

PLASMA PHYSICS

Physical Constants

magnitude of charge on electron $e = 1.60 \times 10^{-19}$ C

mass of electron $m_e = 9.11 \times 10^{-31}$ kg

mass of hydrogen ion $m_i = 1.67 \times 10^{-27}$ kg

Boltzmann's constant $k = 1.38 \times 10^{-23}$ J K⁻¹

velocity of light in vacuum $c = 3.00 \times 10^8$ m s⁻¹

permittivity of vacuum $\epsilon_0 = 8.85 \times 10^{-12}$ F m⁻¹

permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}$ H m⁻¹

1 eV = 1.60×10^{-19} J

I. INTRODUCTION

What is a plasma?

An ionized gas made up of electrons, ions and neutral particles, but electrically neutral.

The word was first used by Irving Langmuir in 1928 to describe the ionized gas in an electric discharge.

Fourth state of matter. Consider the series of phase transitions solid-liquid-gas. If we continue to increase the temperature above, say, 20 000 K (lower if there is a mechanism for ionizing the gas) we obtain a plasma.

(Note solid state physicists talk about electron-hole plasmas.)

A plasma has interesting properties because the electrostatic force is a long range force and every charged particle interacts with many of its neighbours. We can get collective behaviour. We can treat the plasma as an electrical fluid.

Examples

It has been said that 99% of matter in the universe is in the plasma state.

lightning

earth's ionosphere, aurora, earth's magnetosphere, radiation belts

interplanetary medium, solar wind

solar corona

stellar interiors

interstellar medium

laboratory plasmas such as glow discharges, arcs
fluorescent lamps, neon signs
electrical sparks
thermonuclear fusion experiments

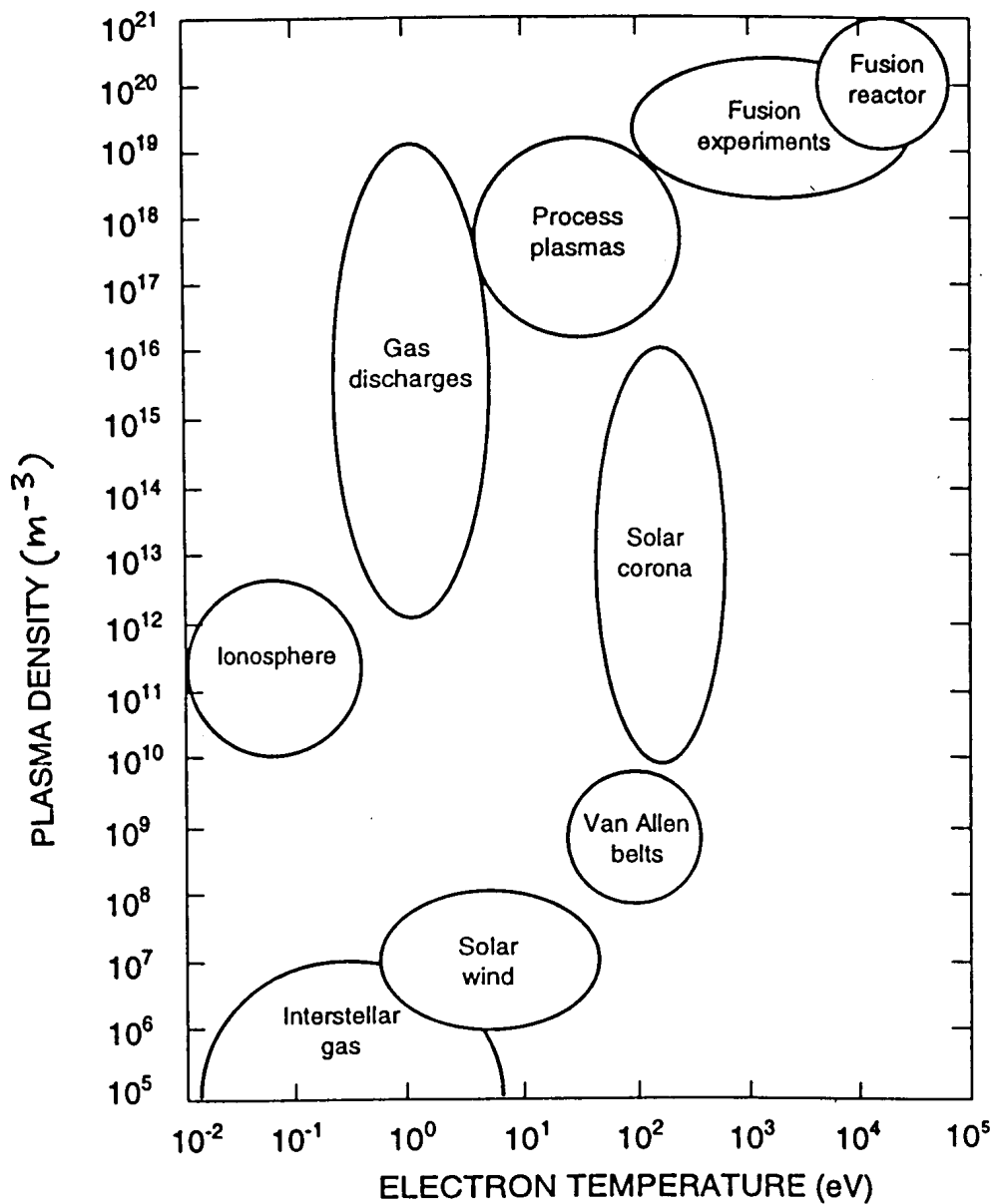
a homely examples: flame

other examples: rocket exhaust

How to characterize a plasma

plasma density n (m^{-3}) (often cm^{-3} . $1 \text{ cm}^{-3} = 10^6 \text{ m}^{-3}$)

temperature T (K or more conveniently eV)



