Strength of Materials

The field of strength of materials deals with forces and deformations that result from their acting on a material. A load applied to a mechanical member will induce internal forces within the member called stresses when those forces are expressed on a unit basis. The stresses acting on the material cause deformation of the material in various manners. Deformation of the material is called strain when those deformations too are placed on a unit basis. The applied loads may be axial (tensile or compressive), or shear. The stresses and strains that develop within a mechanical member must be calculated in order to assess the load capacity of that member. This requires a complete description of the geometry of the member, its constraints, and the loads applied to the member and the properties of the material of which the member is composed. With a complete description of the loading and the geometry of the member, the state of stress and of state of strain at any point within the member can be calculated. Once the state of stress and strain within the member is known, the strength (load carrying capacity) of that member, its deformations (stiffness qualities), and its stability (ability to maintain its original configuration) can be calculated. The calculated stresses may then be compared to some measure of the strength of the member such as its material yield or ultimate strength. The calculated deflection of the member may be compared to deflection criteria that are based on the member's use. The calculated buckling load of the member may be compared to the applied load. The calculated stiffness and mass distribution of the member may be used to calculate the member's dynamic response and then compared to the acoustic environment in which it will be used. There are many types of stresses which are normal stress, shear stress, bearing stress, thermal stress,

Normal Stresses

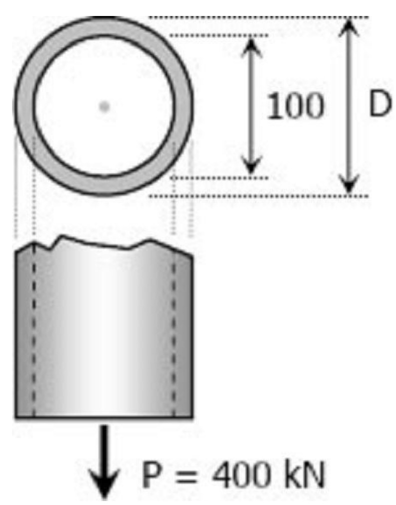
Stress is defined as the strength of a material per unit area of unit strength. It is the force on a member divided by area, which carries the force, formerly express in psi, now in N/mm2 or MPa.

$$\sigma = \frac{P}{A}$$

Where P is the applied normal load in Newton and A is the area in mm^2. The maximum stress in tension or compression occurs over a section normal to the load. Normal stress is either tensile stress or compressive stress. Member subject to pure tension (or tensile force) is under tensile stress, while compression members (members subject to compressive force) are under compressive stress. Compressive force will tend to shorten the member. Tension force on the other hand will tend to lengthen the member.

<u>Example 1</u>

A hollow steel tube with an inside diameter of 100mm must carry a tensile load of 400 kN. Determine the outside diameter of the tube if the stress is limited to 120 MN/m^2.



Sol.:

$$P = \sigma A$$
$$P = \sigma A$$

where:

$$P = 400\text{ kN} = 400\,000\text{N}$$
$$\sigma = 120\text{ MPa}$$
$$A = \tfrac{1}{4}\pi D^2 - \tfrac{1}{4}\pi(100^2)$$
$$A = \tfrac{1}{4}\pi(D^2 - 10\,000)$$

thus,

$$400\,000 = 120\,[\,\tfrac{1}{4}\pi(D^2 - 10\,000)\,]$$

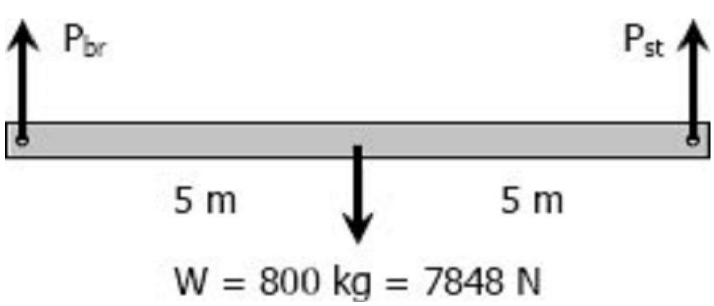
$$400\,000 = 30\pi D^2 - 300\,000\pi$$

$$D^2 = \frac{400\,000 + 300\,000\pi}{30\pi}$$

$$D = 119.35\text{ mm} \rightarrow \textit{answer}$$

Example 2

If the maximum allowable stress for bronze is 90 MPa and maximum allowable stress for steel is 120 MPa, Find the smallest area of bronze and steel cables that required to support the 800 kg bar.



