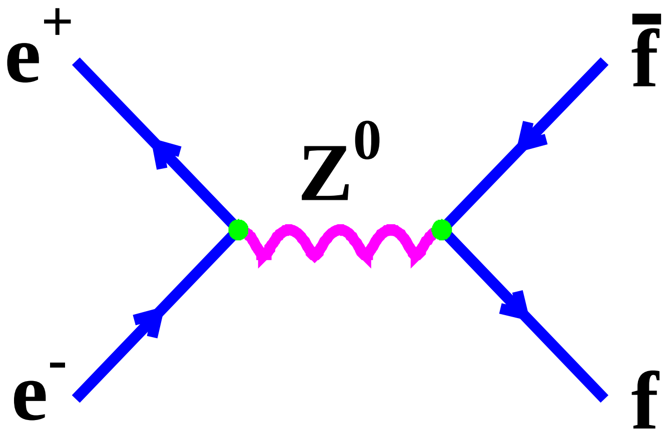
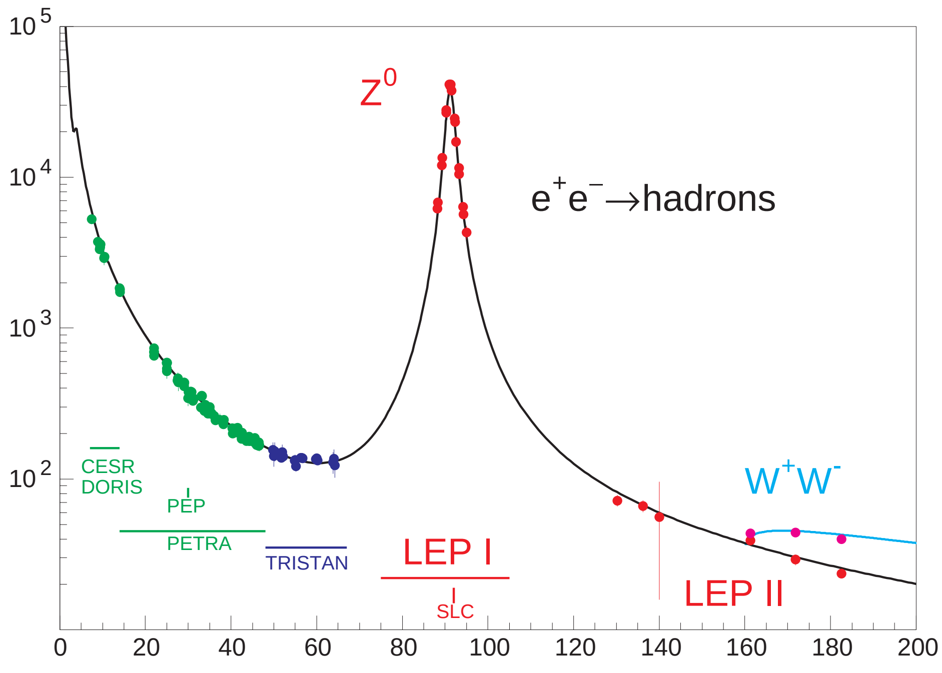
Particle Physics

Dr M.A. Thomson





Part II, Lent Term 2004

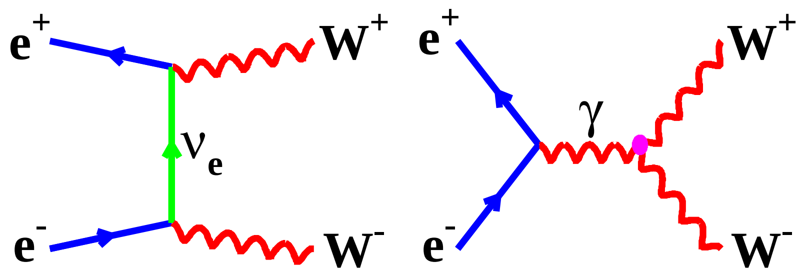
HANDOUT VII

ELECTROWEAK UNIFICATION

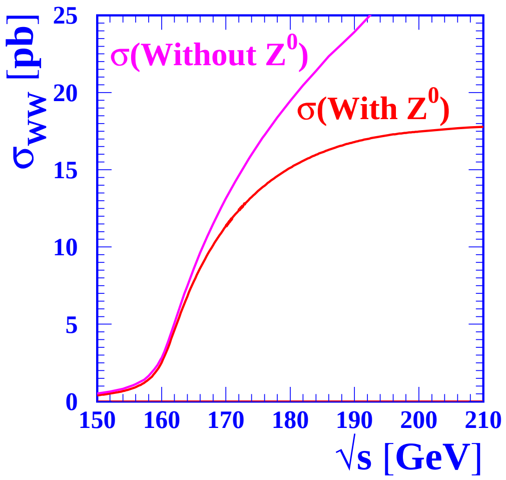
- ★ WEAK Charged Current interactions explained by $\mathrm{W}^{\pm}$ exchange.
- ★ W-bosons are charged $\therefore$ couple to photon.

Consider $\mathrm{e}^{+}\mathrm{e}^{-} \rightarrow \mathrm{W}^{+}\mathrm{W}^{-}$

2 Diagrams (+interference)



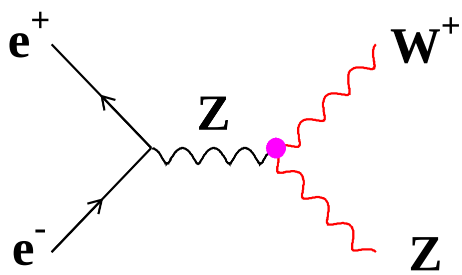
- ★ Cross section diverges at high energy



Divergence CURED by introducing Z^{0}

Extra Diagram

$e^{+}e^{-} \rightarrow \mathrm{W}^{+}\mathrm{W}^{-}$



- ★ Only works if γ, $\mathrm{W}^{\pm}$, Z^{0} couplings related !
- ★ ELECTRO-WEAK UNIFICATION

Electroweak Unification

Glashow (1961), Weinberg (1967) and Salam (1968) model treats EM and WEAK interactions as different manifestations of a UNIFIED force.

- ★ It is somewhat *ad hoc*
- ★ But gives concrete predictions - i.e. a testable theory
- ★ provides perfect description of precise data

"Basic idea" - start with 4 massless bosons, $\{W^+, W^0, W^-\}$ and B^0. The neutral bosons mix to give physical BOSONs, (the particles we see), *i.e.* the $W^\pm$, Z^0 and γ.

$$\begin{pmatrix} W^+ \\ W^0 \\ W^- \end{pmatrix}, B \rightarrow \begin{pmatrix} W^+ \\ Z^0 \\ W^- \end{pmatrix}, \gamma$$

Physical Fields : W^+, W^+ Z^0, A (photon)

$$\begin{aligned} \mathrm{Z}^0 &= W^0 \cos\theta_W - B\sin\theta_W \\ A &= W^0 \sin\theta_W + B\cos\theta_W \end{aligned}$$

θ_W : weak mixing angle

$\mathrm{W}^\pm$ and Z^0 'acquire' mass via the HIGGS MECHANISM

The beauty of the model is that it makes exact predictions:

- ★ Weak coupling constant: $e = g\sin\theta_w$
- ★ The mass of the Z^0 boson

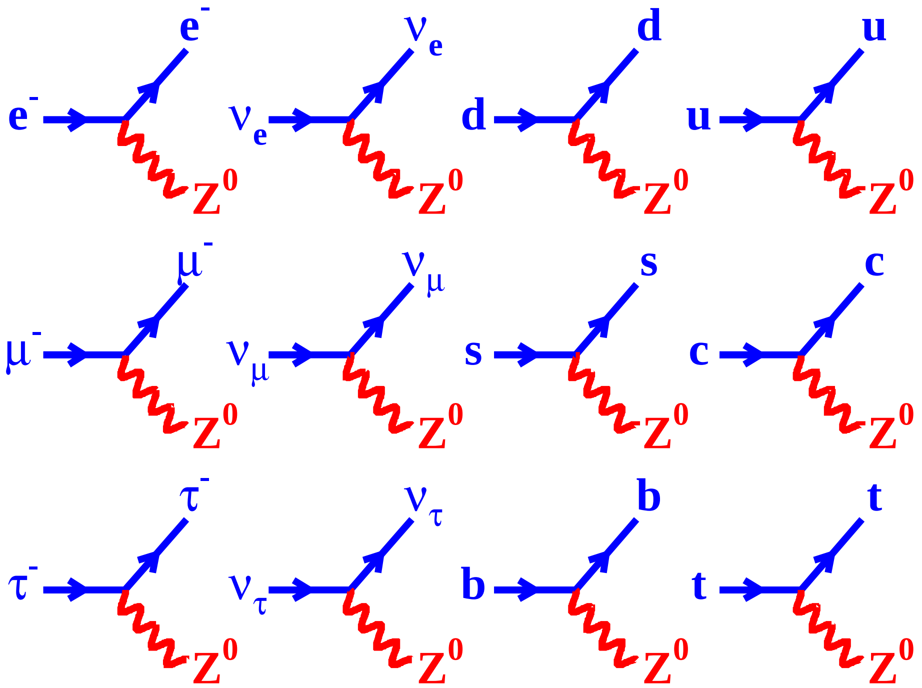
$$M_{\mathrm{Z}^0} = \frac{M_{\mathrm{W}}}{\cos\theta_W}$$

- ★ The couplings of the Z^0 boson
- ★ ONLY 3 free parameters !

IF we know $\{\alpha_{em}, G_{\mathrm{F}}, \sin\theta_W\}$ everything else is FIXED, i.e. predict M_{W}, M_{Z^0}, couplings, etc.

Z^0 Neutral Current

★ WEAK **Neutral Current** (NC) interactions are mediated by the Z^0 boson.