Temperature and energy

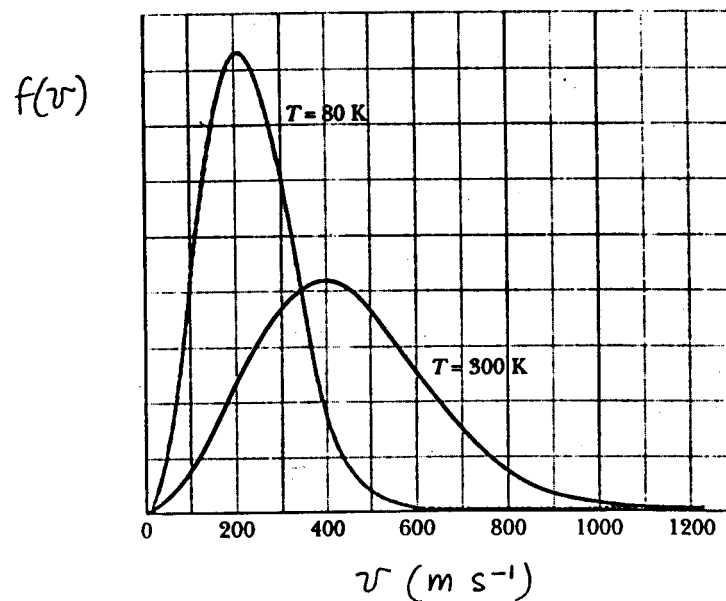
For an ideal gas in thermal equilibrium, the probability that velocity lies in the range

$$dv_x dv_y dv_z \text{ around velocity } (v_x, v_y, v_z) \text{ is proportional to } \exp\left(-\frac{\frac{1}{2}mv^2}{kT}\right) dv_x dv_y dv_z.$$

We can construct the *Maxwellian distribution* function $f(v)$ – distribution of speeds v

$$f(v) = n \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} \exp\left(-\frac{\frac{1}{2}mv^2}{kT}\right),$$

and use it to calculate average values.



e.g., Calculate the particle density

$$n = \int f(v) dv_x dv_y dv_z$$

calculate the mean velocity

$$\mathbf{u} = \frac{1}{n} \int \mathbf{v} f(v) dv_x dv_y dv_z = 0$$

calculate the mean speed

$$\bar{v} = \frac{1}{n} \int v f(v) dv_x dv_y dv_z = \sqrt{\frac{8kT}{\pi m}}$$

calculate the rms speed

$$v_{rms}^2 = \frac{1}{n} \int v^2 f(v) dv_x dv_y dv_z = \frac{3kT}{m} \text{ so } v_{rms} = \sqrt{\frac{3kT}{m}}$$

You do. Show the average kinetic energy per particle is $E_{av} = \frac{3}{2} kT$

Temperature T in K can be expressed in eV simply by calculating the energy kT in J and converting to eV.

You do. Show 1 eV corresponds to 11 600 K.

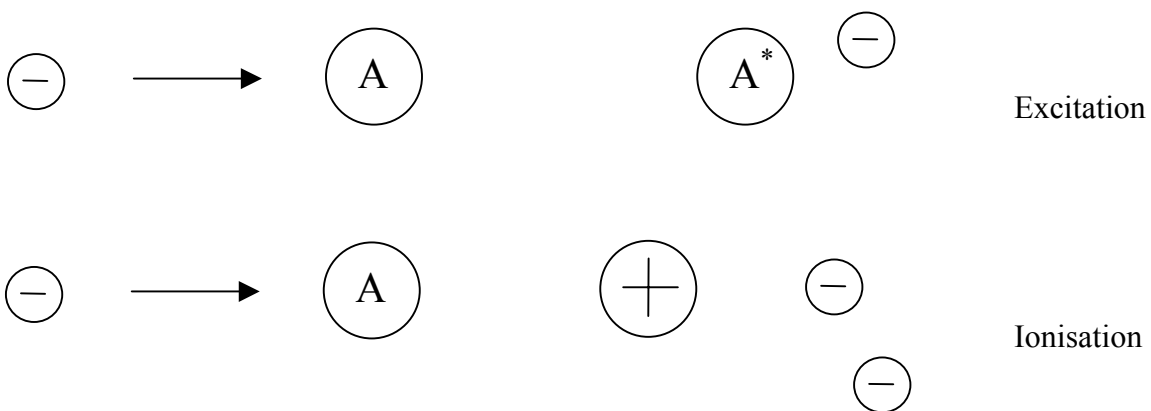
(So, by a 2 eV plasma we mean $T = 23\,200$ K. Note that $E_{av} = 3$ eV.)