Sol.:

By symmetry:

$$P_{br} = P_{st} = \tfrac{1}{2}(7848)$$
$$P_{br} = 3924N$$
$$P_{st} = 3924N$$

For bronze cable:

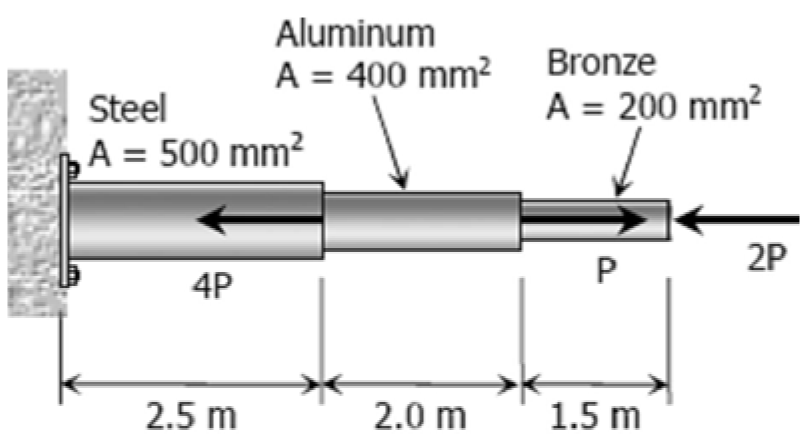
$$P_{br} = \sigma_{br} A_{br}$$
$$3924 = 90\,A_{br}$$
$$A_{br} = 43.6\ \text{mm}^2 \quad \longrightarrow \textbf{\textit{answer}}$$

For steel cable:

$$P_{st} = \sigma_{st} A_{st}$$
$$3924 = 120\,A_{st}$$
$$A_{st} = 32.7\ \text{mm}^2 \quad \longrightarrow \textbf{\textit{answer}}$$

Example 3

For the compound shaft shown, find the maximum safe value of axial load P if the maximum allowable stress for steel is 140 MPa, for aluminum is 90 MPa and for bronze is 100 MPa.



Sol.:

For bronze:

$$\sigma_{br}A_{br} = 2P$$
$$100(200) = 2P$$
$$P = 10\,000\text{ N}$$

For aluminum:

$$\sigma_{al}A_{al} = P$$
$$90(400) = P$$
$$P = 36\,000\text{ N}$$

For Steel:

$$\sigma_{st}A_{st} = 5P$$
$$P = 14\,000\text{ N}$$

For safe P, use $P = 10\,000\text{ N} = 10\text{ kN}$ $\rightarrow$ ***answer***

Shear Stress

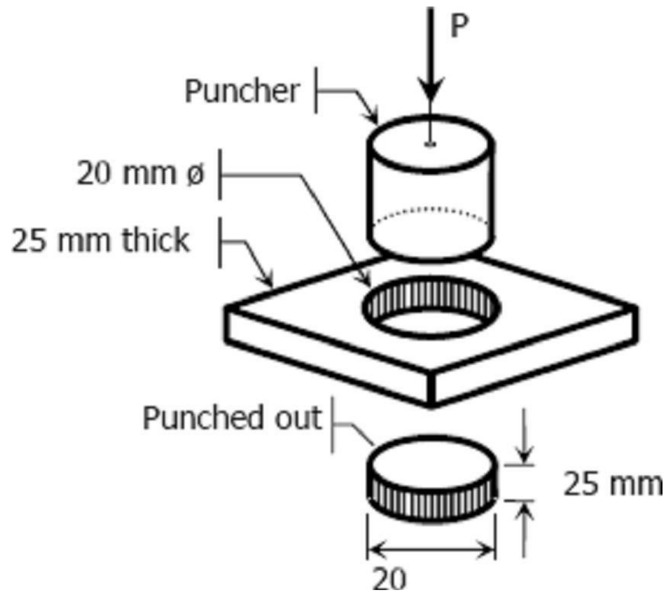
Forces parallel to the area resisting the force cause shearing stress. It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act. Shearing stress is also known as tangential stress.

$$\tau = \frac{V}{A}$$

Where V is the resultant shearing force which passes through the centroid of the area A being sheared.