★ WEAK NC NEVER changes flavour

★ Z^0 couplings are a "MIXTURE" of weak and electro-magnetic couplings

★ WEAK NC couplings therefore depend on $\sin^2\theta_W$

Z^0 couplings are a mixture of EM (VECTOR) and WEAK (VECTOR—AXIAL-VECTOR) couplings

$$\frac{g}{\cos\theta_W}\frac{1}{2}\gamma^\mu(C_V - C_A\gamma^5)$$

Form of neutral current couplings are determined by the WEAK MIXING ANGLE θ_W

ELECTROWEAK CHARGES : NON-EXAMINABLE

EM γ: Charge $Qe = Qg\sin\theta_W$

Fermion	Q	$2C_V$	$2C_A$
ν_e, μ_ν, μ_τ	0	0	0
e^-, μ^-, τ^-	-1	-1	0
u, c, t	$+\frac{2}{3}$	$+\frac{2}{3}$	0
d, s, b	$-\frac{1}{3}$	$-\frac{1}{3}$	0

WEAK CC $W^\pm$: Charge : $g/\sqrt{2}$

Fermion	I_3	$2C_V$	$2C_A$
ν_e, μ_ν, μ_τ	$+\frac{1}{2}$	1	1
e^-, μ^-, τ^-	$-\frac{1}{2}$	1	1
u, c, t	$+\frac{1}{2}$	1	1
d, s, b	$-\frac{1}{2}$	1	1

WEAK NC Z^0: Charge $g/\cos\theta_W$

$$C_V = (I_3 - 2\sin^2\theta_W Q)$$
$$C_A = I_3$$

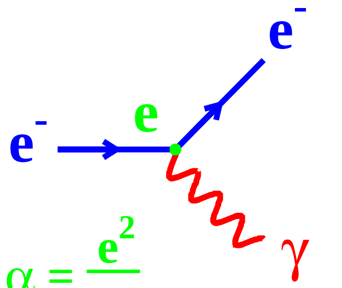
Fermion	Q	I_3	$2C_V$	$2C_A$
ν_e,μ_ν,μ_τ	0	$+\frac{1}{2}$	+1	+1
e^-,μ^-,τ^-	-1	$-\frac{1}{2}$	$-1+4\sin^2\theta_W$	-1
u, c, t	$+\frac{2}{3}$	$+\frac{2}{3}$	$+1-\frac{8}{3}\sin^2\theta_W$	+1
d, s, b	$-\frac{1}{3}$	$-\frac{2}{3}$	$-1+\frac{4}{3}\sin^2\theta_W$	-1

Summary of Standard Model Vertices

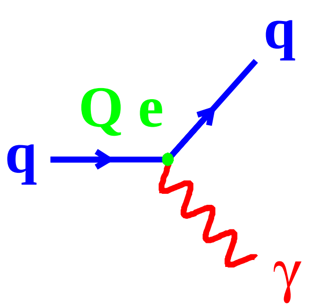
★ At this point have discussed all fundamental fermions and their interactions with the force carrying bosons.

★ Interactions characterized by SM vertices

ELECTROMAGNETIC (QED)



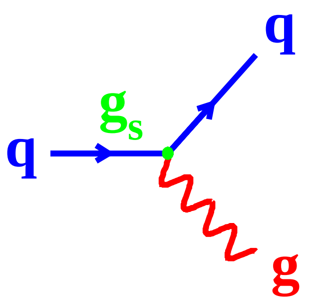
$$\alpha = \frac{e^2}{4\pi}$$



Couples to CHARGE

Does NOT change FLAVOUR

STRONG (QCD)

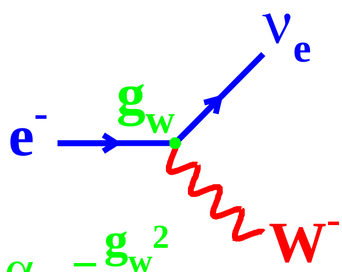


$$\alpha_s = \frac{g_s^2}{4\pi}$$

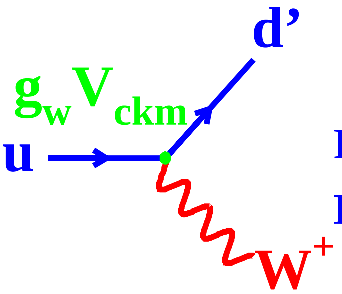
Couples to COLOUR

Does NOT change FLAVOUR

WEAK Charged Current



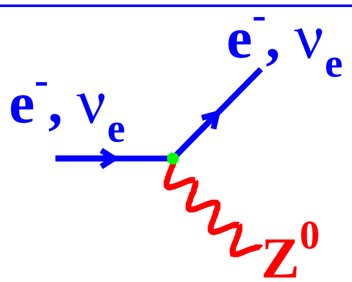
$$\alpha_W = \frac{g_W^2}{4\pi}$$

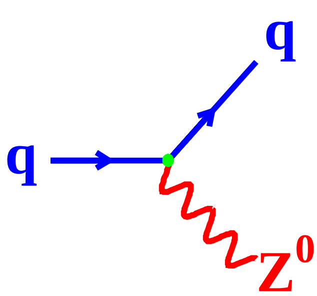


Changes FLAVOUR

For QUARKS: coupling BETWEEN generations

WEAK Neutral Current





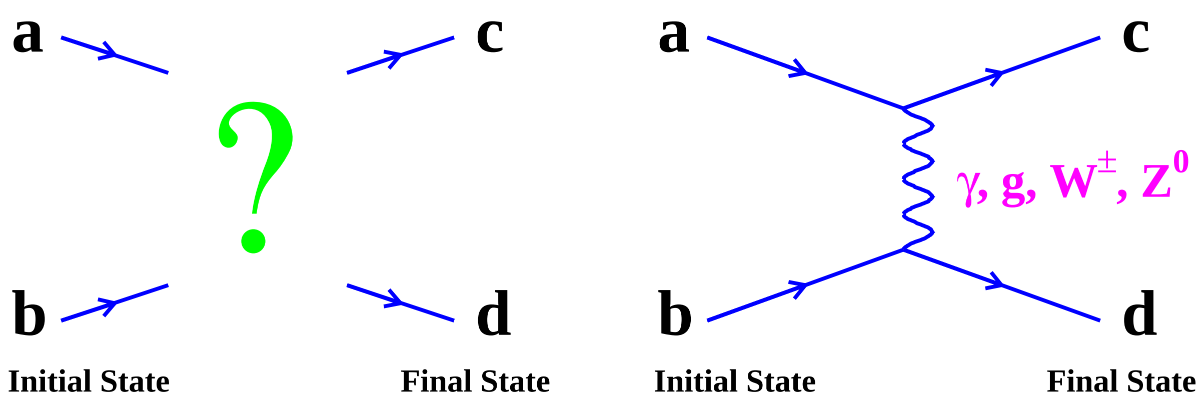
Does NOT change FLAVOUR

Drawing Feynman Diagrams

If all are particles (or all anti-particles) *e.g.*

$$a + b \to c + d$$

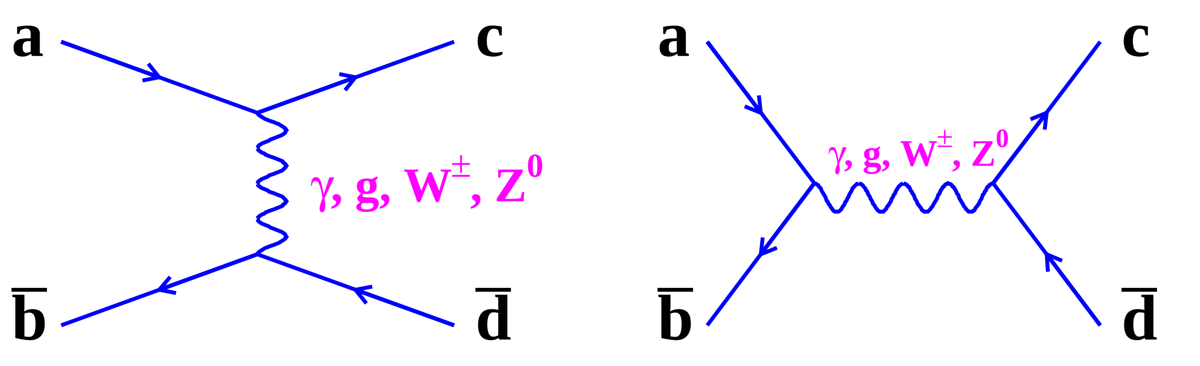
- ★ **First write down initial/final state particles/anti-particles.**
- ★ **Only scattering diagrams involved**
- ★ **Work out which bosons can be exchanged**



If particles and anti-particles *e.g.*

$$a + \overline{b} \to c + \overline{d}$$

- ★ **First write down initial/final state particles/anti-particles.**
- ★ **Can have scattering and/or annihilation diagrams**
- ★ **Again work out which bosons can be exchanged**



- ★ **In all cases only Standard Model vertices allowed.**
- ★ **Try to keep things as simple as possible.**

Does it go ?

or “How to Determine if a process is allowed”

❶ Draw **SIMPLEST** Feynman diagram using Standard Model vertices. Bearing in mind:

- ★ Similar diagrams for particles/anti-particles
- ★ NEVER have a vertex connecting a LEPTON to a QUARK
  - conservation of lepton number
  - conservation of baryon number
- ★ Only the WEAK CC vertex changes FLAVOUR
  - within generations for leptons
  - within/between generations for quarks

❷ Conservation of

- ★ Energy - is it kinematically allowed
- ★ Charge
- ★ Angular Momentum

❸ Parity

- ★ Conserved in EM/STRONG interactions
- ★ CAN be violated in CC and NC WEAK interactions

❹ Check SYMMETRY for IDENTICAL particles in the final state

- ★ BOSONS : $\psi(1,2) = +\psi(2,1)$
- ★ FERMIONS : $\psi(1,2) = -\psi(2,1)$

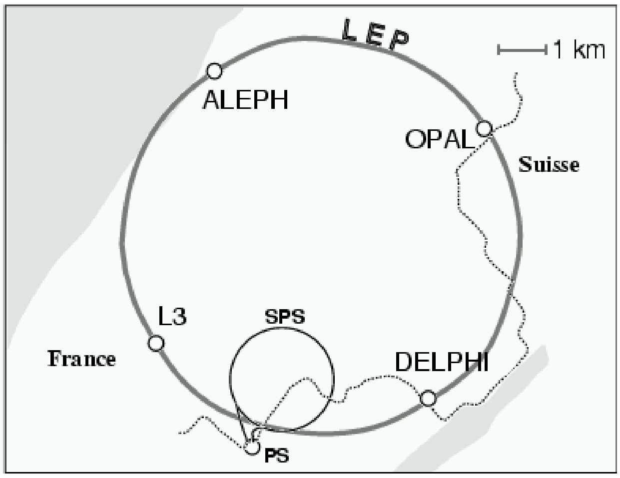
(see Questions 14 and 15 on the problem sheet)

Experimental Tests of the Standard Model

★ The idea of electroweak unification under-pins the modern view of particle physics

★ From 1989-2000 experimental measurements at CERN provided precise tests of the Standard Model

