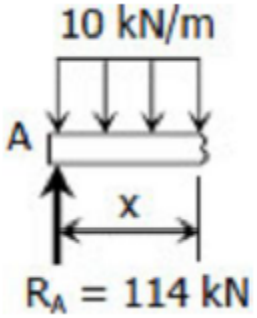
Draw the shear force and the bending moment diagrams for the beam shown in the figure.



Sol.:

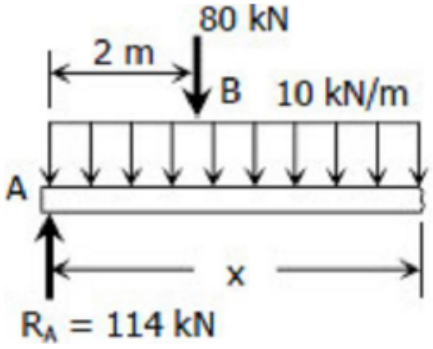
$$\Sigma M_A = 0$$
$$10R_C = 2(80) + 5[10(10)]$$
$$R_C = 66\,\text{kN}$$

$$\Sigma M_C = 0$$
$$10R_A = 8(80) + 5[10(10)]$$
$$R_A = 114\,\text{kN}$$



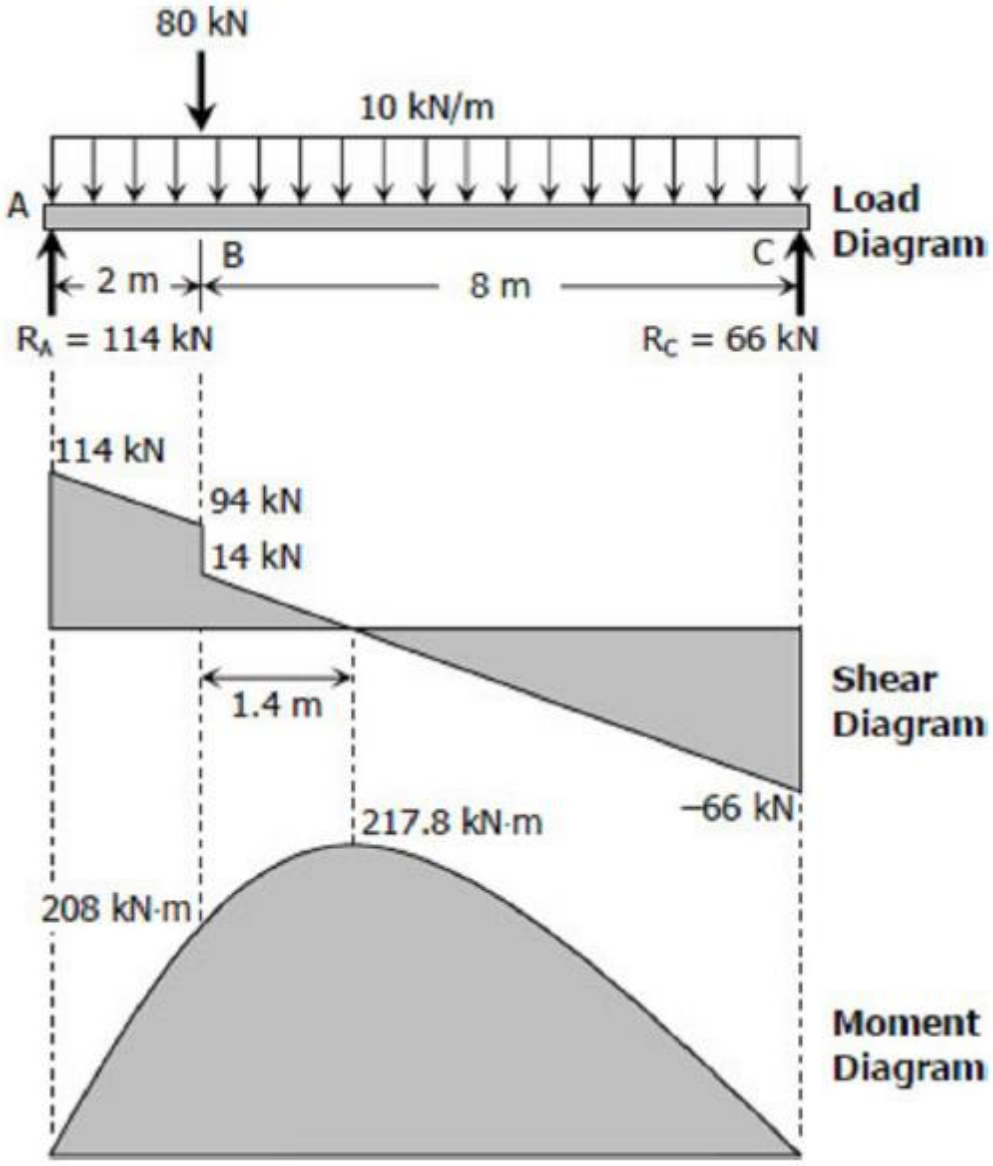
Segment AB:

$$V_{AB} = 114 - 10x \text{ kN}$$
$$M_{AB} = 114x - 10x(x/2)$$
$$M_{AB} = 114x - 5x^2 \text{ kN} \cdot \text{m}$$



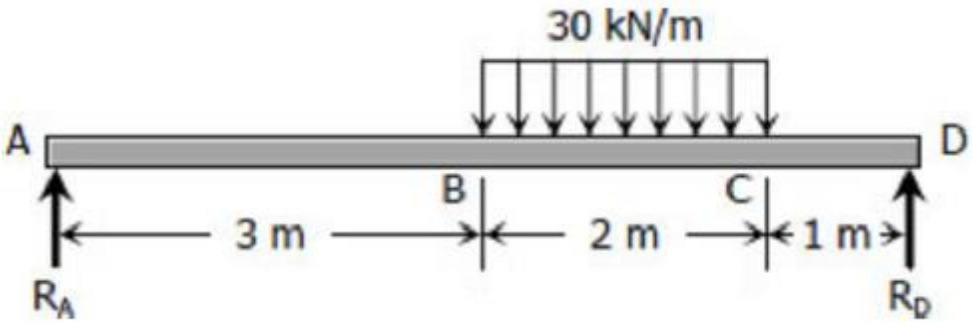
Segment BC:

$$V_{BC} = 114 - 80 - 10x$$
$$V_{BC} = 34 - 10x \text{ kN}$$
$$M_{BC} = 114x - 80(x-2) - 10x(x/2)$$
$$M_{BC} = 160 + 34x - 5x^2 \text{ kN} \cdot \text{m}$$


80 kN
10 kN/m
A
B
C
2 m
8 m
R_A = 114 kN
R_C = 66 kN
Load Diagram
114 kN
94 kN
14 kN
1.4 m
−66 kN
Shear Diagram
217.8 kN·m
208 kN·m
Moment Diagram

Example 4:

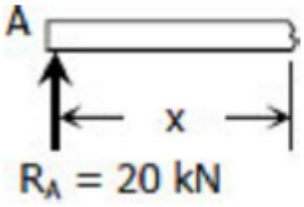
Draw the shear force and the bending moment diagrams for the beam shown in the figure.



Sol. :

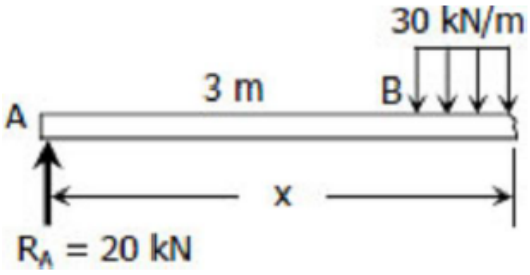
$$\Sigma M_A = 0$$
$$6R_D = 4[2(30)]$$
$$R_D = 40\,\text{kN}$$

$$\Sigma M_D = 0$$
$$6R_A = 2[2(30)]$$
$$R_A = 20\text{kN}$$



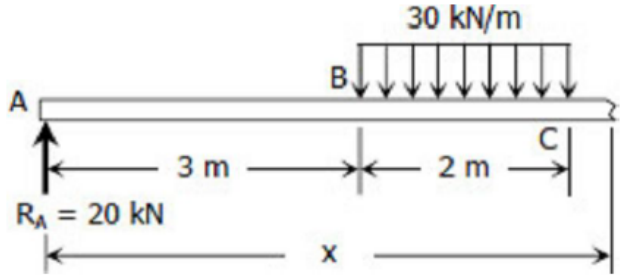
Segment AB:

$$V_{AB} = 20\,\text{kN}$$
$$M_{AB} = 20x\,\text{kN}\cdot\text{m}$$



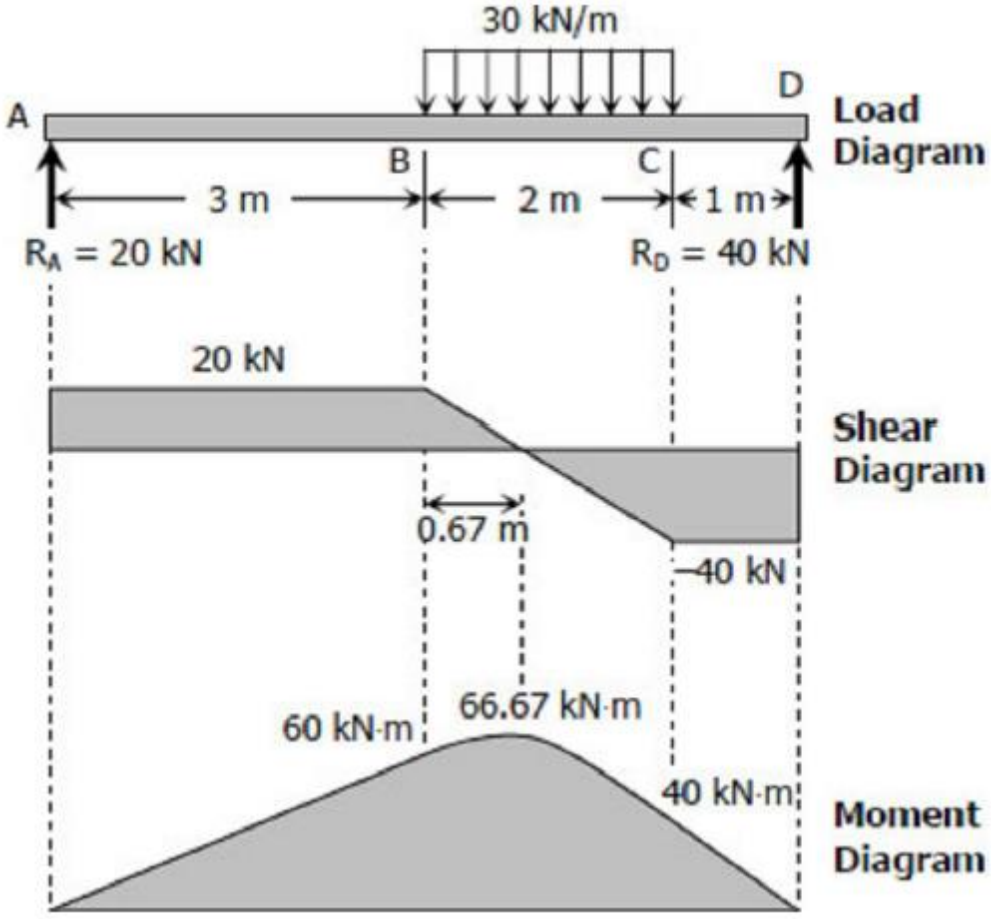
Segment BC:

$$V_{BC} = 20–30(x–3)$$
$$V_{BC} = 110–30x \text{ kN}$$
$$M_{BC} = 20x–30(x–3)(x–3)/2$$
$$M_{BC} = 20x–15(x–3)^2 \text{ kN} \cdot \text{m}$$



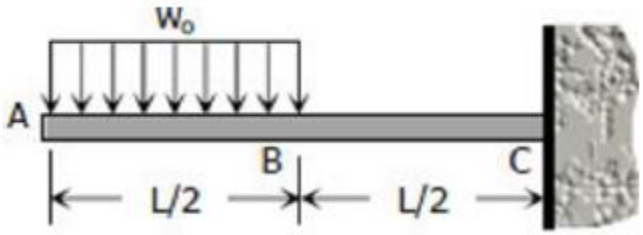
Segment CD:

$$V_{CD} = 20–30(2)$$
$$V_{CD} = –40 \text{ kN}$$
$$M_{CD} = 20x–30(2)(x–4)$$
$$M_{CD} = 20x–60(x–4) \text{ kN} \cdot \text{m}$$

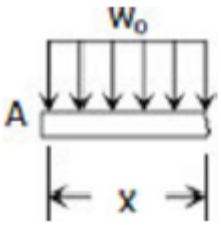


Example 5:

Draw the shear force and the bending moment diagrams for the beam shown in the figure.