Example 4

It is required to make a hole of diameter 20mm in 25mm thickness plate.
If the shear strength of plate is350 MN/m^2, find the force required to punch the hole.



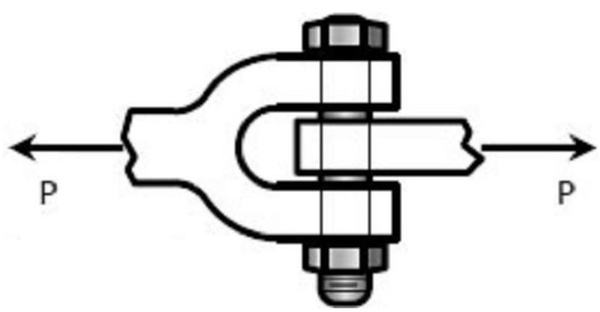
Sol.:

The resisting area is the shaded area along the perimeter and the shear force *V* is equal to the punching force *P*.

$$V = \tau A$$
$$P = 350\,[\,\pi(20)(25)\,]$$
$$P = 549\,778.7\text{ N}$$
$$P = 549.8\text{ kN} \rightarrow \textit{answer}$$

Example 5

Find the smallest diameter of the bolt that can withstand 400N force if the shear strength of the bolts material is 300 MPa.



Sol.:

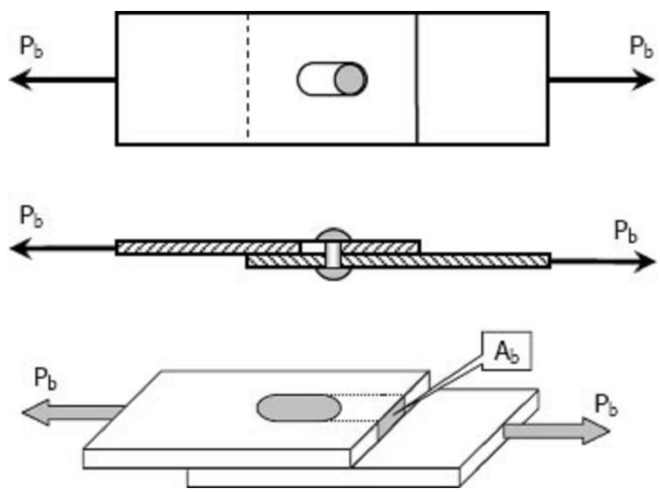
The bolt is subject to double shear.

$$V = \tau A$$
$$400(1000) = 300[\,2(\tfrac{1}{4}\pi d^2)\,]$$
$$d = 29.13\text{ mm} \rightarrow \textit{answer}$$

Bearing Stress

Bearing stress is the contact pressure between the separate bodies. It differs from compressive stress, as it is an internal stress caused by compressive forces. It deals with projectile area instead of the real areas.

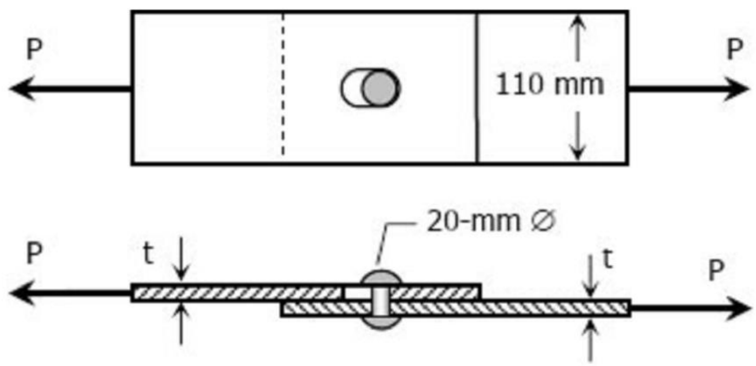
$$\sigma_b = \frac{P_b}{A_b}$$



<u>Example 6</u>

To joint 110 mm wide plates together with each other, a 20-mm-diameter rivet was used. The allowable stresses are 120 MPa for bearing in the plate material and 60 MPa for shearing of rivet. Determine:

a) The minimum thickness of each plate.
b) The largest average tensile stress in the plates.



