PLASMA PHYSICS

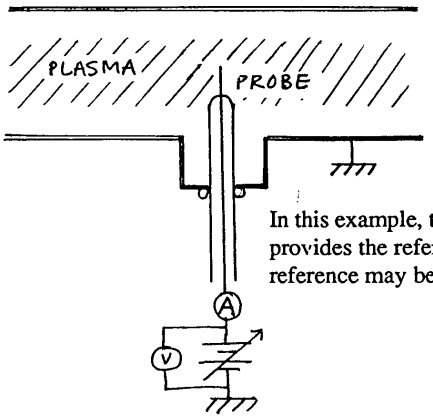
VII. PLASMA DIAGNOSTICS

Electrostatic or Langmuir Probes

Probes are inserted into the plasma to measure T_e and n.
The probes perturb the plasma electrically and may be, in the case of tokamak plasmas, an intolerable source of impurities.

Single probe

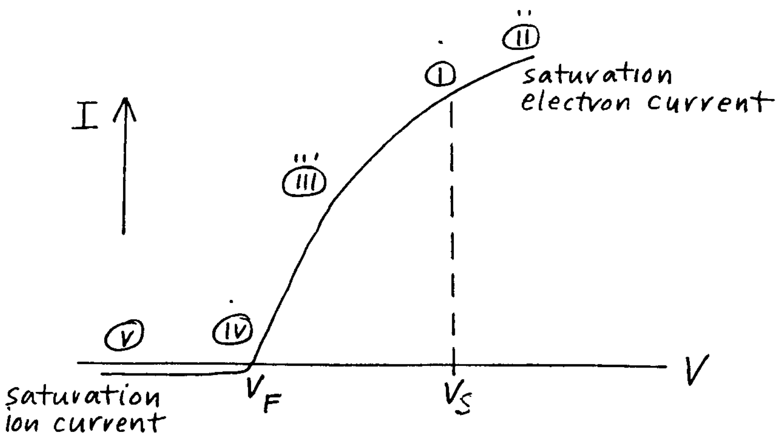
Typically a tungsten probe in a glass tube.



In this example, the conducting wall of the vessel provides the reference potential. In other cases the reference may be the cathode or the anode.

Probe perturbs the plasma but the effects are small outside a thin sheath surrounding the probe. The sheath thickness is of the order of a Debye length $\lambda_D = \sqrt{\frac{\varepsilon_0 k T_e}{n_0 e^2}}$ (see Chapter I) and is << R, the probe radius..

I-V characteristic



(i) V_S is the *space* or *plasma potential* (the potential of the plasma in the absence of a probe). There is no E. The current is due mainly to the random motion of electrons (the random motion of the ions is much slower).

(ii) If the probe is more positive than the plasma, electrons are attracted towards the probe and all the ions are repelled. An electron sheath is formed and *saturation electron current* is reached.

(iii) If the probe is more negative than the plasma, electrons are repelled (but the faster ones still reach the probe) and ions are attracted.

The shape of this part of the curve depends on the electron velocity distribution. For a Maxwellian distribution with $T_e > T_i$, the slope of $ln\ I$ plotted against V is $\left|\frac{e}{kT_e}\right|$

(iv) V_F is the *floating potential* (an insulated electrode would assume this potential). The ion flux = the electron flux so $I = 0$.

(v) All the electrons are repelled. An ion sheath is formed and *saturation ion current* is reached.

Collisions

If there are collisions, particles may have to rely on diffusion to treach the probe.
From before $\lambda_m \approx \frac{D}{v_{rms}}$. Collisions are important if $\lambda_m << R$.

Magnetic field

The effect of a magnetic field is extremely complicated. The ions and electrons gyrate and this affects their random motions and collisions. The behaviour of the probe will depend on its orientation in the magnetic field. The magnetic field can be ignored if $r_{Le} >> R$.

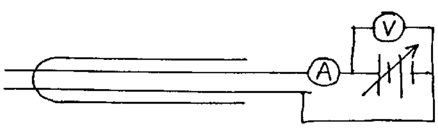
Points to note

Swept Langmuir probes may give time behaviour of plasma parameters, but there are difficulties.

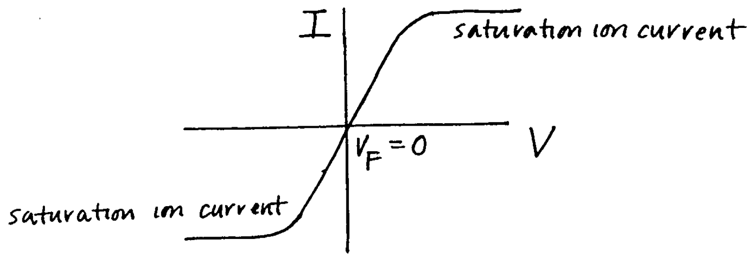
Any secondary emission, photoemission will lead to errors

Care is required if distribution is non-Maxwellian. For instance if there is a drift.