Sol. :

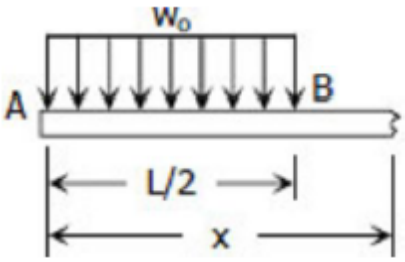


Segment AB:

$$V_{AB} = -w_o x$$

$$M_{AB} = -w_o x(x/2)$$

$$MAB = -\frac{1}{2} w_o x^2$$



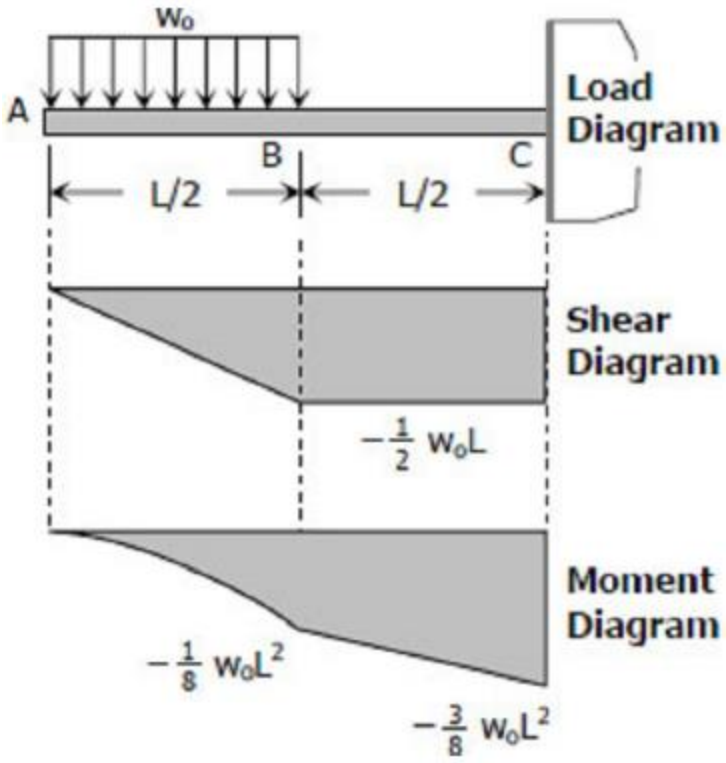
Segment BC:

$$V_{BC} = -w_o(L/2)$$

$$V_{BC} = -\frac{1}{2} w_o L$$

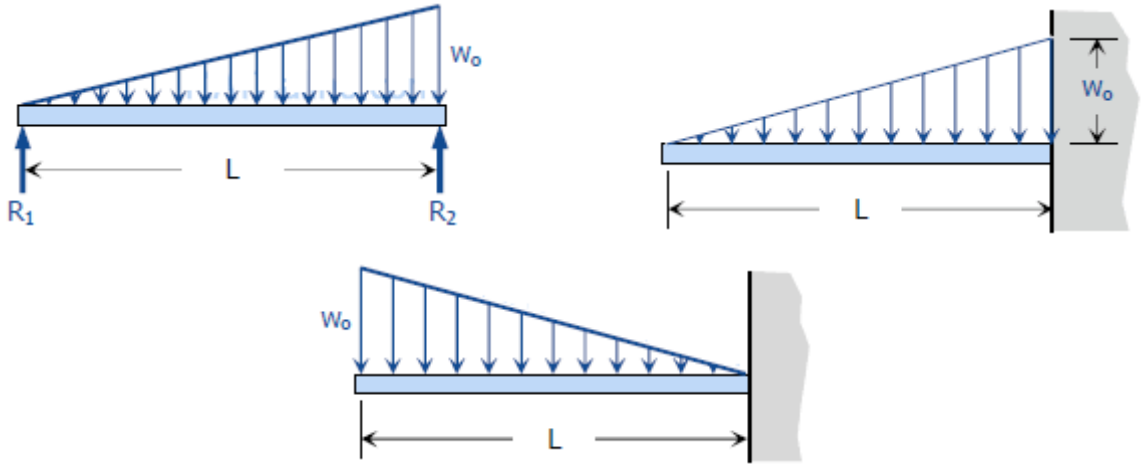
$$M_{BC} = -w_o(L/2)(x - L/4)$$

$$M_{BC} = -\frac{1}{2} w_o L x + \frac{1}{8} w_o L^2$$



Example 6:

Draw the shear force and bending moment diagrams for the following beams.



Example 7:

Draw the shear force and bending moment diagrams for the following beams.