Sol.:

Part (a):

From shearing of rivet:

$$P = \tau A_{rivets}$$
$$P = 60[\,\tfrac{1}{4}\pi(20^2)\,]$$
$$P = 6000\pi\ textN$$

From bearing of plate material:

$$P = \sigma_b A_b$$

$$6000\pi = 120(20t)$$
$$t = 7.85\text{ mm} \rightarrow \textit{answer}$$

Part (b): Largest average tensile stress in the plate:

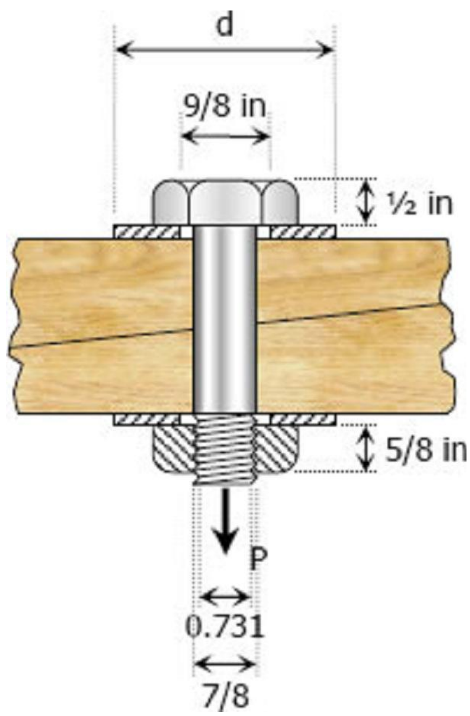
$$P = \sigma A$$
$$6000\pi = \sigma[\,7.85(110-20)\,]$$
$$\sigma = 26.67\text{ MPa} \rightarrow \textit{answer}$$

Example 7

For the assemble shown, if the diameter at the root of the thread (bolt) = 0.731 inches , find:

a) Shearing stress in the head of the bolt
b) Shearing stress in threads of the bolt
c) Outside diameter of the washer

Knowing that the tensile stress in the nut is 18 ksi and the bearing stress is 800 psi.



Sol.:

Tensile force on the bolt:

$$P = \sigma A = 18[\,\tfrac{1}{4}\pi(\tfrac{7}{8})^2\,]$$

$$P = 10.82\ \text{kips}$$

Shearing stress in the head of the bolt:

$$\tau = \frac{P}{A} = \frac{10.82}{\pi(\tfrac{7}{8})(\tfrac{1}{2})}$$

$$\tau = 7.872\ \text{ksi} \rightarrow \textit{answer}$$

Shearing stress in the threads:

$$\tau = \frac{P}{A} = \frac{10.82}{\pi(0.731)(\tfrac{5}{8})}$$

$$\tau = 7.538\ \text{ksi} \rightarrow \textit{answer}$$

Outside diameter of washer:

$$P = \sigma_b A_b$$

$$10.82(1000) = 800\{\,\tfrac{1}{4}\pi\,[\,d^2 - (\tfrac{9}{8})^2\,]\,\}$$

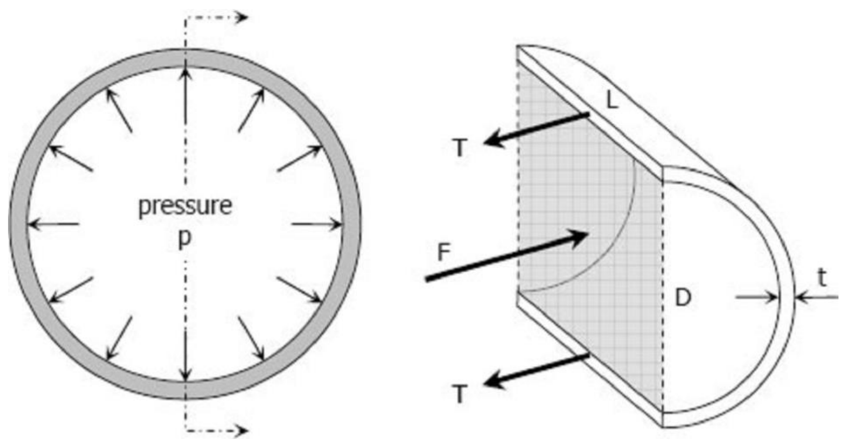
$$d = 4.3\ \text{inch} \rightarrow \textit{answer}$$

Thin-walled Pressure Vessels

They are defined as a tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.

Tangential Stress (Circumferential Stress)

Consider the tank shown being subjected to an internal pressure *p*. The length of the tank is *L* and the wall thickness is *t*. isolating the right half of the tank:



The forces acting are the total pressures caused by the internal pressure *p* and the total tension in the walls *T*.

$$F = pA = pDL$$
$$T = \sigma_t A_{wall} = \sigma_t tL$$

$$\Sigma F_H = 0$$
$$F = 2T$$

$$pDL = 2(\sigma_t tL)$$

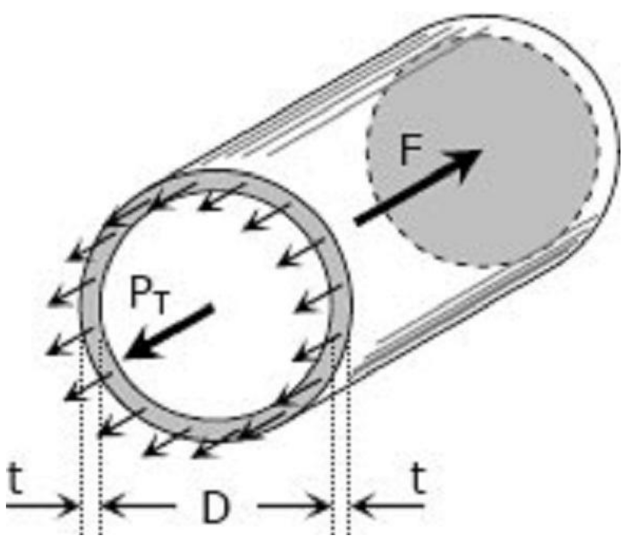
$$\sigma_t = \frac{pD}{2t}$$

If there an external pressure p_o and an internal pressure P_i the formula may be expressed as:

$$\sigma_t = \frac{(p_i - p_o)D}{2t}$$

Longitudinal stress:

Consider the free body diagram in the transverse section of the tank:



The total force acting at the rear of the tank F must equal to the total longitudinal stress on the wall:

$$P_T = \sigma_L A_{wall}$$

Since t is so small compared to D, the area of the wall is close to πD_t

$$F = pA = p\frac{\pi}{4}D^2$$
$$P_T = \sigma_L \pi D t$$
$$\Sigma F_H = 0$$
$$P_T = F$$
$$\sigma_L \pi D t = p\frac{\pi}{4}D^2$$
$$\sigma_t = \frac{pD}{4t}$$

If there an external pressure p_o and an internal pressure p_i the formula may be expressed as:

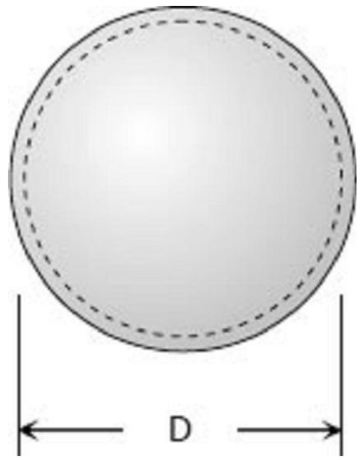
$$\sigma_t = \frac{(p_i - p_o)D}{4t}$$

It can be observed that the tangential stress is twice that of the longitudinal stress.