Double probe



I-V characteristic
Assume symmetry.



The system "floats" and follows any changes in V_S.
If V is slightly positive, there are more electrons reaching 1 and fewer reaching 2.
If V is very positive, saturation ion current into 2. The current into 1 cannot exceed this value.

It can be shown that

$$I = I_{sat\,i} \tanh \frac{eV}{2kT_e}$$

and the slope of the characteristic is

$$\frac{dI}{dV} = \frac{eI_{sat\,i}}{2kT_e}.$$

Points to note

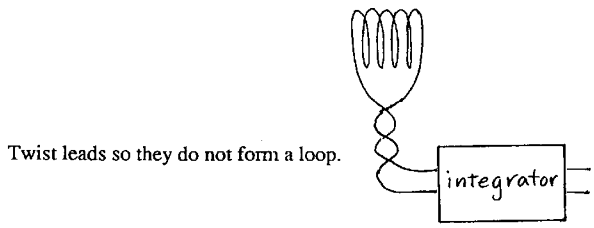
In a magnetic field, shadowing of the probes must be avoided.

Magnetic probes

Magnetic coil

(i) A small coil is inserted into the plasma and oriented to measure a particular component of **B** or to pick up MHD waves.

Typically the small coil is in a glass tube – it has no electrical contact with the plasma.



$$V = NA\frac{dB}{dt}.$$

You do. Derive this.
Use integrator.

$$V_{out} = \frac{NAB}{RC}.$$

(ii) *Flux loop* or *diamagnetic loop*. A large coil surrounding the plasma vessel is used to measure total magnetic flux.

Recall that if $I_z = 0$,

$$p(r) + \frac{B_z(r)^2}{2\mu_0} = \text{ constant}.$$

The presence of the plasma acts to decrease the magnetic field inside it (hence *diamagnetic*).

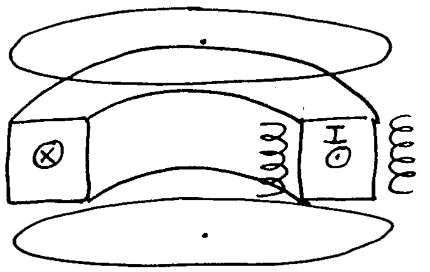
If we measure $B_z(a)$ using a small coil and $<B_z>$ using a diamagnetic loop then

$$< p > = \frac{B_z(a)^2 - < B_z >^2}{2\mu_0}$$

gives an estimate of the total kinetic energy in the plasma (using $p = nkT$).

You do. Show this.

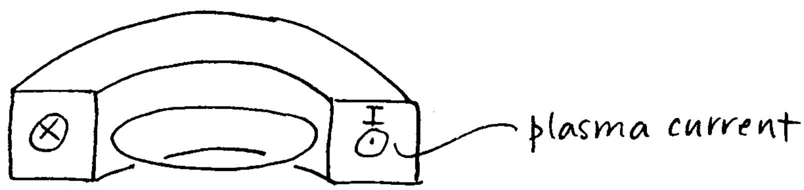
(iii) Monitor position of plasma in a tokamak



If the plasma moves up, the flux in the upper loop increases while the flux in the lower loop decreases. Take difference and integrate.

If the plasma moves out, the flux in the outer solenoid increases while the flux in the inner solenoid decreases. Take difference and integrate.

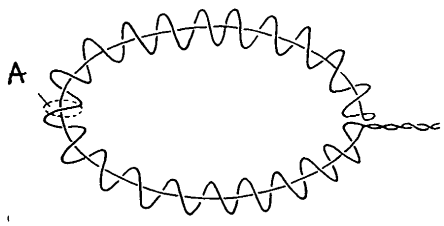
(iv) Loop voltage measurements



The emf induced in the loop equals the emf in the plasma loop. Measure plasma current I independently and calculate the plasma resistivity using $R = \frac{\eta l}{A}$. η depends on electron temperature so T_e can be estimated.

Rogowski coil

To measure I the total current. The large loop completely surrounds I. The measurement is independent of how the current is distributed and it has the advantage of making no electrical contact with the current being measured.