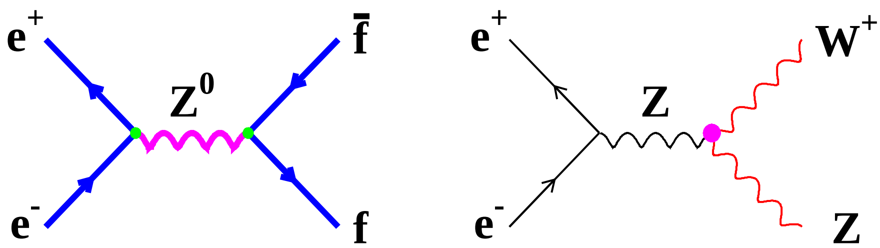
Large Electron Positron Collider



★ Highest energy e^+e^- collider ever built
$\sqrt{s} = 90 - 200\ \mathrm{GeV}$

★ Large = 26 km circumference

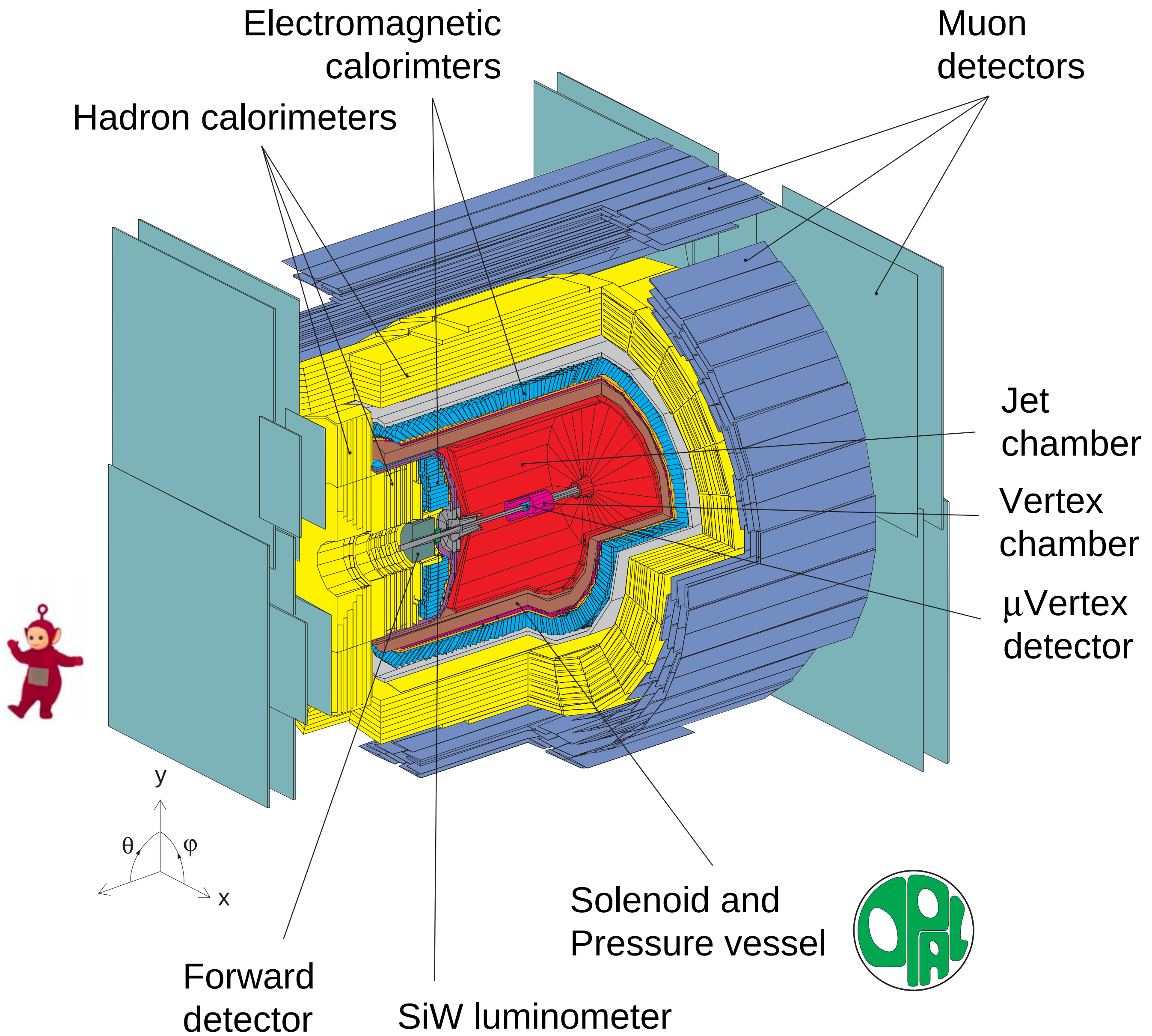
★ Designed as a Z^0 and $W^\pm$ Boson Factory

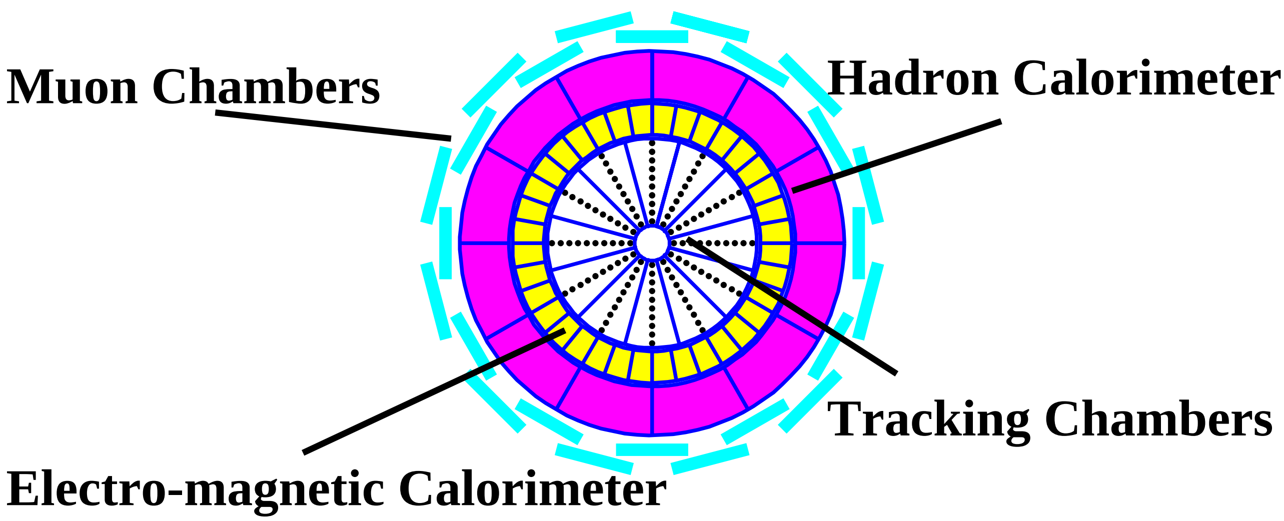


★ Four experiments combined: 16,000,000 Z^0 and 30,000 W^+W^- events

★ Precise measurements of the properties of Z^0 and W bosons provide the most stringent test of our current understanding of particle physics

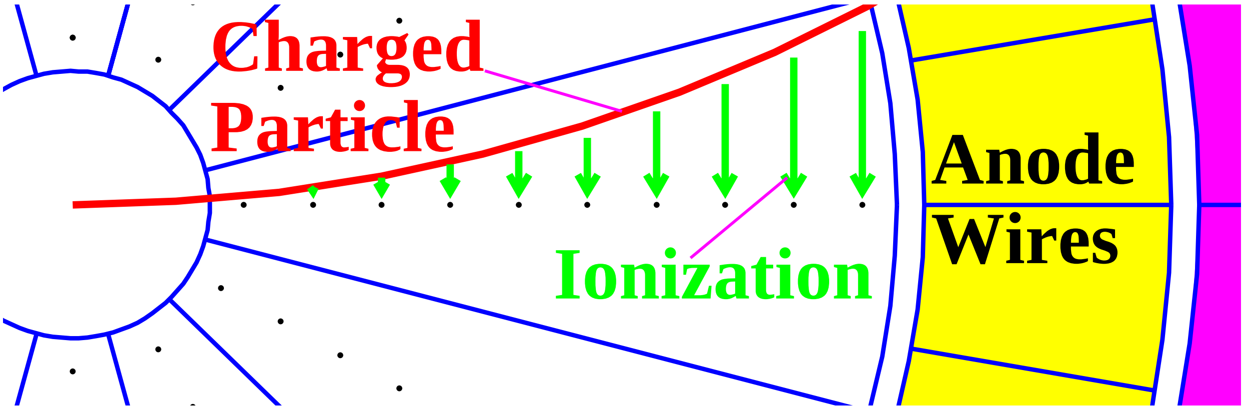
A LEP Detector : OPAL





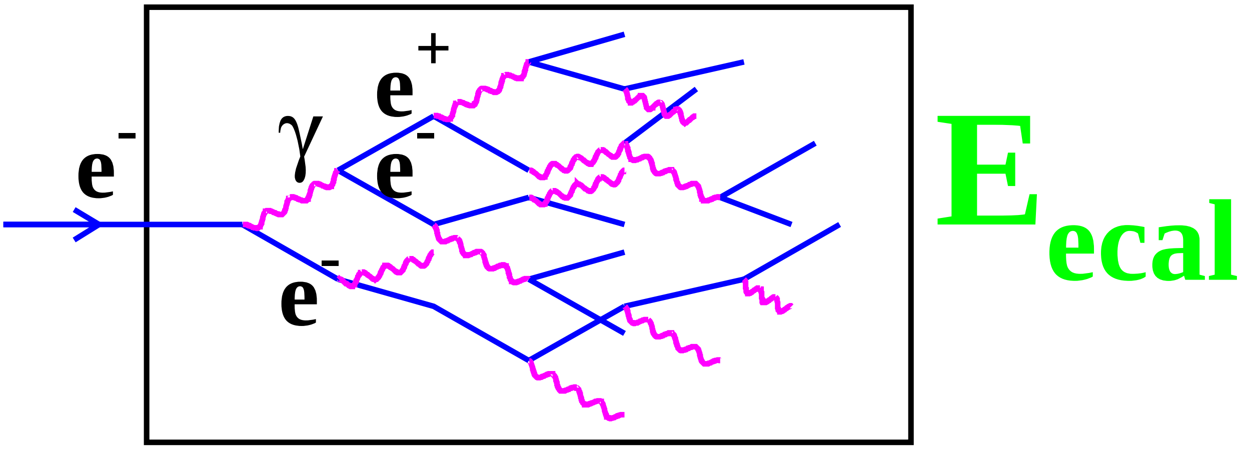
Particle Detection

TRACKING



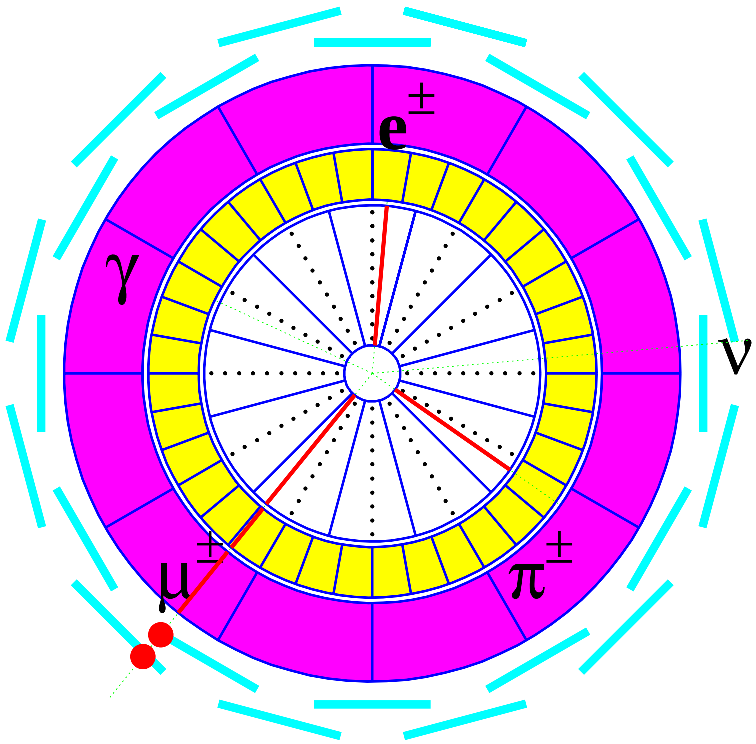
- 0.437 T Axial Magnetic Field
- Track curvature $\propto 1/p_\perp$
- $(\sigma_{p_\perp}/p_\perp)^2 = 0.02^2 + 0.0015^2.(p_\perp/GeV)^2$