Collision processes in plasma

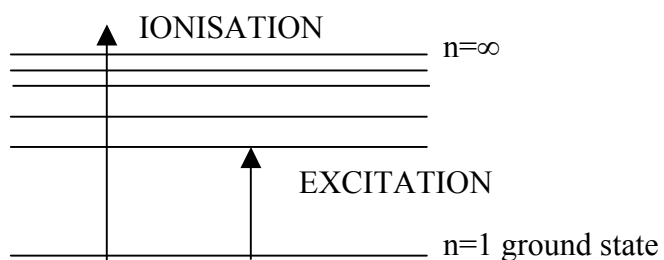
Elastic – total kinetic energy conserved

Many types: Atom-atom, atom-electron, atom-ion, electron-electron, electron-ion, ion-ion.
In some cases these can include charge exchange.

Inelastic – one particle is excited or ionised



For ion-ion collisions there is also the possibility of fusion (this requires a high energy collision).



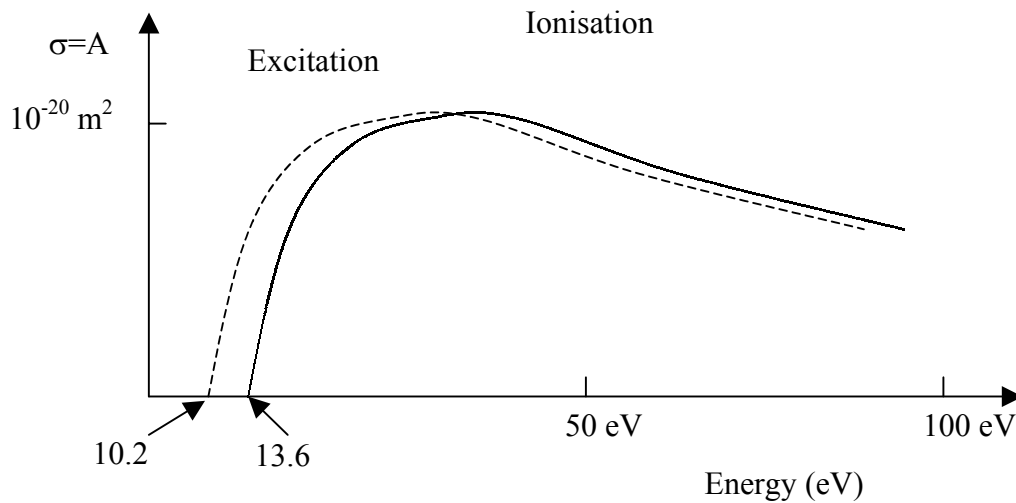
*can also occur on absorption of a photon (photo-ionisation).

For hydrogen the energy needed to excite is 10.2 eV and to ionise 13.6 eV is required.

Radiation occurs on recombination and decay back to the ground state.

Cross-section for ionisation

To ionise the collision energy must exceed ionization potential, 13.6 eV for hydrogen. The effective cross-section for ionisation varies with energy as:

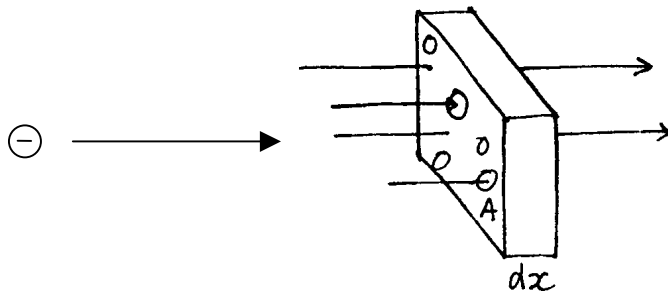


σ drops when $E > 100$ eV because the incident electron doesn't spend much time near the atom.

Knowing σ enables us to calculate the average distance between collisions ("mean free path").

Note that at $T = 1$ eV the plasma is $\sim 50\%$ ionised. The ionisation is due to collisions with electrons in the Maxwellian tail with energies greater than 13.6 eV.

Mean Free Path



Suppose electrons are incident on a slab area A , thickness dx , containing atoms, density n . Number of atoms in volume $A dx$ is $n A dx$. Total area of atoms in the volume is $(n A dx) \sigma$ assuming dx is small enough so one atom doesn't hide behind another i.e. $n A dx \sigma \ll A$.

The fraction of incident electrons having a collision is $\frac{n A dx \sigma}{A} = n \sigma dx$.

Suppose $N(x)$ enter a volume, then $N + dN$ leave, where $dN = -n \sigma dx N$.

So

$$\int_{N_0}^N \frac{dN}{N} = - \int_0^x n \sigma dx$$

$$\ln_e \frac{N}{N_0} = -n\sigma x$$

Thus $N = N_0 e^{-n\sigma x}$ or $N_0 e^{-\frac{x}{\lambda_m}}$.

λ_m is the *mean free path*, the distance for the flux to fall to 1/e of its initial value, ie. after a distance $x = \lambda_m$, $N = \frac{N_0}{e} = \frac{N_0}{2.7}$.

