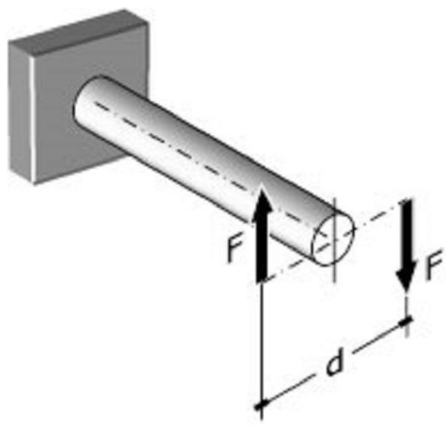
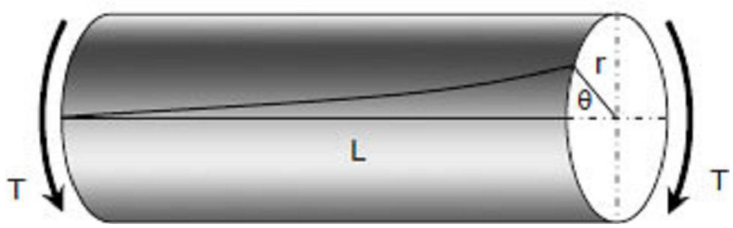
Chapter Three

Torsion

Consider a bar to be rigidly attached at one end and twisted at the other end by a torque or twisting moment T equivalent to F × d, which is applied perpendicular to the axis of the bar, as shown in the figure. Such a bar is said to be in torsion.





TORSIONAL SHEARING STRESS, τ

For a solid or hollow circular shaft subject to a twisting moment T, the torsional shearing stress τ at a distance ρ from the center of the shaft is:

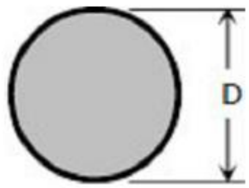
$$\tau = \frac{T\rho}{J} \text{ and } \tau_{max} = \frac{Tr}{J}$$

Where J is the polar moment of inertia of the section and r is the outer radius.

For solid cylindrical shaft:

$$J = \frac{\pi}{32} D^4$$

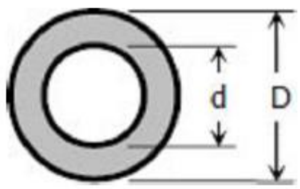
$$\tau_{max} = \frac{16T}{\pi D^3}$$



For hollow cylindrical shaft:

$$J = \frac{\pi}{32} (D^4 - d^4)$$

$$\tau_{max} = \frac{16TD}{\pi(D^4 - d^4)}$$



Angle of Twist

The angle θ through which the bar length L will twist is

$$\theta = \frac{TL}{JG} \text{ in radians}$$

Where T is the torque in N·mm, L is the length of shaft in mm, G is shear modulus in MPa, J is the polar moment of inertia in mm^4, D and d are diameters in mm, and r is the radius in mm.

Power Transmitted By the Shaft

A shaft rotating with a constant angular velocity ω (in radians per second) is being acted by a twisting moment T. The power transmitted by the shaft is:

$$P = T\omega = 2\pi T f$$

Where T is the torque in N·m, f is the number of revolutions per second, and P is the power in watts.

Problem 1

What is the minimum diameter of a solid steel shaft that will not twist through more than 3° in a 6-m length when subjected to a torque of 12 kN·m? What maximum shearing stress is developed? Use G = 83 GPa.

Sol.:

$$\theta = \frac{TL}{JG}$$

$$3^\circ \left(\frac{\pi}{180^\circ}\right) = \frac{12(6)(1000^3)}{\frac{1}{32}\pi d^4 (83\,000)}$$

$$d = 113.98 \text{ mm} \quad \textit{answer}$$

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16(12)(1000^2)}{\pi(113.98^3)}$$

$$\tau_{max} = 41.27 \text{ MPa} \quad \textit{answer}$$

Problem 2

A steel propeller shaft is to transmit 4.5 MW at 3 Hz without exceeding a shearing stress of 50 MPa or twisting through more than 1dig if its length is 26 times of diameters. Compute the proper diameter if G = 83 Gpa.