ELECTRO-MAGNETIC CALORIMETRY



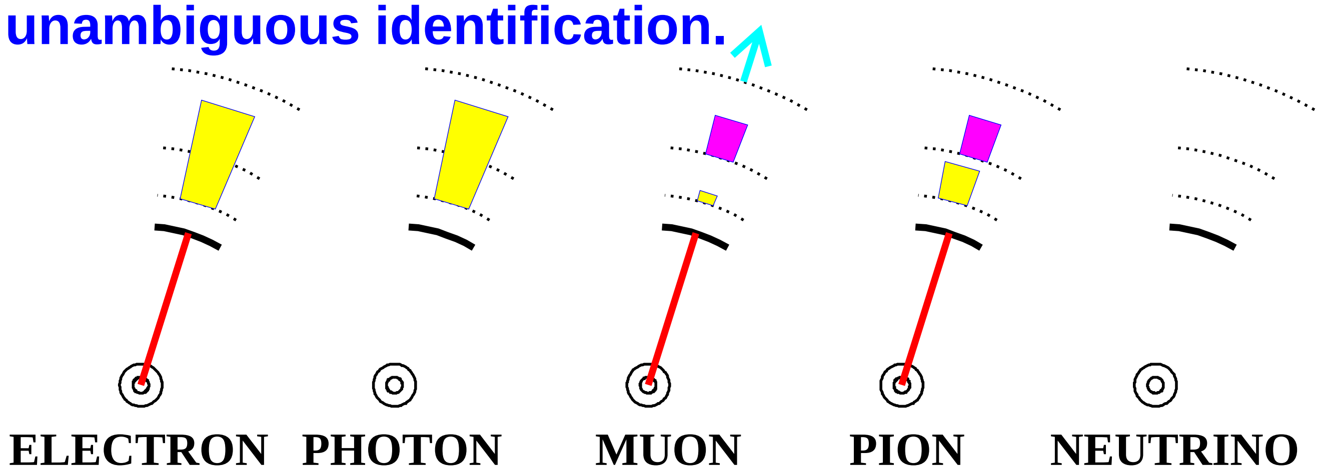
- ECAL = 11705 Lead-Glass blocks
- ECAL coverage $\sim 98\ \%\ 4\pi$
- ECAL Energy Resolution ~ 3 % at 45 GeV

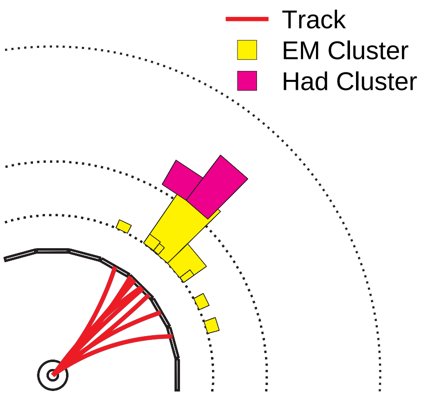
Particle Identification



- ★ **PHOTONS** energy in ECAL, no track
- ★ **ELECTRONS** energy in ECAL and track
- ★ **MUONS** track, little energy deposit and penetrate to outer muon chambers
- ★ **TAUS** decay - observe decay products
- ★ **QUARKS** seen as jets
- ★ **NEUTRINOS** not detected

Different particles leave different signals in the various detector components allowing almost unambiguous identification.

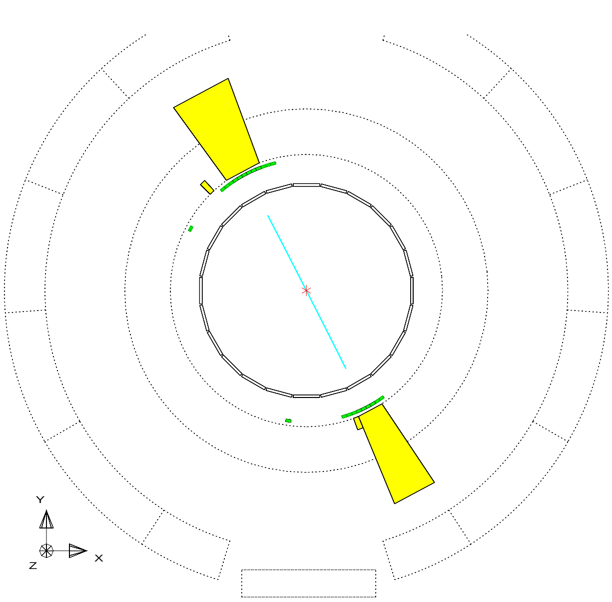




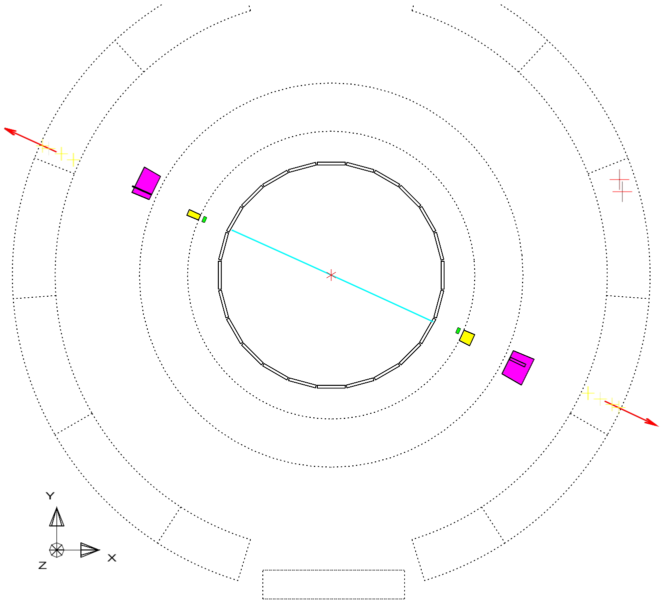
Quarks, which appear as jets of hadrons, look very different to leptons which (usually) appear as a single track

Typical $e^+e^- \to Z^0$ Events

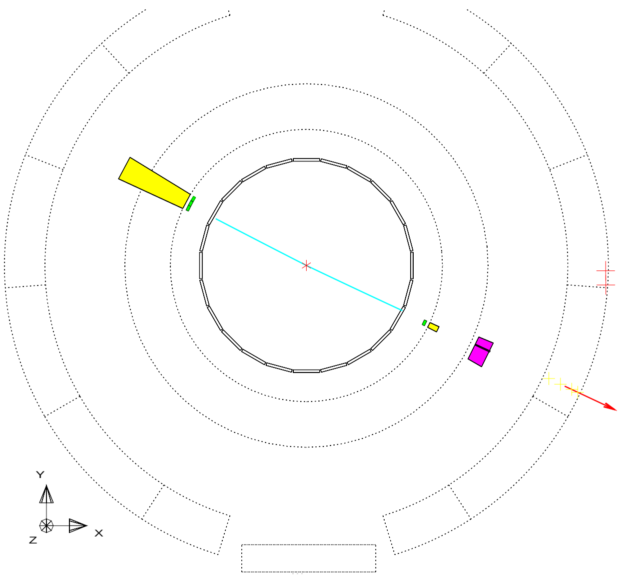
$e^+e^- \to Z^0 \to e^+e^-$



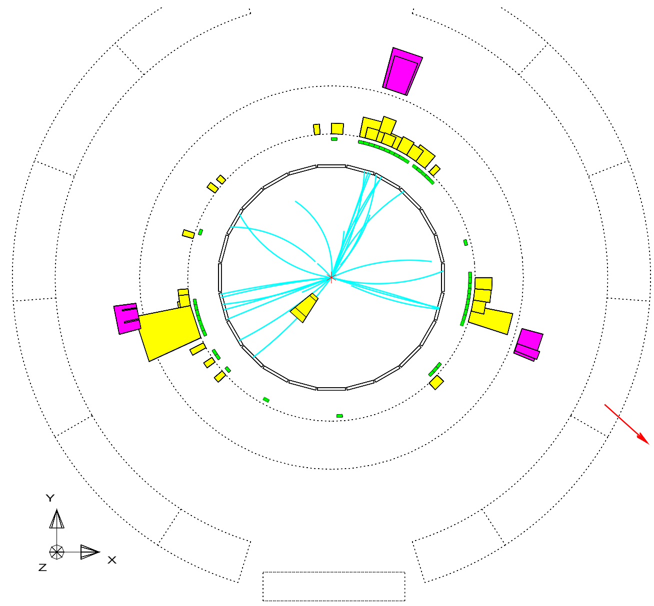
$e^+e^- \to Z^0 \to \mu^+\mu^-$



$e^+e^- \to Z^0 \to \tau^+\tau^-$



$e^+e^- \to Z^0 \to q\overline{q}$



In $e^+e^- \to Z^0 \to \tau^+\tau^-$ event, the tau leptons decay within the detector (lifetime $\sim 10^{-13}$ s), here $\tau^- \to e^-\overline{\nu}_e\nu_\tau$ and $\tau^+ \to \mu^+\nu_\mu\overline{\nu}_\tau$.