Note the return wire to minimize the effect of any flux threading the large loop.

$$V = nA\mu_0 \frac{dI}{dt}.$$

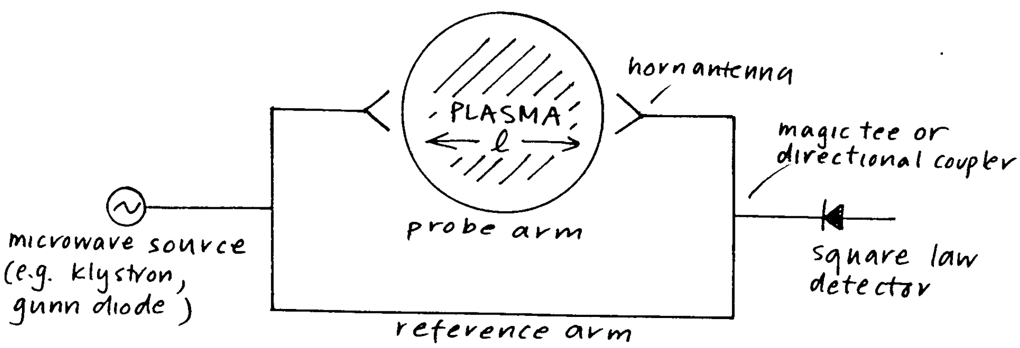
where n is the number of turns per unit length, A is the area of the small loop.

You do. Derive this. You have to assume B is uniform across A.

Use integrator.
The coil should be shielded to avoid electrostatic pickup from the plasma.

(Rogowski coil can be used as a current probe in non-plasma applications.)

Microwave interferometry



This is a non-perturbing way of measuring n. The interferometer shown is the microwave version of the Mach-Zehnder interferometer. The optical path length in the probe arm changes as the plasma density varies.

The waves in the plasma are usually the *O* waves whose refractive index

$$N^2 = 1 - \frac{n}{n_c}$$

depends only on the density and not on the magnetic field.

Field at detector due to signal passing through probe arm

$$E_1 = A_1 \cos\left(\omega t + \phi_1 + \phi_{\text{plasma}}\right)$$

where $\phi_{\text{plasma}} = \int_0^l (k - k_0) dx$, $k_0 = \frac{\omega}{c}$, $k = \mu \frac{\omega}{c}$.

Field at detector due to signal passing through reference arm

$$E_2 = A_2 \cos\left(\omega t + \phi_2\right).$$

The output of the square-law detector is

$$\begin{aligned} V &= \left(E_1 + E_2\right)^2 \\ &= A_1^2 \cos^2\left(\omega t + \phi_1 + \phi_{\text{plasma}}\right) + 2A_1 A_2 \cos\left(\omega t + \phi_1 + \phi_{\text{plasma}}\right)\cos\left(\omega t + \phi_2\right) \\ &\quad + A_2^2 \cos^2\left(\omega t + \phi_2\right) \\ &= \frac{A_1^2 + A_1^2 \cos 2\left(\omega t + \phi_1 + \phi_{\text{plasma}}\right)}{2} \\ &\quad + A_1 A_2 \cos\left(2\omega t + \phi_1 + \phi_{\text{plasma}} + \phi_2\right) + A_1 A_2 \cos\left(\phi_1 + \phi_{\text{plasma}} - \phi_2\right) \\ &\quad + \frac{A_2^2 + A_2^2 \cos 2\left(\omega t + \phi_2\right)}{2} \end{aligned}$$

The capacitance of the detector shorts the microwave frequency signals to ground.

The remaining slowly-varying part is

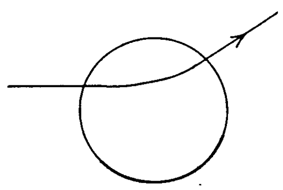
$$V = \frac{A_1^2}{2} + A_1 A_2 \cos\left(\phi_1 + \phi_{\text{plasma}} - \phi_2\right) + \frac{A_2^2}{2}.$$

Points to note

As the derivation shows, the interferometer measures average density along the chord. To obtain a density profile would require measurements along many chords.

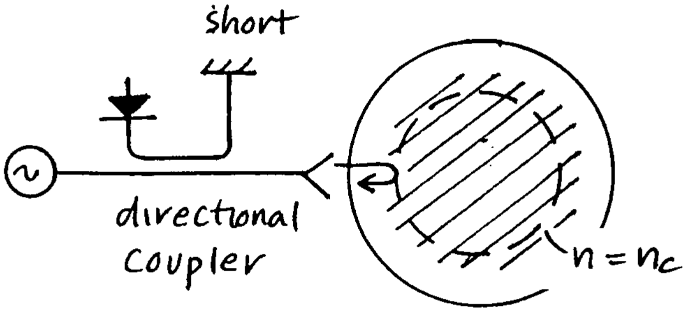
The longer the wavelength the better is the sensitivity to small densities but the poorer is the spatial resolution.

Beam bending may occur.



Microwave reflectometry

Measures n. Different frequencies will be reflected from different layers in the plasma.