$$\sigma_t = 2\sigma_L$$

Spherical Shell

If a spherical tank of diameter *D* and thickness *t* contains gas under a pressure of *p*, the stress at the wall can be expressed as:



$$\sigma_t = \frac{(p_i - p_o)D}{4t}$$

<u>Example 8</u>

A cylindrical pressure vessel of 400 mm diameter and 20 mm Wall thickness currying an internal pressure of 4.5 MN/m^2. If the allowable stress is 120 MN/m^2, find:

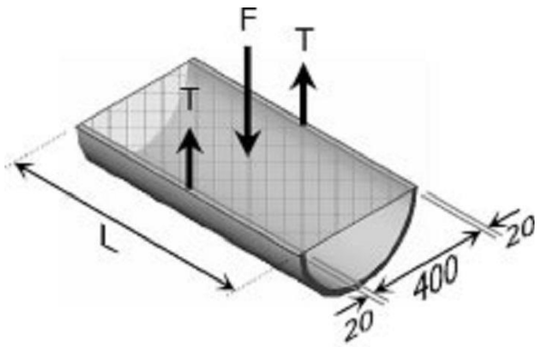
a) Longitudinal and tangential stresses.

b) Maximum amount of internal pressure that can be applied and the expected fracture if failure occurs

Sol.:

Part (a)

Tangential stress (longitudinal section):

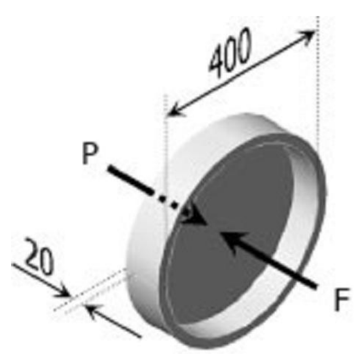


Longitudinal Section

$$F = 2T$$
$$pDL = 2(\sigma_t tL)$$
$$\sigma_t = \frac{pD}{2t} = \frac{4.5(400)}{2(20)}$$

$$\sigma_t = 45\text{ MPa} \rightarrow \textit{answer}$$

Longitudinal Stress (transverse section):



Transverse Section

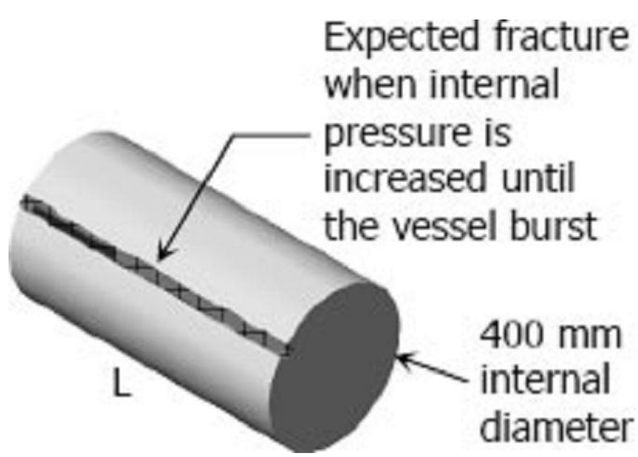
$$F = P$$
$$\tfrac{1}{4}\pi D^2 p = \sigma_l(\pi D t)$$
$$\sigma_l = \frac{pD}{4t} = \frac{4.5(400)}{4(20)}$$
$$\sigma_l = 22.5 \text{ MPa} \rightarrow \textbf{\textit{answer}}$$

Part (b)

From (a), $\sigma_t = \dfrac{pD}{2t}$ and $\sigma_l = \dfrac{pD}{4t}$ thus, $\sigma_t = 2\sigma_l$, this shows that tangential stress is the critical.