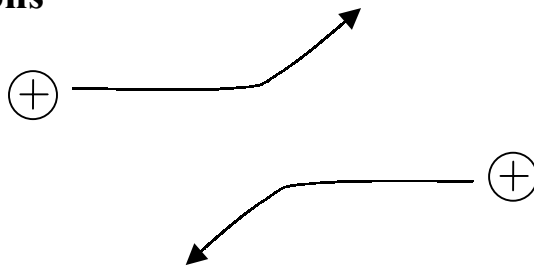
$$\lambda_m = \frac{1}{n\sigma}.$$

The *mean time between collisions* for particles with velocity v is $\tau = \frac{\lambda_m}{v}$.

The *collision frequency* $\nu = \frac{1}{\tau}$. (Strictly, we should average over the velocity distribution.)

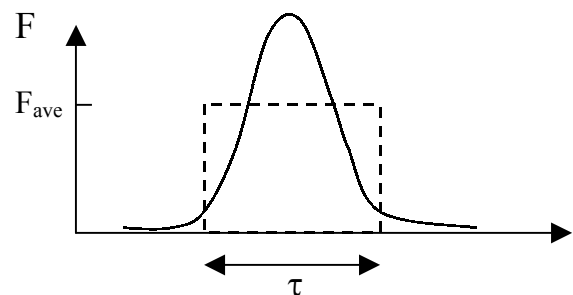
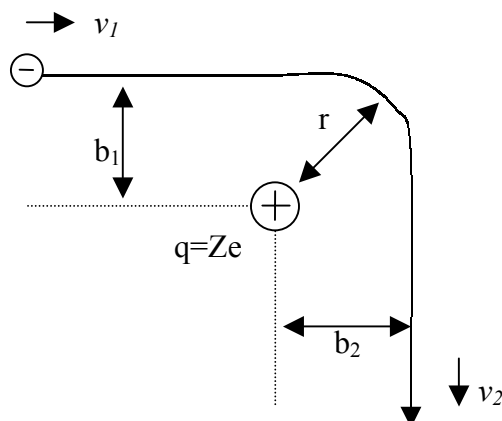
You do. Show $\lambda_m = \frac{v}{\nu}$.

Coulomb collisions



An ion nearing another will feel a repulsive force. What is the cross-section for a collision? The answer is ∞ , since no matter how far apart the ions are, they will be deflected. However we can define a cross-section for 90° scattering - *cross-section for momentum transfer*.

We can derive an *approximate* expression for the 90° scattering cross-section. The correct result is similar but the derivation is much more complicated, it takes into account the accumulated effect of many small angle collisions.



Assume ion remains at rest and the electron is incident with impact parameter, b , such that it is deflected 90° ($b_1=b_2$, $v_1=v_2$). The attractive force is given by:

$$F = \frac{Ze^2}{4\pi\epsilon_o r^2} = \frac{d}{dt}(mv)$$

F varies with time because r varies with time, so change in momentum is given by:

$$\int_0^\infty F dt = \int d(mv) = \Delta \vec{p}$$

$$\int_0^\infty F dt = F_{ave} \tau = \frac{Ze^2}{4\pi\epsilon_o (2b)^2} \frac{4b}{v} = \Delta p = \sqrt{2}mv$$

so get

$$b = \frac{Ze^2}{4\sqrt{2}\pi\epsilon_o mv^2}$$

when the integral is done properly the answer is:

$$b = \frac{Ze^2}{4\pi\epsilon_o mv^2}$$

The important point to note is that $b \propto v^{-2}$. If an electron is moving slowly, the force acts for a long time so the electron will experience a larger deflection than a fast electron at the same b .

The cross-section for deflections larger than 90° can be written as:

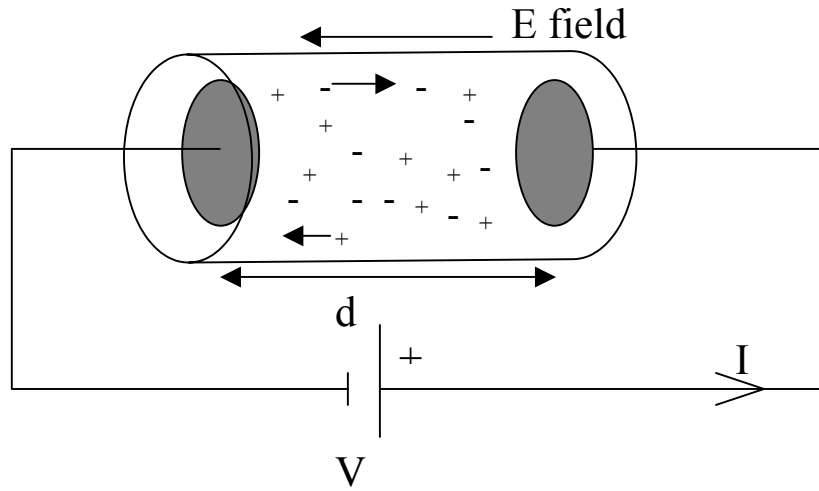
$$\sigma = \pi b^2 = \frac{Z^2 e^4}{16\pi\epsilon_o^2 m^2 v^4}$$

And so the collision frequency for Coulomb collisions of an electron with ions is:

$$\nu_{ei} = n_i \sigma v = n_i \pi b^2 v = \frac{n_i Z^2 e^4}{16\pi\epsilon_o^2 m^2 v^3}$$

Electrical Resistivity

Using v_{ei} we can estimate the electrical resistivity of a plasma column.



Resistance of the plasma is $R=V/I$ with $R=\eta \frac{d}{A}$ where η is the plasma resistivity.