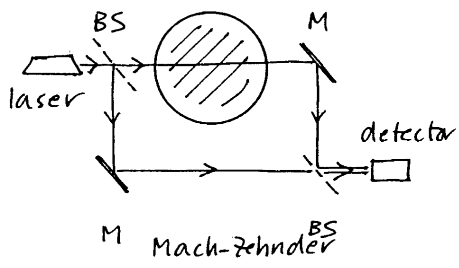
Point to note

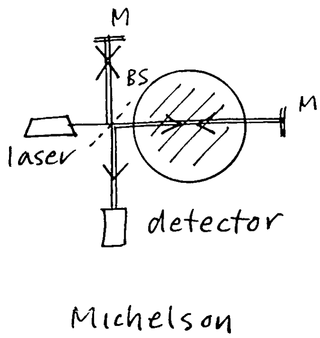
There is a problem if density is not monotonically increasing.

Laser interferometry

Measures $\langle n \rangle$.

Two forms of interferometers are the Mach-Zehnder interferometer and the Michelson interferometer.



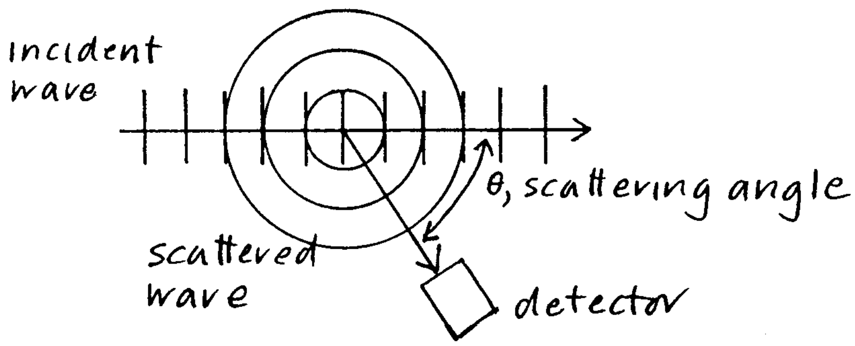


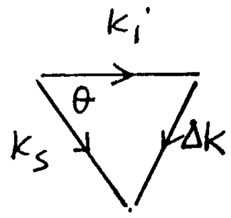
Some lasers that have been used in the School of Physics are He-Ne laser (visible 633 nm, infrared 3.39 μm), CO_2 laser (infra-red 10.6 μm), formic acid molecular vapour laser (far-infrared 433 μm), HCN laser (far-infrared 337 μm).

Scattering of electromagnetic radiation from a plasma

(a) Thomson scattering

Scattering of laser light from electrons in the plasma to make a non-perturbing measurement of T_e.





The scattered wave satisfies

$$\mathbf{k}_s = \mathbf{k}_i + \Delta\mathbf{k} \text{ and } \omega_s = \omega_i + \Delta\omega$$

Since $k_s = k_i$,

$$\Delta k = 2k_i \sin\frac{\theta}{2}.$$

Thomson scattering can be described classically. The principle is that the incident wave accelerates the electrons; In the non-relativistic case,

$$\dot{\mathbf{v}} = -\frac{e}{m_e}\mathbf{E}_i \cos(\mathbf{k}_i \cdot \mathbf{r} - \omega t),$$

and because the electrons are accelerating, they radiate; At large distances, in the far field,

$$\mathbf{E}_s = -\frac{e}{c^2}\frac{\mathbf{r}\times(\mathbf{r}\times\dot{\mathbf{v}})}{r^3}$$

where the quantities on the rhs are evaluated at retarded time.
(At high energies a quantum mechanical treatment is more appropriate. *Compton scattering*.)

The ions are too massive to radiate - they still have an effect however.

The contributions from all the electrons must be combined.

$\alpha = \dfrac{1}{\Delta k\, \lambda_D} << 1$. There are two ways of looking at this case

(a) Δk is large so the resolution is high, individual electrons can be seen.
(b) λ_D is large so plasma effects are unimportant.

In other words, the effects of individual electrons are important. The phases of the wave arriving at the different electrons is random as will be the phases of their scattered waves. So the scattering is *incoherent*. The total scattered intensity is given by adding the scattered intensities from each electron.

(If we think of $\frac{1}{\Delta k}$ as a scale length, then $\alpha = \frac{\text{scale length}}{\text{Debye length}}$.)

$$\alpha = \frac{1}{\Delta k\, \lambda_D} >> 1$$

(a) Δk is large so the resolution is low, only a 'cloud' can be seen,

(b) the scattering is from fluctuations in charge density (ion acoustic waves) and is *collective* or *coherent*. The total scattered intensity is found by calculating the total far field and squaring it.