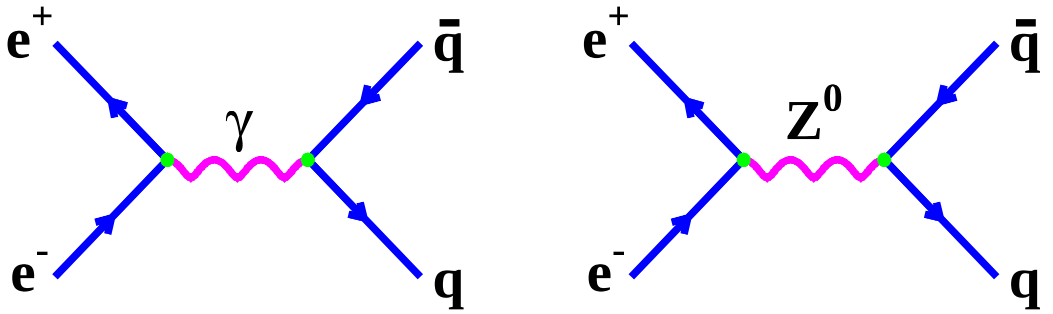
(see Question 16 on the problem sheet)

The Z^0 Resonance

Consider the process of $e^+e^- \rightarrow q\bar{q}$

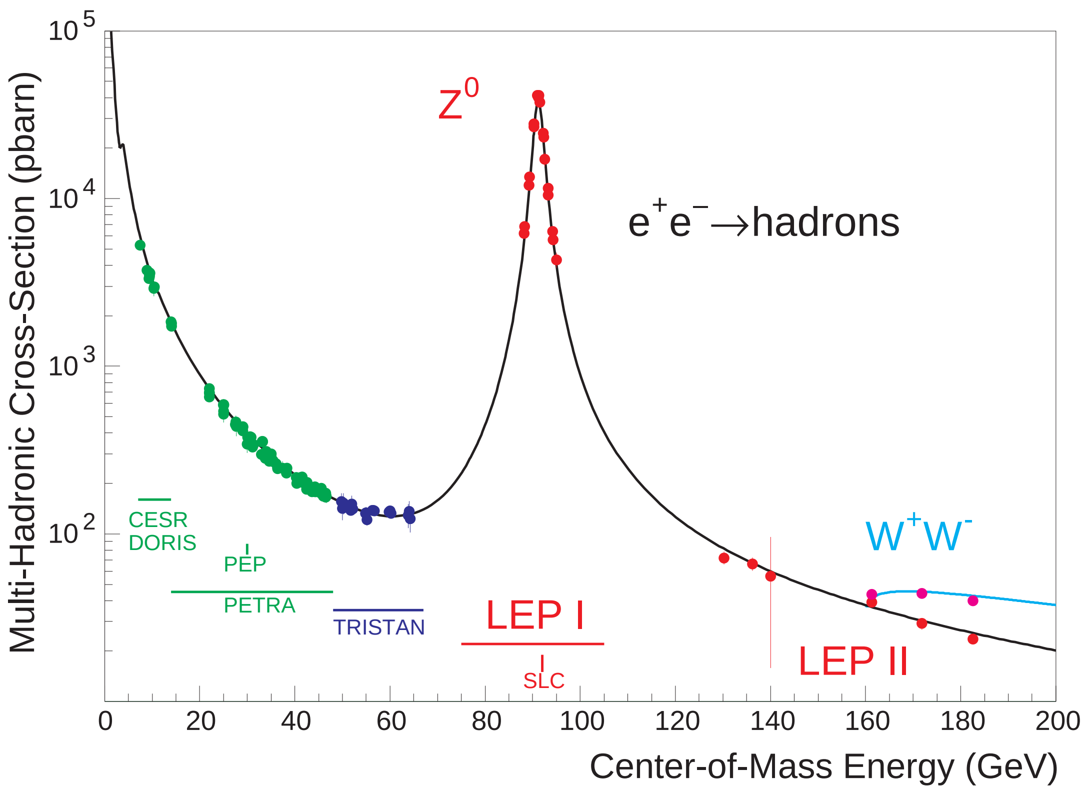
★ Previously, $\sqrt{s} < 50\ \mathrm{GeV}$ only considered an intermediate photon.

★ At higher energies also have the Z^0 exchange diagram (plus $Z^0\gamma$ interference).



★ The Z^0 is a decaying intermediate massive state (lifetime $\sim 10^{-25}$ s)

$\therefore$ BREIT-WIGNER resonance



★ At $\sqrt{s} \sim M_{Z^0}$ the Z^0 diagram dominates.

BREIT-WIGNER formula for $e^+e^- \to Z^0 \to f\bar{f}$ (where $f\bar{f}$ is any fermion-antifermion pair)

centre-of-mass energy $\sqrt{s} = E_{\rm CM} = E_{e^+} + E_{e^-}$

$$\sigma(e^+e^- \to Z^0 \to f\bar{f}) = g\frac{\pi}{E_e^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(E_{\rm CM} - M_Z)^2 + \Gamma_Z^2/4}$$

$$\text{with} \quad g = \frac{2J_Z + 1}{(2S_{e^+} + 1)(2S_{e^-} + 1)}$$

giving

$$\sigma(e^+e^- \to Z^0 \to f\bar{f}) = \frac{3\pi}{4E_e^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(\sqrt{s} - M_Z)^2 + \Gamma_Z^2/4}$$

$$= \frac{3\pi}{\sqrt{s}}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(\sqrt{s} - M_Z)^2 + \Gamma_Z^2/4}$$

★ Γ_Z is the TOTAL DECAY WIDTH, *i.e.* the sum over the partial widths for the different decay modes.

$$\boxed{\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + \Gamma_{\nu\bar{\nu}}}$$

At peak of the resonance $\sqrt{s} = M_Z$

$$\sigma(e^+e^- \to Z^0 \to f\bar{f}) = \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

NOTE: There are a number of equivalent forms sometimes quoted in the textbooks, *e.g.*

$$\frac{12\pi M_Z^2}{\sqrt{s}}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$

In the limit that $\Gamma \ll M_Z$ these are all equivalent.

(see Question 12 on the problem sheet)

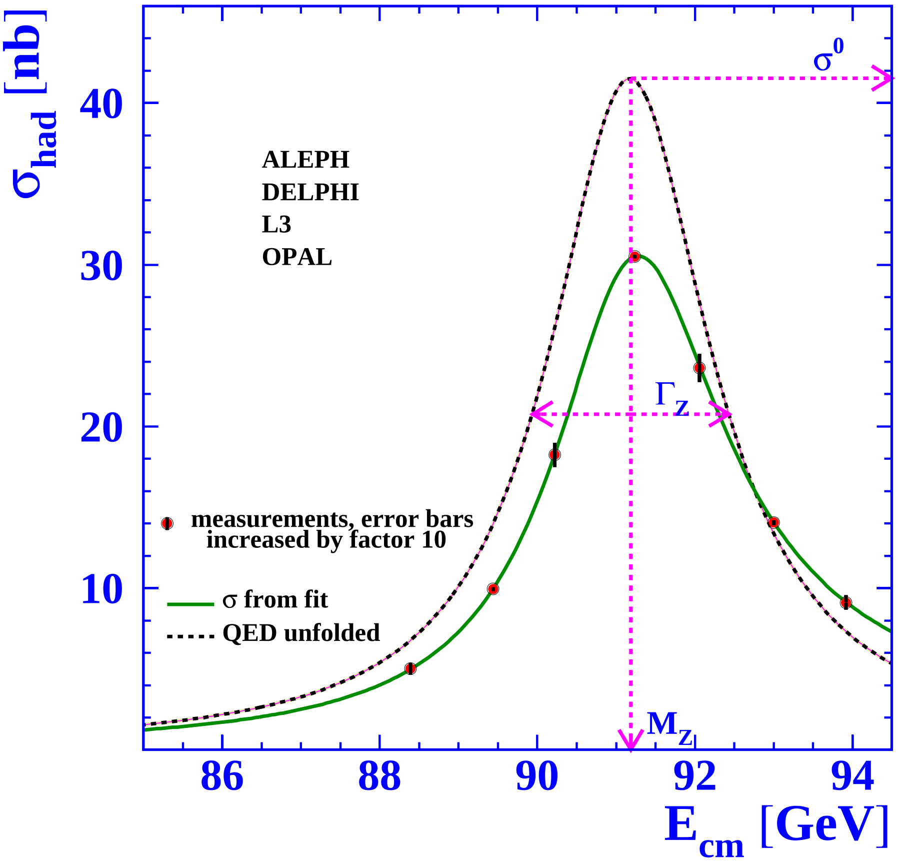
Measurement of M_Z and Γ_Z

- ★ Run accelerator at various centre-of-mass energies ($\sqrt{s}$) close to the peak of the Z^0 resonance and measure $\sigma(e^+e^- \to q\overline{q})$
- ★ Determine the parameters of the resonance.

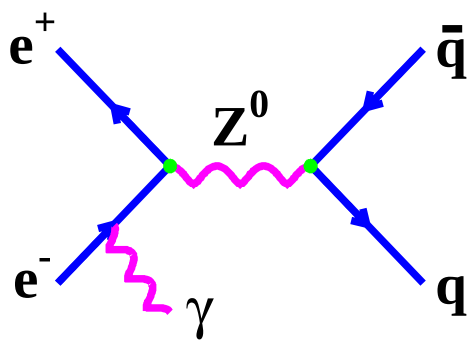
  Mass of the Z^0, M_Z

  Total decay width, Γ_Z

  Peak cross section, σ_0



One subtle feature : the measurements have to be corrected for well-known QED effects due to radiation from the e^+e^- beams. This radiation has the effect of reducing the centre-of-mass energy of the e^+e^- collision which smears out the resonance.

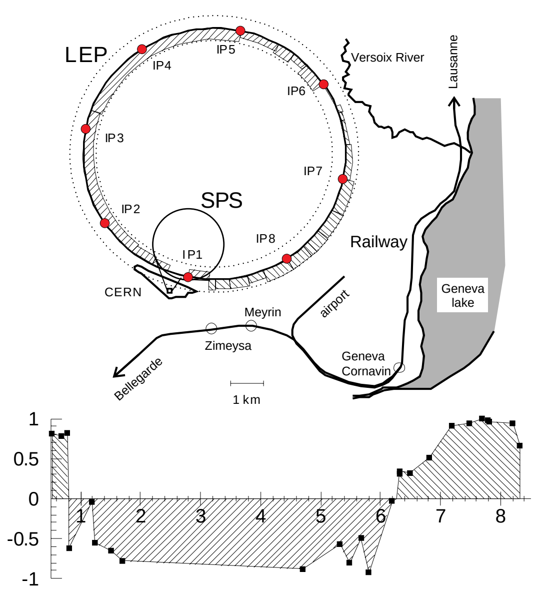


(see Question 11 on the problem sheet)

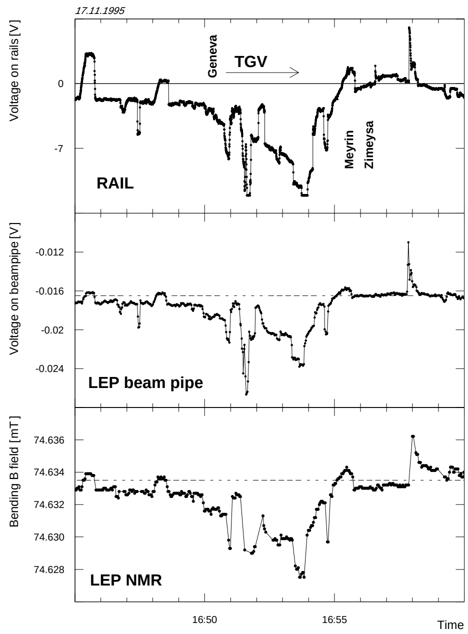
M_Z measured with precision 2 parts in 10^5

★ To achieve this required a detailed understanding of the accelerator and astrophysics ! Tidal distortions of the earth by the moon cause the rock surrounding the accelerated to be distorted. The nominal radius of LEP changes by 0.15 mm compared to radius of 4.3 km. This is enough to change the centre-of-mass energy !