If A is the cross-sectional area of the plasma perpendicular to the plane of the electrodes then

$j = \frac{I}{A}$ is the current density and $E = \frac{V}{d}$ is the electric field strength in the plasma.

So we can write Ohm's law as:

$$R = \eta \frac{d}{A} = \frac{V}{I} = \frac{Ed}{jA} \quad \text{or} \quad \boxed{E = \eta j} \quad (\text{dimensionless})$$

The current is determined by the charge passing any point in the circuit per unit time.

So the ion current can be written as: $I_i = n_i q_i v_i A$

which means that the current density due to ions is $j_i = n_i q_i v_i$, and similarly

$$I_e = -n_e e v_e A \quad \text{so that} \quad j_e = -n_e e v_e$$

and thus the total current density is $j = n_i q_i v_i - n_e e v_e$

(note: the v_i and v_e are averages over the acceleration-collision cycle)

Because the plasma is quasi-neutral, if $q_i = Z_i e$ then $n_e = Z_i n_i$ so

$$\vec{j} = n_e e (\vec{v}_i - \vec{v}_e) \quad [1]$$

An electron loses all of its forward momentum with respect to the ion, in the collision at the end of each acceleration-collision cycle. Thus the force on it must be large enough to provide that momentum each cycle. That is:

$$F = -eE = m_e v_{ei} (\vec{v}_i - \vec{v}_e) \quad [2]$$

(if E is positive v_e is negative)

Combining [1] and [2] gives the following expression for resistivity:

$$\boxed{\eta = \frac{E}{j} = \frac{m_e v_{ei}}{n_e e^2}}$$

If you use the expression we derived previously for v_{ei} you will see that the resistivity is **independent of n_e !!!!!** This is quite different to the situation with other types of conductors.

Now use $\frac{1}{2} m_e v_e^2 = \frac{3}{2} k T_e$ to express the resistivity in terms of the electron temperature.

$$m_e v_e^3 = \frac{(3kT_e)^{\frac{3}{2}}}{\sqrt{m_e}}$$

$$\eta = \frac{Z_i e^2 \sqrt{m_e}}{16\pi\epsilon_0^2 (3kT_e)^{\frac{3}{2}}} = 23.3 \frac{Z_i}{T_e^{\frac{3}{2}}} \quad (\Omega m)$$

Note that as plasma gets hotter, the resistivity falls so we need a larger current to maintain the temperature or heat the plasma. This is a major problem for controlled fusion.

If we do the calculation properly, taking into account the Maxwell distribution and cumulative effects of small angle collisions and electron-electron collisions, we get the **Spitzer resistivity**

$$\boxed{\eta = \frac{65.3 (\ln \Lambda) Z_i}{T_e^{\frac{3}{2}}} \quad (\Omega m)}$$

where $\ln_e \Lambda$ (known as the Coulomb logarithm) is given by $\Lambda = 1.239 \times 10^7 \left(\frac{T_e^3}{n_e} \right)^{1/2}$ with n_e in m^{-3} and T in $^\circ\text{K}$. For nearly all plasmas of interest, $\ln_e \Lambda \sim 10$ (usually between 8 and 12).

Formation of a plasma

Ionization at high temperatures

We have said that a sufficiently hot gas becomes a plasma.

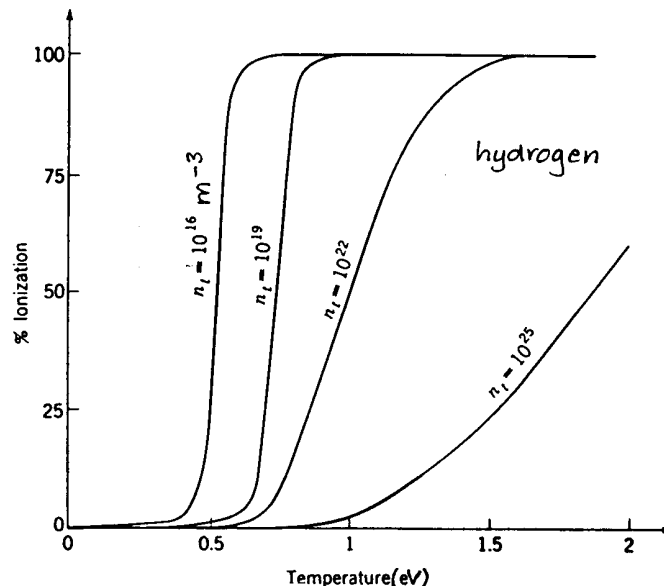
Atoms in a gas have a spread in thermal energy and they collide with each other. Sometimes there is a collision with high enough energy to knock an electron out of the atom and ionize it. Energy must exceed ionization potential, 13.6 eV for hydrogen.

In a cold gas such collisions are very infrequent, in a hot gas more likely.

From the Maxwellian velocity distribution function we can derive the *Saha equation* which gives the fraction of ionization we can expect in a gas in thermal equilibrium at temperature T ,

$$\frac{n_i}{n_n} \cong 3 \times 10^{27} \frac{T^{\frac{3}{2}}}{n_i} \exp\left(-\frac{U_i}{T}\right)$$

where n_i is the ion density, n_n is the neutral particle density and U_i is the ionization energy. (In this expression U_i and T are both in eV.)