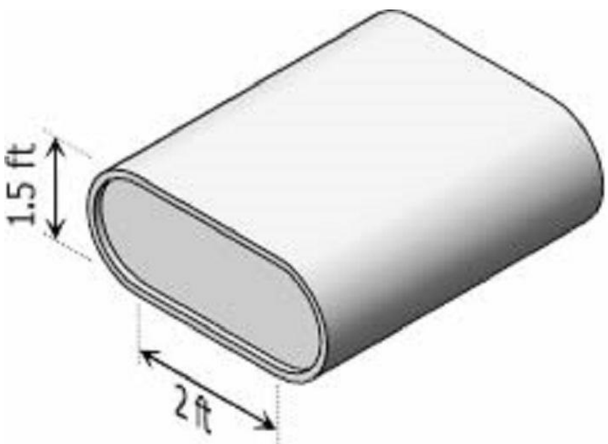
$$\sigma_t = \frac{pD}{2t}$$
$$120 = \frac{p(400)}{2(20)}$$
$$p = 12 \text{ MPa} \rightarrow \textbf{\textit{answer}}$$

The bursting force will cause a stress on the longitudinal section that is twice to that of the transverse section. Thus, fracture is expected as shown.



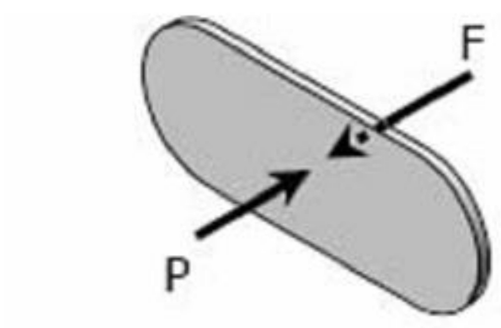
Example 9

Find the maximum longitudinal and circumferential stress for the pressure vessel shown if the wall thickness is 1/8 inch and the internal pressure = 125 psi.



<u>Sol.:</u>

Longitudinal Stress:

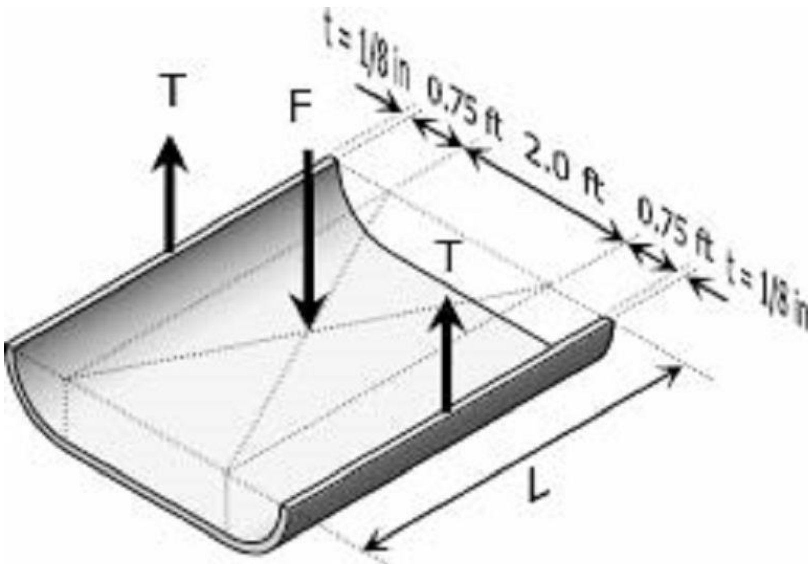


thickness, t = 1/8 in.

$$F = pA = 125[\,1.5(2) + \tfrac{1}{4}\pi(1.5)^2\,](12^2)$$
$$F = 85\,808.62\,\text{lbs}$$

$$
\begin{aligned}
P &= F \\
\sigma_l[\,2(2 \times 12)(\tfrac{1}{8}) + \pi(1.5 \times 12)(\tfrac{1}{8})\,] &= 85\,808.62 \\
\sigma_l &= 6\,566.02 \text{ psi} \\
\sigma_l &= 6.57 \text{ ksi} \;\rightarrow \textit{answer}
\end{aligned}
$$

Circumferential Stress:



$$
\begin{aligned}
F &= pA = 125[\,(2 \times 12)L + 2(0.75 \times 12)L\,] \\
F &= 5250L \; textlbs
\end{aligned}
$$

$$
\begin{aligned}
2T &= F \\
2[\,\sigma_t(\tfrac{1}{8})L\,] &= 5250L \\
\sigma_t &= 21\,000 \text{ psi} \\
\sigma_t &= 21 \text{ ksi} \;\rightarrow \textit{answer}
\end{aligned}
$$