★ Also need a train timetable ! Leakage currents from the TGV rail via lake Geneva follow the path of least resistance... using LEP as a conductor.



Correlation versus IP



Accounting for these effects (and many others):

$$\boxed{M_Z = 91.1875 \pm 0.0021\ \mathrm{GeV}}$$

★ An incredible achievement and powerful test of our understanding of the Standard Model of particle physics.

Shape of measured Breit-Wigner distribution also gives:

$$\Gamma_Z = 2.4952 \pm 0.0023\ \mathrm{GeV}$$
$$\sigma^0_{q\bar{q}} = 41.540 \pm 0.037\ \text{nb}$$

Number of Generations

★ So far only discussed 3 generations of fermions, *e.g.* $\{e^-, \mu^-, \tau^-\}$

★ What about a possible fourth generation ?

$$\begin{pmatrix} e^- & d \\ \nu_e & u \end{pmatrix}, \begin{pmatrix} \mu^- & s \\ \nu_\mu & c \end{pmatrix}, \begin{pmatrix} \tau^- & b \\ \nu_\tau & t \end{pmatrix}, +?$$

★ The Z^0 boson couples to ALL fermions, including neutrinos. Therefore the total decay width, Γ_Z has contributions from all fermions $m_f < M_Z/2$

$$\begin{aligned} \Gamma_Z &= \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{qq} + \Gamma_{\nu\overline{\nu}} \\ \text{with } \Gamma_{\nu\overline{\nu}} &= \Gamma_{\nu_e\overline{\nu}_e} + \Gamma_{\nu_\mu\overline{\nu}_\mu} + \Gamma_{\nu_\tau\overline{\nu}_\tau} \end{aligned}$$

★ If there were an additional generation, it seems likely that the fourth generation neutrino would be light and, if so, would be produced at LEP, $\mathrm{e^+e^-} \rightarrow \mathrm{Z}^0 \rightarrow \nu\overline{\nu}$

★ Wouldn't observe the neutrinos directly, but could infer their presence from the effect on the Z^0 resonance curve

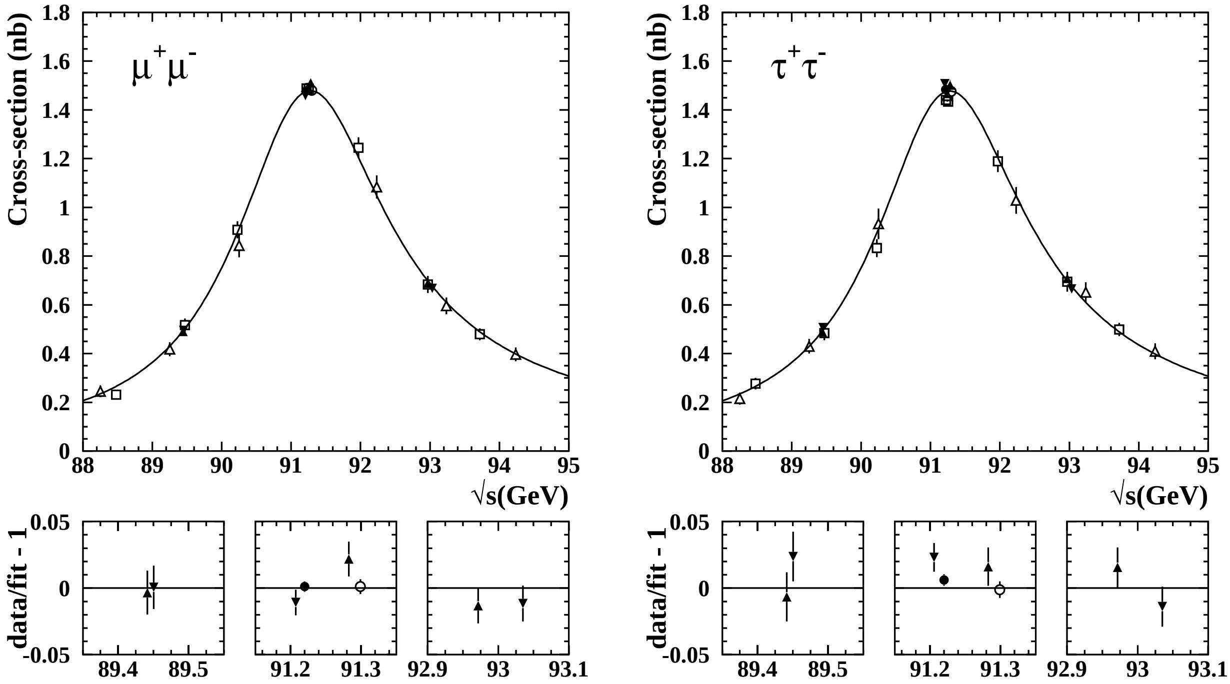
At the peak of the Z^0 resonance $\sqrt{s} = M_Z$

$$\sigma^0_{\mathrm{f\overline{f}}} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{\mathrm{f\overline{f}}}}{\Gamma_Z^2}$$

A fourth generation neutrino would INCREASE the Z^0 decay rate and thus increase Γ_Z. As a result one would observe a DECREASE the measured peak cross sections for the visible final states.

★ Measure the $e^+e^- \to Z^0 \to f\bar{f}$ cross-sections for all visible decay modes (*i.e.* all fermions apart from $\nu\bar{\nu}$)

EXAMPLES:



★ Have already measured M_Z and Γ_Z from the shape of the Breit-Wigner resonance. Therefore obtain $\Gamma_{f\bar{f}}$ from the peak cross-sections in each decay mode using

$$\sigma^0_{f\bar{f}} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

Note, obtain Γ_{ee} from

$$\sigma^0_{ee} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}^2}{\Gamma_Z^2}$$

★ Can relate the partial widths to the measured TOTAL width (from the resonance curve)

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{qq} + N_\nu \Gamma_{\nu\nu}$$

where N_ν is the number of neutrinos species and $\Gamma_{\nu\nu}$ is the partial width for a single neutrino species.

The difference between the measured value of Γ_Z and the sum of the partial widths for all visible final states gives the "invisible" width.

Γ_Z	2494.8 ± 4.1 MeV
Γ_{ee}	83.7 ± 0.2 MeV
$\Gamma_{\mu\mu}$	84.0 ± 0.3 MeV
$\Gamma_{\tau\tau}$	83.9 ± 0.4 MeV
$\Gamma_{q\bar{q}}$	1745.3 ± 3.5 MeV
$N_\nu \Gamma_{\nu\nu}$	497.3 ± 3.5 MeV

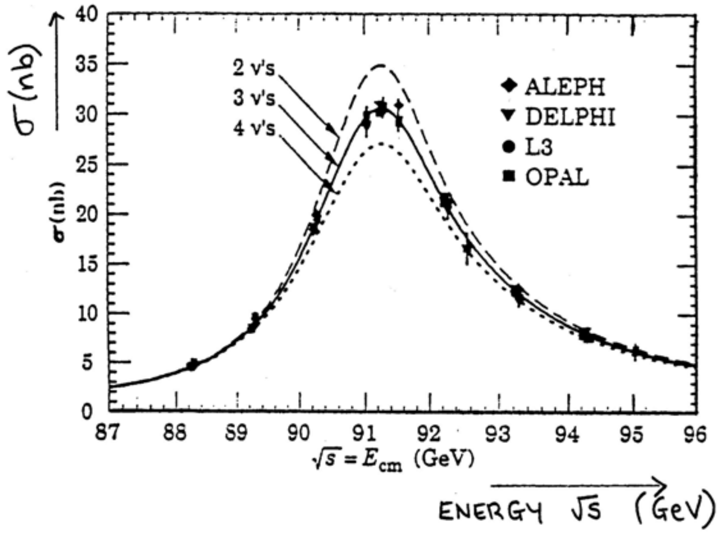
In the Standard Model calculate

$$\Gamma_{\nu\nu} = 167 \text{ MeV}$$

therefore
$$N_\nu = \frac{497.3 \pm 3.5}{167} = 2.98 \pm 0.02$$

3 generations of light neutrinos ($m_\nu < \frac{M_{Z^0}}{2}$)

$\Rightarrow$ Probably only 3 GENERATIONS !



In addition:

- ★ $\Gamma_{ee}, \Gamma_{\mu\mu}, \Gamma_{\tau\tau}$ are consistent $\Rightarrow$ universality of the lepton couplings to the Z^0
- ★ $\Gamma_{q\bar{q}}$ is consistent with the expected value which assumes 3 COLOURS - yet more evidence for colour

Parity Violation in Z^0 Decays

EXAMPLE: $e^+e^- \rightarrow \mu^+\mu^-$

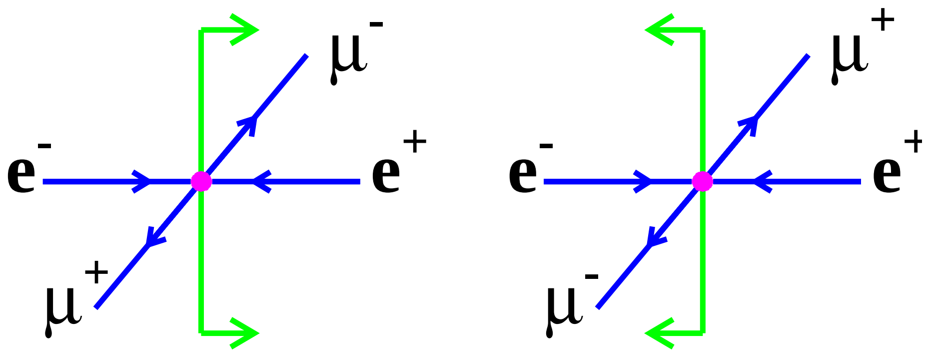
- ★ Parity is conserved in the strong and EM interactions
- ★ Parity is maximally violated in the WEAK charged current interaction.

  W-bosons mainly couple to LH particles

What about the WEAK neutral current ?

- ★ Parity IS violated in the WEAK neutral current
- ★ The Z^0 is a 'mixture' of a parity conserving VECTOR field and a parity violating 'W-like' field.