Note: There is significant ionization below 13.6 eV.

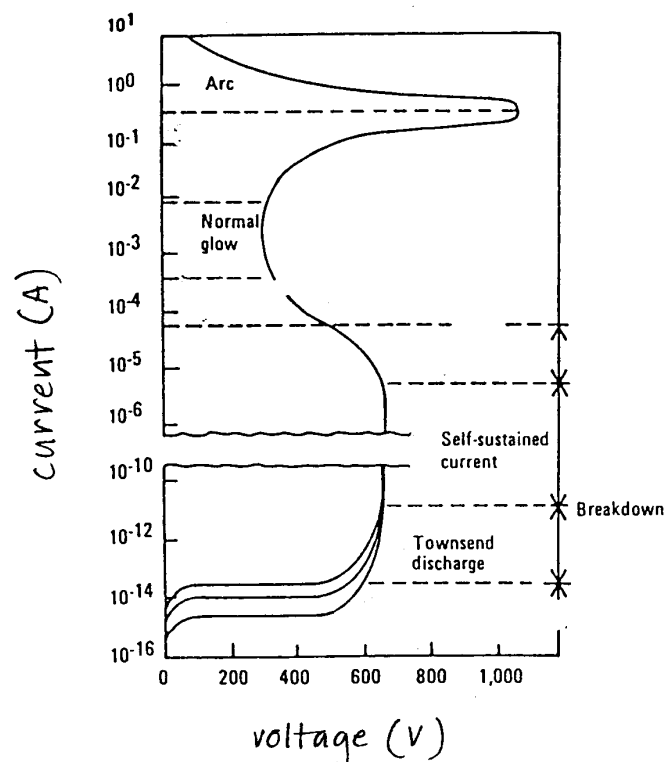
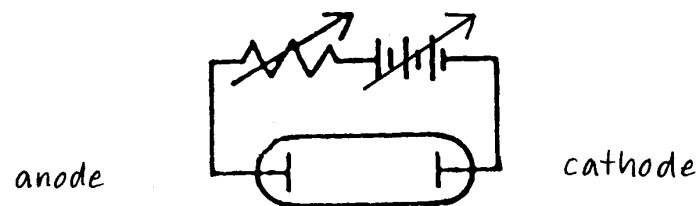
Ionization in an electric field, gas discharges

Another way of achieving ionization.

The term *discharge* was first used when a capacitor was discharged across the gap between two electrodes placed close to each other. If the voltage is sufficiently high, electric breakdown of the air occurs. The air is ionized and the conducting path closes the circuit and a current can flow. Later the term was applied to any situation where a gas was ionized by an electric field and a current flowed.

Discharge may give off light.

The simplest gas discharge is a glass tube with a metal electrode sealed into each end. The tube is evacuated and filled with various gases at different pressures. The electrodes are connected to a dc supply.



Raise the voltage.

(i) Low voltage (10s of volts)

no visible effect.

very small currents ($\approx 10^{-15}$ A), ionization by cosmic rays and natural radioactivity.

The discharge is called *non-self-sustaining* as an external ionizing agent is required.

increase voltage and a saturation current is reached when all the charges are collected.

Townsend discharge.

(ii) Increase voltage

breakdown occurs. e.g., if the gap is 10 mm and the pressure is 1 torr then this happens at 400 V, if 1 atmosphere then 30 kV.

current increases by several orders of magnitude, but voltage does not change.

discharge becomes independent of an external ionizing source; it is *self-sustaining*.

ionization is caused by electrons colliding with atoms.

This is one of the most important mechanism in an electric discharge. So we will examine it in some detail.

Electrons are accelerated by the electric field and gain energy.

They collide with atoms. If their energy is small, the collision will be *elastic* and they will lose only a small fraction $\approx m/M$ of their energy in the encounter.

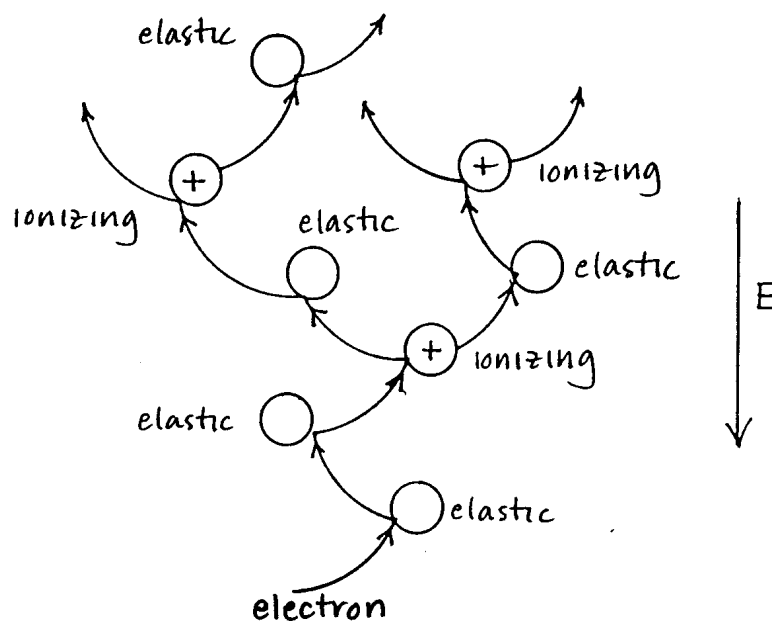
After the collision they will gain more energy from the field.