The intensity in the incoherent case is $\propto n$ and in the coherent case is much larger, $\propto n^2$.

The intensity of the scattered radiation is given by

$$I(\omega) = I_0 r_e^2 \left(1 - \cos^2\phi\right) n S(\Delta\mathbf{k}, \Delta\omega)$$

where I_0 is the incident intensity, $r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}$ is the classical radius of the electron (which is very small), $\left(1-\cos^2\phi\right)$ is a geometrical factor in which ϕ is the angle between $\mathbf{E}_i$ and $\mathbf{k}_s$, n is the plasma density and $S(\Delta\mathbf{k}, \Delta\omega)$ is the dynamic form factor.

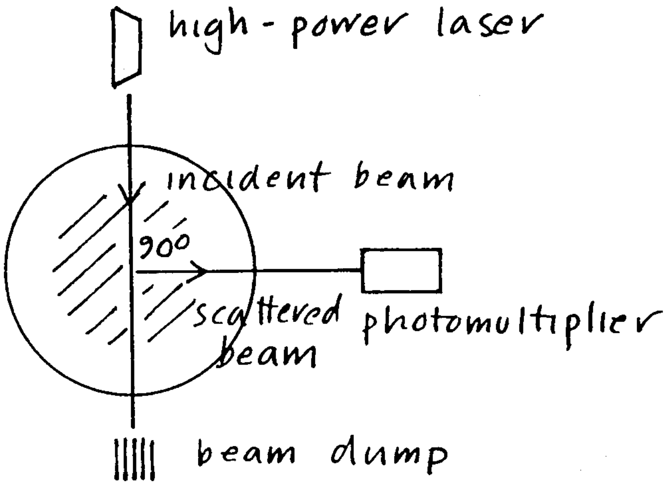
From this expression we see

(i) that the scattered intensity is greatest for $\phi = 90°$

So choose $\phi = 90°$.

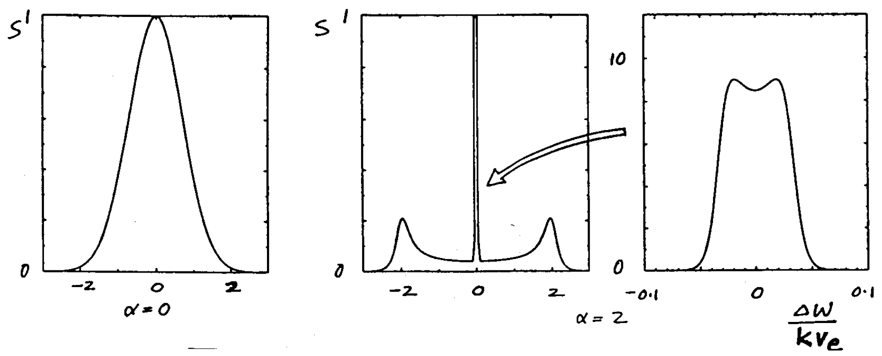
(ii) the scattered intensity is extremely small

So use an energetic pulsed laser (ruby laser).



(iii) the spectrum goes as $S(\Delta\mathbf{k}, \Delta\omega)$ which depends on α.

$S(\Delta\mathbf{k}, \Delta\omega)$ has been plotted below for two values of α.



$S(\Delta\mathbf{k}, \Delta\omega)$ has an electron term and an ion term. If $\alpha << 1$, the ion term is negligible and the shape is due to Doppler broadening. Electron thermal motion gives a doppler shift and the width of the line gives T_e. n could be obtained from the absolute value of the scattered intensity.

If $\alpha \geq 1$ the electron peak can give n but at still higher α depends on n and T_e. At $\alpha \geq 1$ the ion term becomes important and, since the peak is interpreted as the existance of an ion acoustic wave, depends on T_e. Under different conditions the width can depend on T_i or T_e. The condition $\alpha \geq 1$ can be obtained by using a long wavelength and/or a small scattering angle.

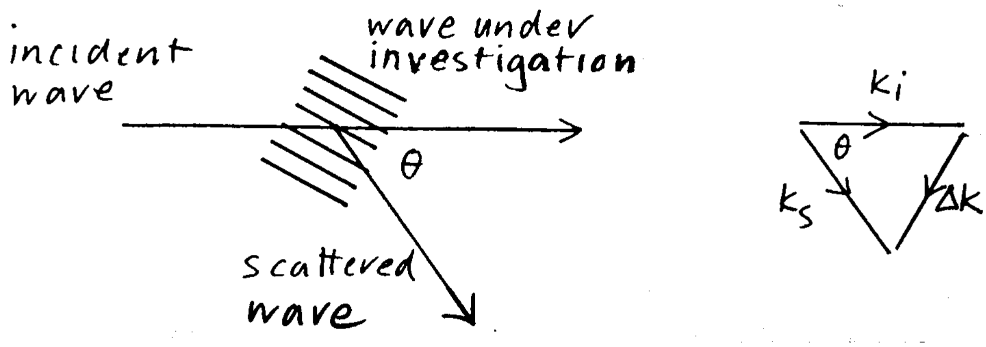
Points to note

Avoid parasitic light. But note the light of interest is at a slightly different wavelength from the incident light. Use a beam dump (a near perfect absorber).