Perform a 'parity' violation experiment analogous to that of Handout VI page 13 :
FORWARD-BACKWARD asymmetry



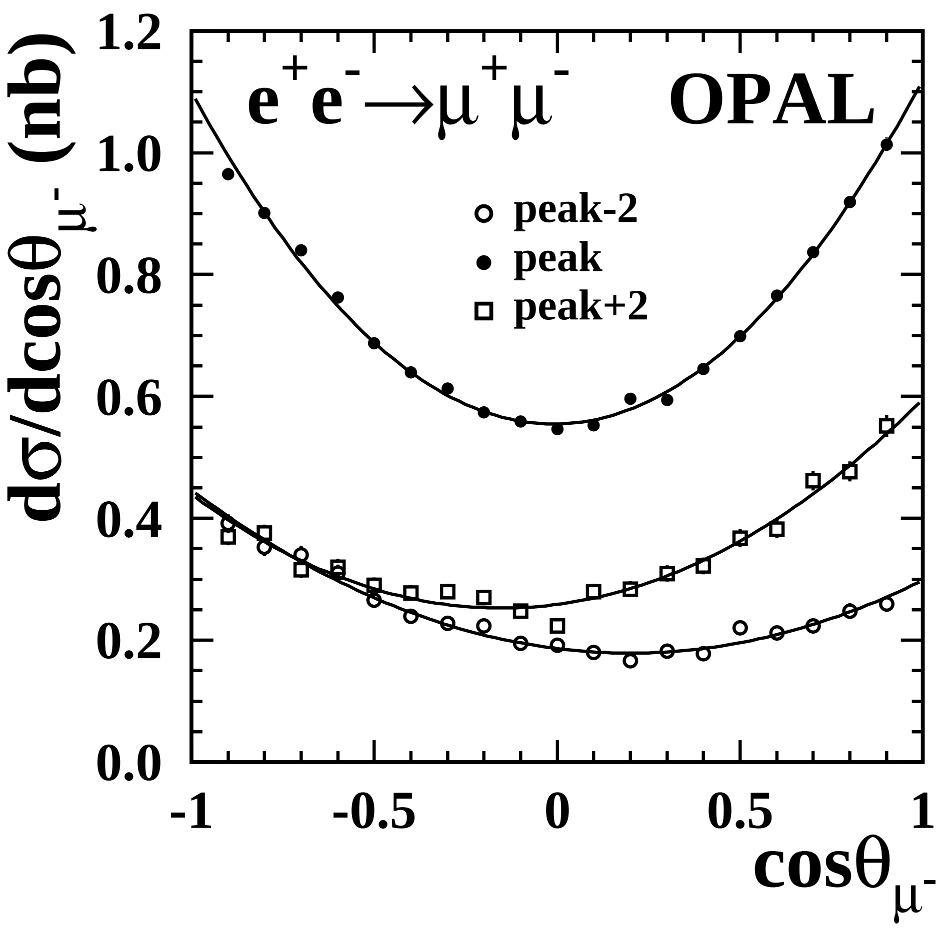
μ^- in FORWARD hemisphere

μ^- in BACKWARD hemisphere

If parity is conserved the number of μ^- observed in FORWARD hemisphere will be equal to number observed in BACKWARD hemisphere

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$
$$= 0 \quad \text{IF parity conserved}$$

OPAL $e^+e^- \to \mu^+\mu^-$ data



For data recorded at $\sqrt{s} = M_{Z^0}$:

$$A_{FB} = 0.0171 \pm 0.0010$$

i.e. a small but statistically significant non-zero asymmetry $\Rightarrow$ PARITY VIOLATED

EXPLANATION

$Z^0 f\bar{f}$ **coupling is a mixture of VECTOR and VECTOR—AXIAL-VECTOR couplings.**

$$\frac{g}{\cos\theta_W}\frac{1}{2}\gamma^\mu(C_V - C_A\gamma^5)$$

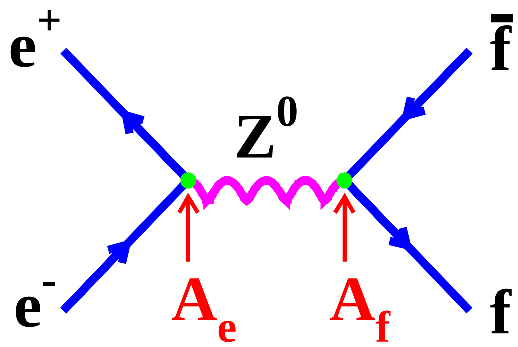
Mixture determined by WEAK MIXING ANGLE θ_W**. For leptons**

$$\begin{aligned} C_V &= (1 - 4\sin^2\theta_W) \\ C_A &= 1 \end{aligned}$$

The measured asymmetry:

$$A_{FB} = A_e A_\mu$$

where $$A_{e/\mu} = \frac{2C_V C_A}{C_V^2 + C_A^2}$$



Small asymmetry implies $(1 - 4\sin^2\theta_W) \sim 0.$

By measuring the asymmetry measure $\sin^2\theta_W$

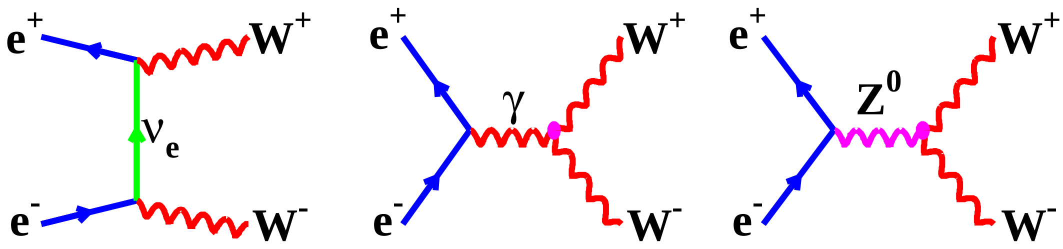
ALL LEP A_{FB}**:**

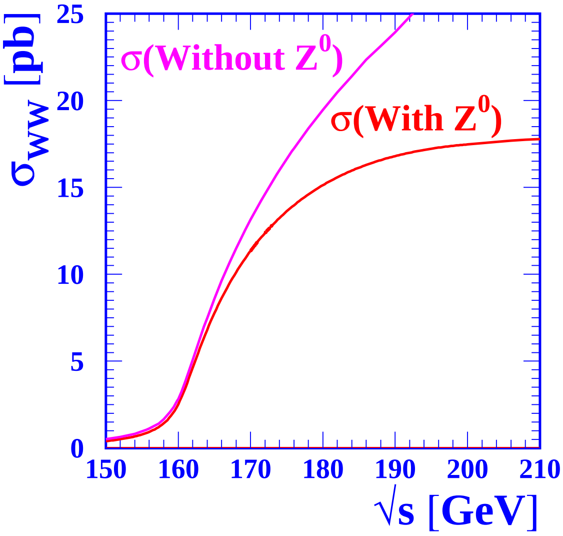
$$\sin^2\theta_W = 0.23099 \pm 0.0053$$

$$\boxed{\text{LEP} \longrightarrow M_{Z^0} \text{ and } \sin^2\theta_W}$$

$\mathrm{W^+W^-}$ at LEP

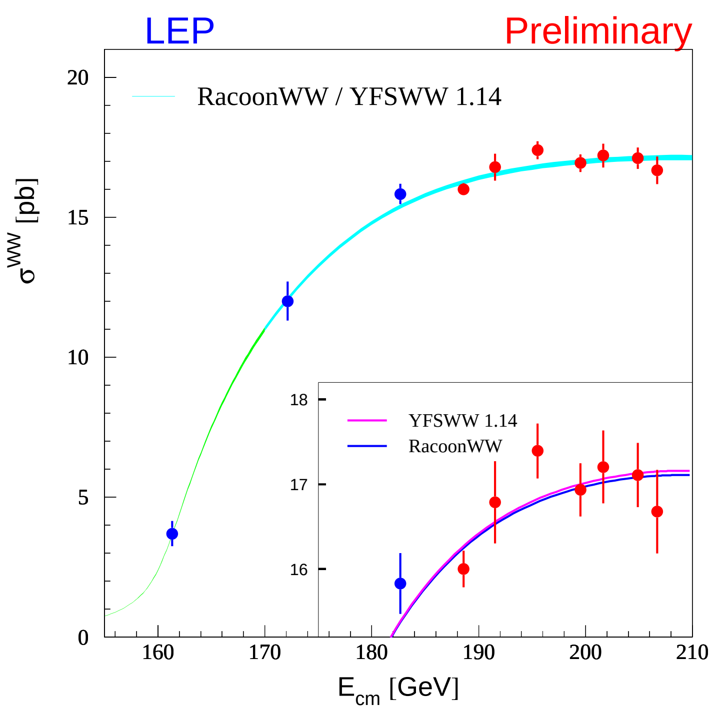
- ★ $\mathrm{e^+e^-}$ collisions Ws produced in pairs.
- ★ In Standard Model 3 possible diagrams for $\mathrm{e^+e^-} \rightarrow \mathrm{W^+W^-}$





Cross section sensitive to presence of the Triple Gauge Boson vertex $\mathrm{Z^0W^+W^-}$

<u>1996-2000</u>, LEP operated above the threshold for $\mathrm{W^+W^-}$ production $\sqrt{s} > 2M_\mathrm{W}$



Cross section agrees with Standard Model prediction. Confirmation of the existence of the $\mathrm{Z^0W^+W^-}$ vertex

W^+W^- Decay at LEP

In Standard Model: $W^\pm \ell\nu$ and $W^\pm q\bar{q}$ couplings are equal.