Their energy increases until it is large enough that the collision is *inelastic*; the atom is excited or ionized. For ionization, the electron energy must exceed the ionization potential of the atom.

(Note that positive ions lose a large fraction of their energy in each elastic collision and it is much more unlikely that their energy will increase sufficiently to ionize.)

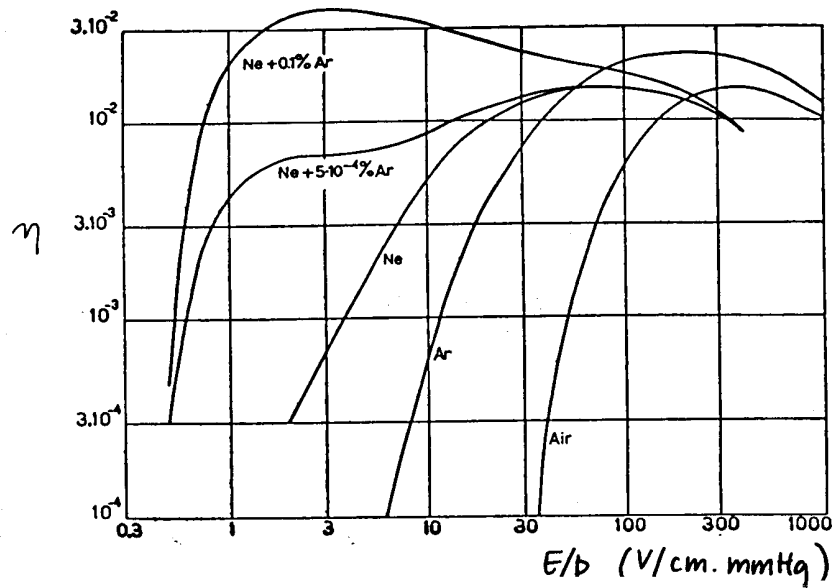
You do. Estimate fractions of energy lost by collisions between (a) electrons with atoms and (b) ions with atoms.

After an ionizing collision the second electron is then available to ionize. There is an avalanche effect.



the gas is appreciably ionized in μs to ms .

We can plot *Townsend's ionization coefficient*, η (in V^{-1}), the number of ionizing collisions caused by an electron as it falls through a p.d. of 1 V



Note the units. 1 V/cm. mmHg or $1 \text{ V/cm. torr} = 0.75 \text{ V m}^{-1} \text{ Pa}^{-1}$.

$$\eta = a \frac{p}{E} \exp\left(-b \sqrt{\frac{p}{E}}\right)$$

η depends on E/p (a common parameter in gas discharge work – it allows scaling), the gas.

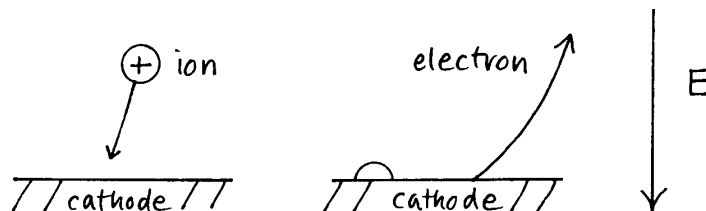
η is low at low pressures because an electron encounters hardly any atoms.

η is low at high pressures because elastic collisions are more frequent and it is more difficult for the electron to gain sufficient energy to ionize.

The figure shows the *Penning effect* in a gas mixture where argon is ionized by metastable excited neon atoms.

Photoionization is not important in this case but might provide the initial ionization to start things off.

The second important process is positive ions bombarding the cathode and knocking off electrons.



This is described by γ , the *secondary ionization coefficient*, defined as the number of electrons knocked off the cathode by a single positive ion.

γ depends on E/p , the gas and the cathode material (and the state of the cathode surface, whether it is a pure metal or has an oxide layer).

Consideration of these two processes allow us to estimate the p.d. for breakdown.

Suppose the p.d. between the electrodes is V .

One electron leaving the cathode becomes $e^{\eta V}$ electrons arriving at the anode.

And $e^{\eta V} - 1$ positive ions heading back towards the cathode.

One ion produces γ electrons.

So one electron gives rise to $\gamma (e^{\eta V} - 1)$ electrons.