A full treatment must include magnetic fields and relativistic effects.

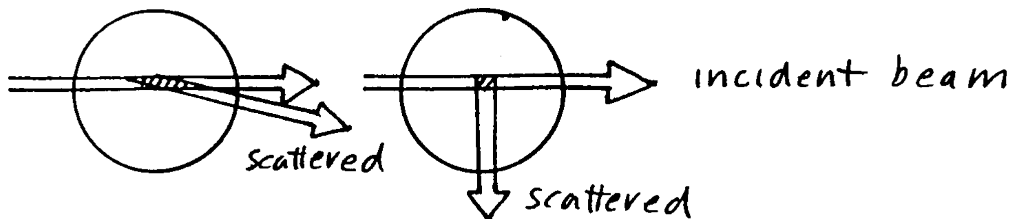
(b) Scattering from macroscopic density fluctuations

Scattering from waves, instabilities, turbulence.



Choice of frequency/wavelength. Assume you know what range of frequencies/wavelengths you want to study.

Given λ_i, the minimum $\Delta\lambda$ that can be investigated is $\lambda_i/2$ (or maximum Δk is $2k_i$).
If $\Delta\lambda$ is too large (or if Δk is too small), the scattering angle θ is small and spatial resolution is poor.

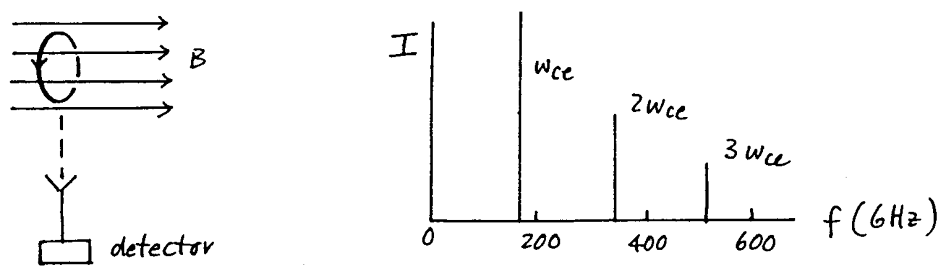


If $\Delta\omega$ is too low (or the wavelength of the waves you are studying is too large), beam bending may occur (see above),
the width of the beam which is set by diffraction may be too large and spatial resolution is poor.

You do. Find a reference to gaussian beams. What determines how the beam spreads with distance

Electron cyclotron emission

Electron cyclotron emission from a single gyrating electron is at the electron cyclotron frequency and its harmonics.



Electron cyclotron emission from the plasma.
(i) The spectrum is broadened principally because the magnetic field is not uniform.

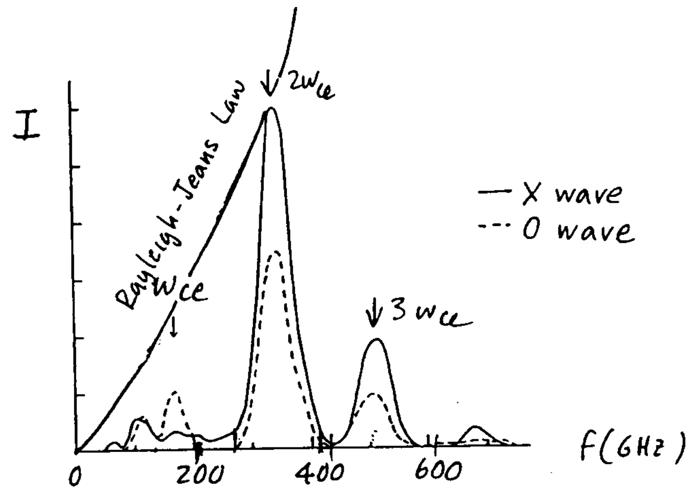
e.g., A tokamak whose major radius R = 0.54 m, minor radius a = 0.10 m and in which the magnetic field B = 6.1 T.
At the inside wall of torus B = 7.2 T, at the outside wall B = 5.0 T.

You do. Show this. (See Chapter II) What range of frequencies might we expect for each harmonic?