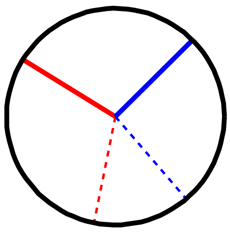
			3 TIMES		
e	μ	τ	u	c	~~t~~
ν_e	ν_μ	ν_τ	d	s	~~b~~

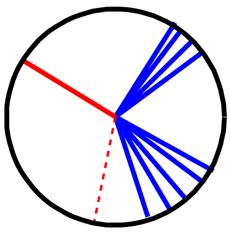
<u>EXPECT</u> (assuming 3 COLOURS)

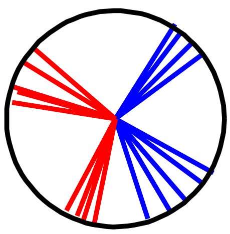
★ Br($W^\pm \rightarrow q\bar{q}$) = $\frac{2}{3}$

★ Br($W^\pm \rightarrow \ell\nu$) = $\frac{1}{3}$

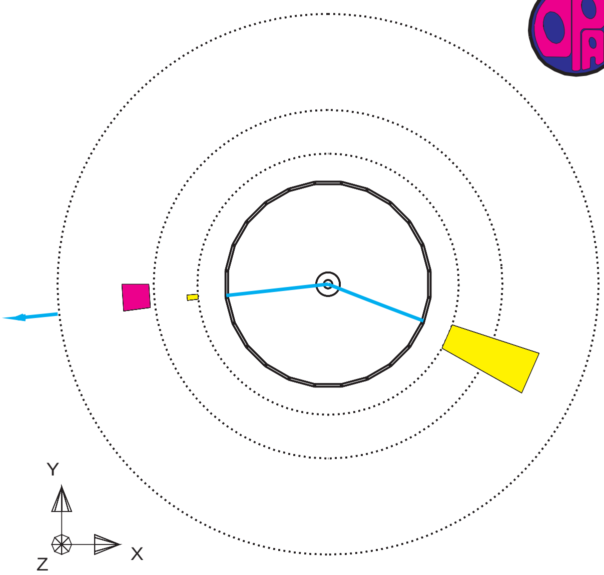
QCD corrections $\sim (1 + \alpha_s/\pi)$ ➡ Br($W^\pm \rightarrow q\bar{q}$) = 0.675

WW→lνlν  **10.5 %**

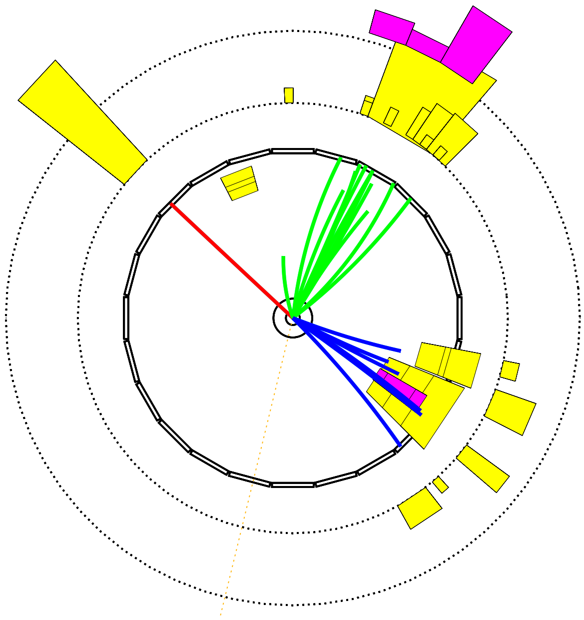
WW→qqlν  **43.9 %**

WW→qqqq  **45.6 %**

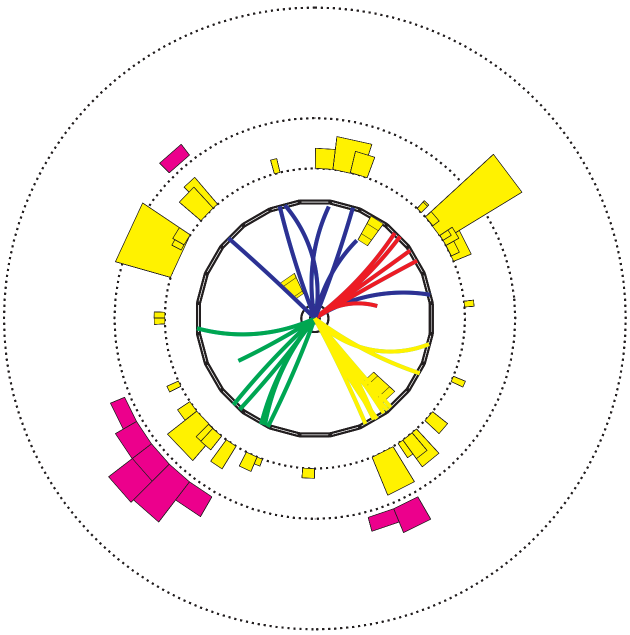
$\mathrm{W^+W^-}$ Events in OPAL



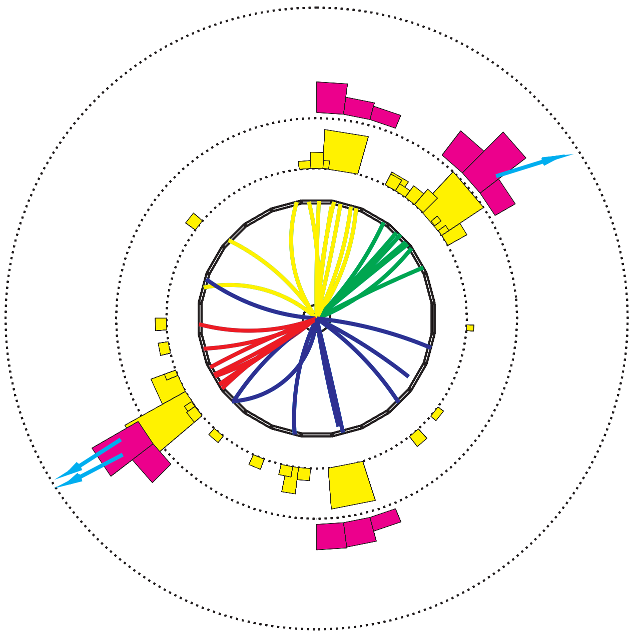
$$\mathrm{W^+W^-} \rightarrow \mathrm{e}\nu\mu\nu$$



$$\mathrm{W^+W^-} \rightarrow \mathrm{q\bar{q}}e\nu$$



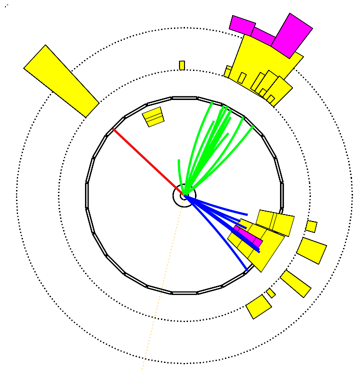
$$\mathrm{W^+W^-} \rightarrow \mathrm{q\bar{q}q\bar{q}}$$



$$\mathrm{W^+W^-} \rightarrow \mathrm{q\bar{q}q\bar{q}}$$

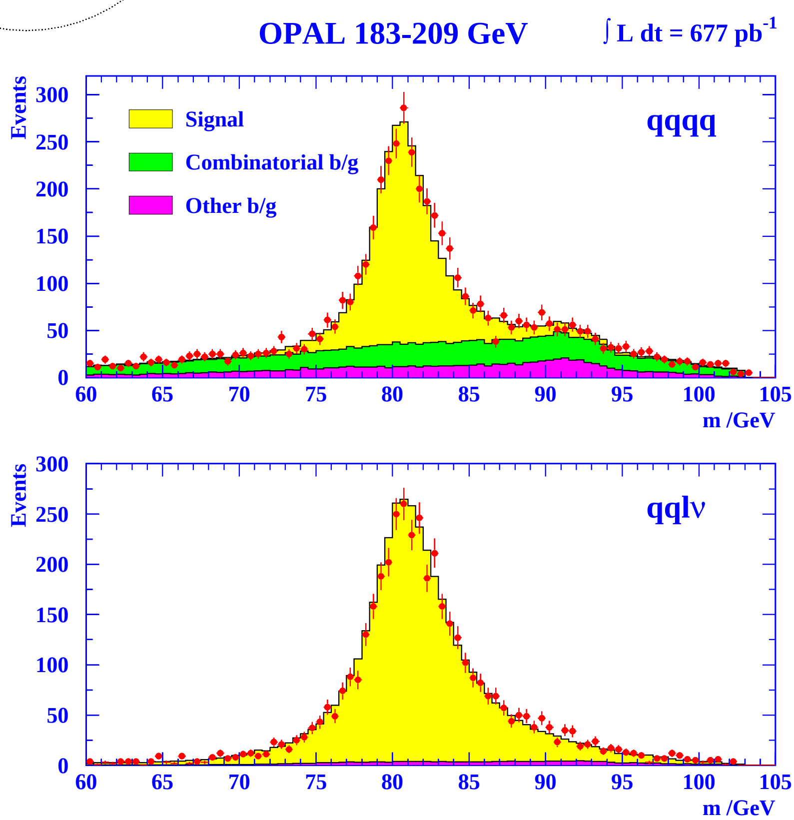
W-Boson Mass and Width

- ★ Unlike $e^+e^- \to Z^0$, W boson production at LEP is not a resonant process
- ★ M_W measured differently.
- ★ Reconstruct invariant mass distribution.
- ★ Use measured lepton/jet momenta and energies to estimate M_W on an event-by-event basis



➡ $\vec{p}_{q_1}, \vec{p}_{q_2}, \vec{p}_e, \vec{p}_\nu$

➡ $M_W = \frac{1}{2}(M_{q\bar{q}} + M_{ln})$



$$\Gamma_W = 2.12 \pm 0.11 \text{ GeV}$$

$$M_W = 80.423 \pm 0.038 \text{ GeV}$$

W-Boson Decay Width

In the Standard Model the W-boson decay width is given by:

$$\Gamma(W^- \to e^- \overline{\nu}_e) = \frac{g_w^2 M_W}{48\pi} = \frac{G_F {M_W}^3}{6\sqrt{2}\pi}$$

From μ-decay : $G_F = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$.

From LEP measure : $Mw = 80.423 \pm 0.038\,\mathrm{GeV}$.

Therefore predict partial width

$$\Rightarrow \quad \Gamma(W^- \to e^- \overline{\nu}_e) = 227\,\mathrm{MeV}$$

Total width is the sum over all partial widths:

$$\begin{aligned} W^- &\to e^- \overline{\nu}_e \\ W^- &\to \mu^- \overline{\nu}_\mu \\ W^- &\to \tau^- \overline{\nu}_\tau \\ W^- &\to d\bar{u} \\ W^- &\to s\bar{c} \end{aligned}$$

Consequently, IF the W-coupling to leptons and quarks is equal, and there are 3 colours

$$\Gamma = \sum_i \Gamma_i = (3 + 2 \times 3)\Gamma(W^- \to e^- \overline{\nu}_e) \approx 2.1\,\mathrm{GeV}$$

Compare with measured value (LEP) $2.1 \pm 0.1\,\mathrm{GeV}$

- ★ Universal coupling strength
- ★ Yet more evidence for colour !

(see Question 13 on the problem sheet)

Summary

Now have 5 precise measurements of fundamental parameters of the Standard Model

- ★ α_{em}
- ★ $G_{\mathrm{F}}=(1.16632\pm0.00002)\times10^{-5}\ \mathrm{GeV}^{-2}$
- ★ $M_{\mathrm{W}}=(80.423\pm0.038)\ \mathrm{GeV}$
- ★ $M_{\mathrm{Z^0}}=(91.1875\pm0.0021)\ \mathrm{GeV}$
- ★ $\sin^2\theta_W=0.23143\pm0.00015$

In the Standard Model, ONLY 3 are independent.

Their consistency is an incredibly powerful test of the Standard Model of Electroweak Interactions !

This (in)consistency is the subject of the first part of the last lecture (HANDOUT VIII)