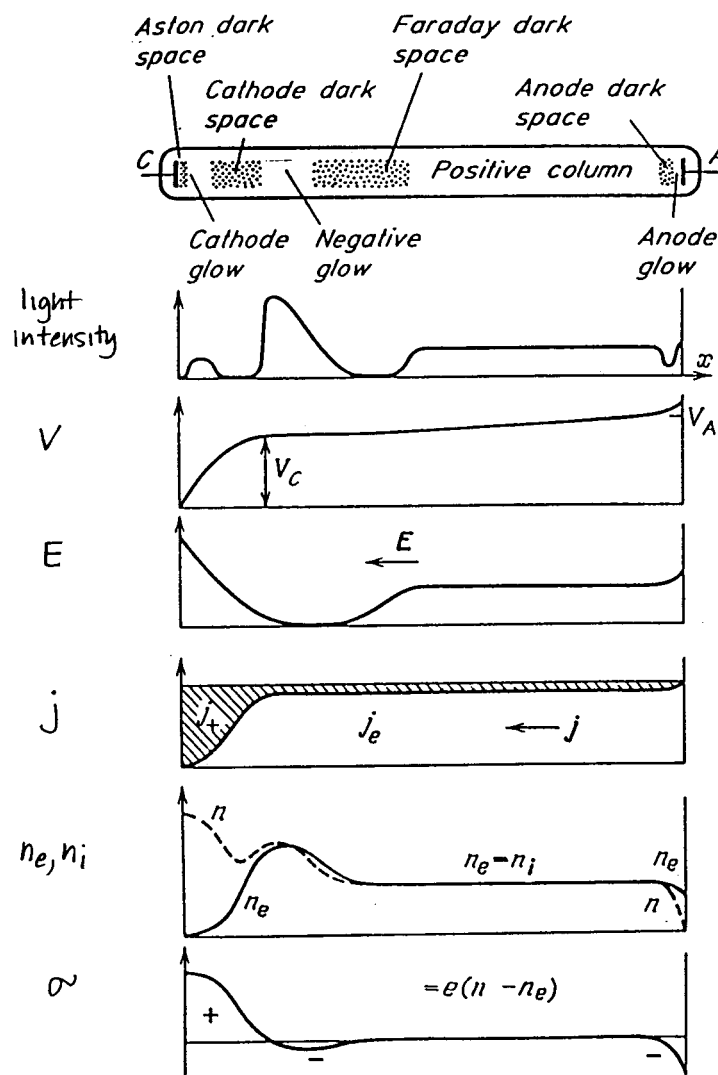
For breakdown, this must be ≥ 1 .

(iii) Now increase current

if the tube is long can get a beautiful radiant column, *glow discharge*.

voltage 100s of volts, current milliamperes.

discharge is maintained by positive ions bombarding the cathode and knocking off electrons.



the ion and electron densities are equal only in the positive column. This plasma is weakly-ionized, fraction ionized is 10^{-8} to 10^{-6} , T_e is 10^4 K but T_i and T_n are 300 K.

Increase pressure to 100 torr, positive column becomes longer and thinner.

Increase electrode distance, higher voltage required, positive column longer to occupy the extra length.

Increase current, cathode glow covers more of the cathode surface so the current density and the voltage remain fairly constant.

Different gases yield different colours. “neon” signs

(iv) Suppose the pressure is high and any series resistance is low

arc discharge

voltage can be low 10 V, current > 1 A

fraction ionized is 10^{-3} to 10^{-1} , T_e and T_i are 10^4 K

types of arcs

thermionic arc - emission of electrons is due to cathode being heated by the large current of ions bombarding it. Cathode must withstand very high temperature, e.g., carbon, tungsten. This arc is self-sustaining.

(Emission of electrons from a hot surface is described by

$$j = aT^2 \exp\left(-\frac{e\phi}{kT}\right)$$

where ϕ is the work function.)

e.g., carbon and tungsten arc lamps

thermionic arc with cathode heated by external source - non-self-sustaining.

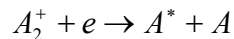
field emission arc - emission of electrons is due to very high E at cathode.
e.g., mercury arc lamp, mercury arc rectifier.

metal arc - heating the cathode vapourizes the metal.

high-pressure arc $p > 1$ atm; *low pressure arc* < 1 atm.

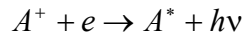
(v) Processes of deionization

Dissociative recombination



is the fastest recombination process in a weakly-ionized gas like a glow discharge

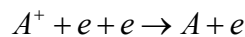
Radiative recombination



is not important for electron removal but may be important for light emission.

Diffusion to wall is slower in a well-developed discharge.

Three-body electron-ion recombination



is main process in high density, low temperature laboratory plasmas.

Introduction to the major applications for plasma: Fusion and Processing Discharges

Fusion is the nuclear reaction that powers the sun. The discipline of Plasma Physics was developed as a result of our efforts to realise fusion as an energy source on Earth. These efforts began in the 1950's. Although the program hasn't resulted in a fusion reactor in the time frame originally proposed, considerable progress in our understanding of the behaviour of plasmas has been made. The fusion program continues with negotiations to build the biggest test reactor ever, ITER, in a major collaboration involving the European Union, Canada and Japan. The use of powerful lasers for plasma confinement is also being investigated in Inertial Confinement projects.

The 1970's saw the development of laboratory discharges for materials deposition and surface modification. This has become a very broad and important field with direct applications in many industries, such as micro-electronics, optics, machining and more recently the biomedical industry.

It is important to realise that although both these fields depend on plasma physics, the types of plasmas studied and the phenomena of primary interest are quite different. In fusion, high temperature and high density plasmas are required. To achieve this combination of high temperature and density it is necessary to develop very good methods of plasma confinement. Plasma waves and instabilities are of great concern because they lead to major losses of plasma and energy. In contrast, the materials processing field is more concerned with the interactions of the plasma with solid surfaces in contact with the plasma. Sheaths and wall boundaries are closely studied. Although high densities are beneficial in increasing processing speed, the plasmas used in materials processing are generally cold (i.e. the ions have energies corresponding to a gas at room temperature and the electrons have energies of a few eV).