(ii) Intensity of the radiation.
At low frequencies, the plasma is optically thick. Any radiation is reabsorbed. The plasma is like a black-body

$$I(\omega) = I_{bb}(\omega) = \frac{\omega^2 kT_e}{8\pi^3 c^2}$$

so T_e can be estimated from the intensity.



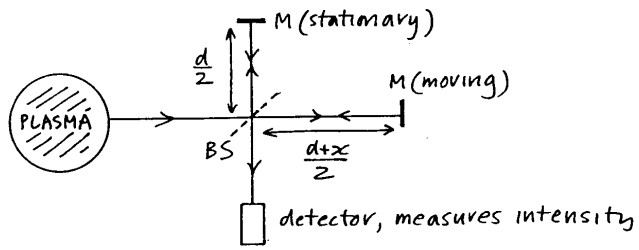
At high frequencies, the plasma is optically thin. Reflections from the walls become important.

$$I(\omega) = I_{bb}(\omega)\frac{1-e^{-\tau(\omega)}}{1-re^{-\tau(\omega)}}$$

where τ is the optical thickness and r is the reflection coefficient. Note that optical thickness cannot be deduced from the earlier cold plasma dispersion relations.

(iii) Usually observe at 90° to the magnetic field. In this direction, X waves dominate.

The spectrum of electron cyclotron emission is obtained using *Fourier transform spectroscopy*.



Suppose plasma emits a single frequency
Field at detector due to beam reflected off stationary mirror

$$E_1 = A\cos(kd-\omega t).$$

Field at detector due to beam reflected off moving mirror

$$E_2 = A\cos(k(d+x)-\omega t).$$

The output of the square-law detector is

$$\left(E_1+E_2\right)^2 = 4A^2\cos^2\left(k\frac{x}{2}\right)\cos^2\left(k\left(d+\frac{x}{2}\right)-\omega t\right)$$

$$= 4A^2\cos^2\left(k\frac{x}{2}\right)\frac{1}{2}\left(1+\cos2\left(k\left(d+\frac{x}{2}\right)-\omega t\right)\right)$$

The low-frequency part is

$$V = 2A^2\cos^2\left(k\frac{x}{2}\right) = A^2(1+\cos kx).$$

Express this as

intensity $I = S(k)(1+\cos kx)$.

The plasma emits a range of frequencies. Using a similar approach

$$\text{intensity } I(x) = \int_0^\infty S(k)(1+\cos kx)\,dk$$

$S(k)$ is the spectrum.

In a measurement, $I(x)$ is recorded for a range of x, the average value is subtracted out leaving what is called the *interferogram*

$$Int(x) = \int_0^\infty S(k)\cos kx\,dk\,.$$

This is a Fourier transform. Carry out the inverse transform to obtain the spectrum $S(k)$

$$S(k) = \frac{2}{\pi}\int_0^\infty Int(x)\cos kx\,dx\,.$$

We do not have readings for $x = 0$ to ∞, only to x_m. This means that the resolution of the instrument is limited to $\Delta k = \frac{\pi}{x_m}$.

Plasma spectroscopy

Important processes that determine populations include

1 *Radiative*, involving photons

bound-bound transitions - absorption and (its inverse process) emission of photons

$$A + h\nu \leftrightarrow A^*$$

free-bound transitions - photoionization and recombination with the emission of a photon

$$A + h\nu \leftrightarrow A^+ + e$$

2 *Collisional*, involving electrons. Since the processes involve electrons T_e is important.

bound-bound transitions - electron impact causing excitation or deexcitation

$$A + e \leftrightarrow A^* + e$$

free-bound transitions - electron impact causing ionization or three-body recombination

$$A + e \leftrightarrow A^+ + e + e$$

Two models are of particular interest.

1. *Local thermal equilibrium* (LTE)

High density, low temperature plasmas. Collisions excite and deexcite. The populations of the energy levels is given by Boltzmann distribution for temperatute T.

For two levels n and m

$$\frac{n_n}{n_m} = \frac{g_n}{g_m}\mathrm{e}^{-\frac{E_n - E_m}{kT_e}}$$

where the ns are the number densities, the gs are the statistical weights and the Es are the energies.

This can be generalized to give ratios of number densities of energy levels for atoms in different states of ionization and as a special case the Saha equation (see Chapter I).

2. *Coronal equilibrium*. Like the sun's corona.

Low density, high temperature plasmas. Collisions excite and photons deexcite.

Since collision rate depends on density, which is low, the only way for a downward transition is by spontaneous emission

A low density plasma is optically thin so any photons escape before they can excite another atom and the only way for an upward transition is by collisions.

Most plasmas lie outside LTE. The coronal model is appropriate for tokamak plasmas as long as there has been sufficient time for equilibrium to have been reached. Coronal equilibrium may apply to lower energy levels but LTE may still apply to the higher.