Sol.:

$$T = \frac{P}{2\pi f} = \frac{4.5(1\,000\,000)}{2\pi(3)}$$

$$T = 238\,732.41\ \text{N}\cdot\text{m}$$

Based on maximum allowable shearing stress:

$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$50 = \frac{16(238\,732.41)(1000)}{\pi d^3}$$

$$d = 289.71\ \text{mm}$$

Based on maximum allowable angle of twist:

$$\theta = \frac{TL}{JG}$$

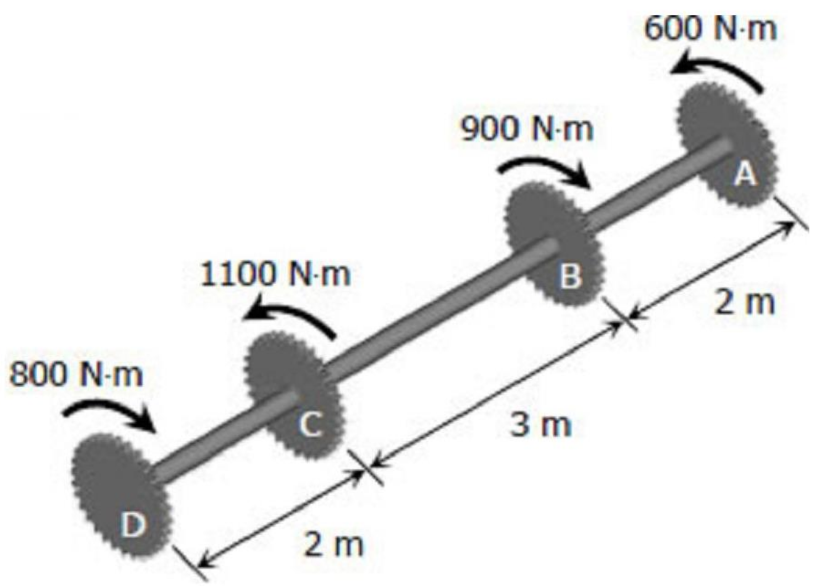
$$1^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{238\,732.41(26d)(1000)}{\frac{1}{32}\pi d^4(83\,000)}$$

$$d = 352.08\ \text{mm}$$

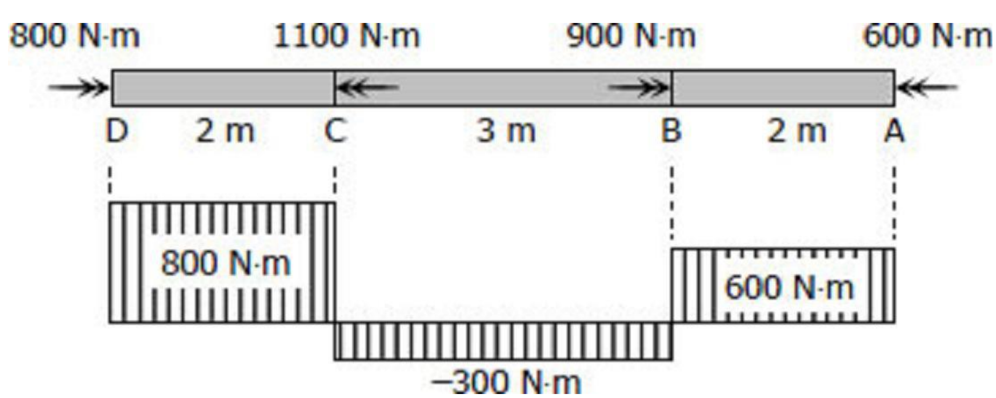
Use the bigger diameter, **d = 352 mm** ***answer***

Problem 3

An aluminum shaft with a constant diameter of 50 mm is loaded by torques applied to gears attached to it as shown. Using G = 28 GPa, determine the relative angle of twist of gear D relative to gear A.



Sol.:



$$\theta = \frac{TL}{JG}$$

Rotation of D relative to A:

$$\theta_{D/A} = \frac{1}{JG}\Sigma TL$$

$$\theta_{D/A} = \frac{1}{\frac{1}{32}\pi(50^4)(28\,000)}\,[\,800(2)–300(3) + 600(2)\,]\,(1000^2)$$

$$\theta_{D/A} = 0.1106 \text{ rad}$$

$$\theta_{D/A} = 6.34° \quad \textbf{\textit{answer}}$$

Problem 4

Determine the maximum torque that can be applied to a hollow circular steel shaft of 100-mm outside diameter and an 80-mm inside diameter without exceeding a shearing stress of 60 MPa or a twist of 0.5 deg/m. Use G = 83 GPa.

Sol.:

Based on maximum allowable shearing stress:

$$\tau_{max} = \frac{16TD}{\pi(D^4 - d^4)}$$

$$60 = \frac{16T(100)}{\pi(100^4 - 80^4)}$$

$$T = 6\,955486.14 \text{ N} \cdot \text{mm}$$

$$T = 6\,955.5 \text{ N} \cdot \text{m}$$

Based on maximum allowable angle of twist:

$$\theta = \frac{TL}{JG}$$

$$0.5^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{T(1000)}{\frac{1}{32}\pi(100^4 - 80^4)(83\,000)}$$

$$T = 4\,198\,282.97\ \text{N}\cdot\text{mm}$$

$$T = 4\,198.28\ \text{N}\cdot\text{m}$$

Use the smaller torque, **T = 4 198.28 N·m** ***answer***