Line spectra

Neutral atoms and ions that still have bound electrons emit radiation whenever they make a transition from a higher energy state to a lower one. This provides a way of studying the working gas and any impurities in the plasma. However interpretation (besides simply revealing which species are present) is generally very difficult. We need to know the populations in the various possible states (complicated functions of n and T_e and the composition of the plasma) and understand the processes that maintain them.

$$A^{**} \rightarrow A^{*} + h\nu$$

Diagnostics frequently use a *monochromator* to measure the

absolute intensity of line

ratio of intensities of two lines

line shape and/or width

Line broadening

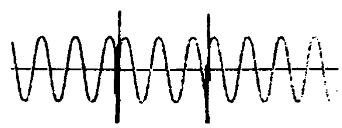
The width of spectral lines is principally due to

Doppler broadening - particle thermal motion gives a doppler shift. This would give T_a or T_i.

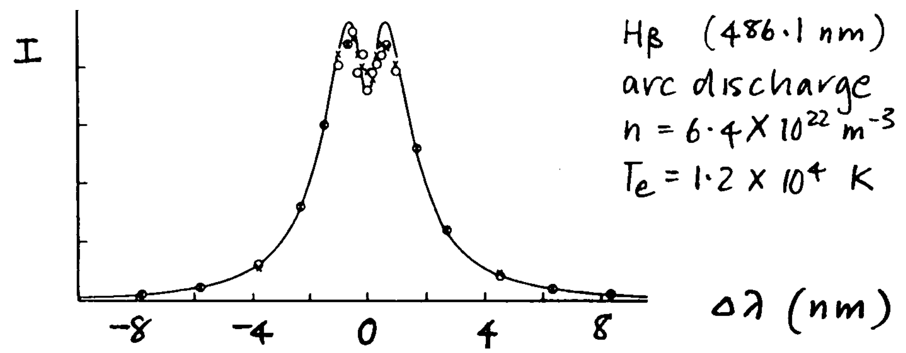
Pressure broadening which includes collisional broadening and Stark broadening.

depends on the influence of nearby particles on the emitting atom.

Collisional broadening. Most of the time the atom radiates undisturbed, occasionaly there is a collision and an abrupt phase change. $\Delta f_{FWHM} \approx \dfrac{1}{\pi\tau}$ where τ is the mean time between collisions.



Stark broadening. The most important perturbing effect is the E field of nearby atoms.



Instrumental broadening. The measuring instrument has a finite resolution.

Use convolution to combine the effects of the different kinds of broadening

Continuum spectra

Diagnostics may be based on

absolute intensity

ratio of intensities at two wavelengths

In visible, uv and x-ray.

bremsstrahlung

free-free transitions - interaction of electrons with ions of effective nuclear charge *Z*.

emissivity (radiated power /unit volume /unit angular frequency /unit solid angle)

$$j = 8.6 \times 10^{-53} n_e n_i Z^2 T_e^{-\frac{1}{2}} \, \mathrm{e}^{-\frac{\hbar\omega}{kT_e}}$$

No bremsstrahlung from non-relativistic like particle interactions.
Radiation losses by a fusion plasma are worse if the impurities make the $Z_{effective}$ higher.

recombination radiation

$$A^+ + e \to A^*$$

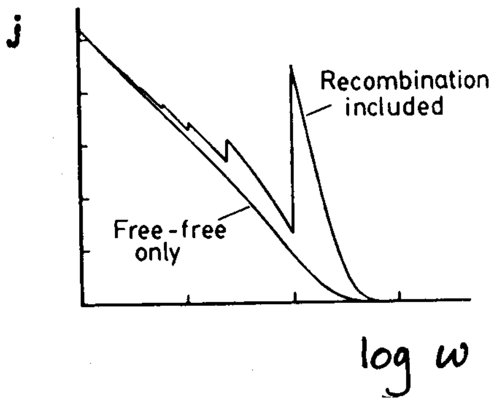
j is similar but the exponent is replaced by a series of terms with

$$G\mathrm{e}^{-\frac{\hbar\omega - \frac{Z^2}{n^2}E_1}{kT_e}}$$

where E_1 is the ionization potential of hydrogen and $G = 1$ if $\hbar\omega >> \frac{Z^2}{n^2}E_1$, otherwise 0.

Recombination radiation is less important than bremsstrahlung if $kT_e > 3E_1Z^2$.

The combined spectrum is shown below.



Note

at low frequencies $j \approx T_e^{-\frac{1}{2}}$,

the recombination edges. The highest frequency edge is when the atom ends up in the ground state after the electron is captured.

Points to note

Need to calibrate setup to find its sensitivity at the wavelengths being used.
Need to be aware of: any lines in the wavelength range, radiation from vessel walls.